Band Structure and Sound Transmission Loss of Infinite Periodic Partitions: Numerical Studies with COMSOL

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Introduction

The sound transmission loss of periodic structures has been studied by approximate analytical methods as well as by exact approaches for special configurations [1]. Meanwhile, the finite element method became a powerful tool for the analysis of periodic structures. Compared to analytical modeling the finite element method is less limited in terms of geometric configurations and material properties. By applying Bloch's theorem, the study of an infinite periodic structure can be confined to a unit cell. Furthermore, boundary effects of finite specimens are eliminated in the infinite model. This kind of analysis enhances physical insight and can finally be used as a guidance for desirable modifications of periodic structures.

This paper deals with two periodic structures of infinite extent: (i) a sinusoidally corrugated plate and (ii) a metamaterial-like double-leaf partition. It is shown that with COMSOL not only Bloch waves and band structures, but also the sound transmission loss of infinite periodic partitions can be calculated. The band structure is used for interpretation of the sound transmission loss curve.

Basics

Bloch waves

COMSOL is able to handle infinite structures with rectangular periodicity (rectangular unit cell and no offset between neighboring unit cells). According to Bloch's theorem, the displacement vector **u** of a periodic structure can be expressed as [2, Chapt. 7]

$$\mathbf{u}(\mathbf{r}, t) = \tilde{\mathbf{u}}(\mathbf{k}, \mathbf{r})\exp[\mathrm{i}(\mathbf{k} \cdot \mathbf{r} - \omega t)]. \quad (1)$$

This solution is referred to as a Bloch wave. The above mentioned periodic structures possesses two-dimensional periodicity with periods of L_x and L_y entailing a Bloch wave vector $\mathbf{k} = (k_x, k_y, 0)$ with zero z-component. $\mathbf{r} = (x, y, z)$ denotes the position vector and $\tilde{\mathbf{u}}(\mathbf{k}, \mathbf{r})$ is a periodic function, which satisfies $\tilde{\mathbf{u}}(\mathbf{k}, \mathbf{r}) = \tilde{\mathbf{u}}(\mathbf{k}, \mathbf{r} + \mathbf{L})$ with arbitrary lattice vector **L**. Generally, a Bloch wave is not spatially periodic. In the limit of a homogeneous plate, the periodic function $\tilde{\mathbf{u}}(\mathbf{k}, \mathbf{r})$ is constant, and thus the Bloch waves becomes a plane waves.

Using Eq. (1) the displacements at parallel boundaries of the unit cell can be related to each other. They only differ in their phases:

$$\mathbf{u}(x_0 + L_x, y, z) = \mathbf{u}(x_0, y, z)\exp[\mathrm{i}k_x L_x],$$
$$\mathbf{u}(x, y_0 + L_y, z) = \mathbf{u}(x, y_0, z)\exp[\mathrm{i}k_y L_y]. \quad (2)$$

These boundary conditions are called "Floquet Periodicity" in COMSOL.

Band structure

The frequencies ω of Bloch waves with prescribed wave vector **k** can be numerically computed in COMSOL using eigenvalue analysis. Varying the wave vector results in dispersion relations ω(**k**), which form the band structure. Setting all components of the wave vector to zero generates standing waves. The dispersion relations are symmetric with respect to **k** = 0 [3, Sec. 5], i.e. $\omega(\mathbf{k}) = \omega(-\mathbf{k})$. Thus it is sufficient to compute and visualize the dispersion relations only within the positive half of the first Brillouin Zone [3, Sec. 27]. In case of a 2D rectangular periodic cell, the reduced Brillouin zone is defined as

$$0 \le k_x \le \frac{\pi}{L_x},\ 0 \le k_y \le \frac{\pi}{L_y}. \quad (3)$$

Transmission loss

The Bloch-wave description can also be applied to the sound transmission problem of an infinite 2D periodic partition between two homogeneous fluid half-spaces, since parallel to the partition the half-spaces trivially possess the same periodicity. With the additional items "Background Pressure Field" and "Perfectly Matched Layer (PML)" for the infinite extension perpendicular to the partition a COMSOL model for the sound transmission under plane wave excitation is readily obtained by extending the unit cell into the fluid half-spaces.

In this way, the computational load for the prediction of the sound transmission loss of periodic structures is comparable to that for duct calculations with the same cross-sectional area as the unit cell. The discretized model follows the rule of six elements per minimum wavelength at maximum frequency. The structure and its vicinity is segmented by finer elements because of near field effects.

Acoustic trace wave

The wavenumber of the acoustic wave, which excites the periodic structure, is given by $k = \omega/c_0$ with the speed of sound c_0. The spatial dependence of the excitation of the structure is determined by the trace wave, the wavenumber of which, k_{trace}, is related to k by the sine of the polar angle θ of the incident wave. Therefore, the dispersion relation of the acoustic trace wave follows as

$$\omega = \frac{c_0 k_{\mathrm{trace.}}}{\sin\theta}. \quad (4)$$

Adding this dispersion relation to the band structure diagram (with appropriate azimuthal angle ϕ) results in intersection points with Bloch-wave branches. At these intersections the x- and y-components of the wave vectors and the frequencies are the same for both the trace wave and the Bloch wave. This is called coincidence. It is well known that in the case of bending waves in homogeneous plates this coincidence has a strong effect on the transmission loss: the acoustic energy is efficiently transmitted from the sending side to the receiving side. But not all coincidences of frequency and wave vector have such an effect on the transmission. In particular, if a Bloch wave shows no displacement perpendicular to the surface of the partition, it is not excited by the airborne sound wave. Thus there is coincidence, but no coincidence effect. In general, the efficiency of Bloch-wave excitation has to be studied for each intersection point.

Singly curved shell

Physical setting

The middle-surface of the sinusoidally corrugated shell is expressed by

$$z_{\sin} = Z_0 \sin\left(\frac{2\pi x}{L_0}\right), \qquad (5)$$

where L_0 is the period, and Z_0 is the half-amplitude of the corrugation. The x-axis runs across the ridges and troughs, while the y-axis runs along them. In this study, L_0 = 76 mm, Z_0 = 9 mm. It is assumed that the thickness of the panel is everywhere the same: 1 mm. The material properties of stainless steel are used in the calculation.

As the shape of the sinusoidal shell does not change with the y-coordinate, the dimension L_y of the unit cell along the homogeneous direction is not uniquely determined. Theoretically, the dimension of the unit cell along the homogeneous axis is not relevant for the analysis, because a homogeneous structure can be considered as a periodic structure with an arbitrary period. In this paper, L_y is set to be a quarter of L_0. A smaller L_y has the advantage of smaller computational load. As the thickness is much smaller than the other dimensions, this structure is modelled by shell elements.

Results

The Sound Transmission Loss (STL) of the infinite sinusoidal shell under a plane wave excitation with $\theta = 60°$ and $\phi = 30°$ is shown in Fig. 1 by a black solid line. As a reference, the mass law (blue solid line) and the STL of the infinite homogeneous panel with equivalent thickness and equivalent anisotropic stiffness (red dotted line) are plotted. The equivalent stiffness of the corrugated panel is analytically determined according to [5].

Figure 2 illustrates the band structure of this shell (blue curves) for

$$\mathbf{k} = (k_x, \quad k_y, \quad 0) = p\left(\frac{\pi}{L_0}\cos\phi\,, \quad \frac{\pi}{L_0}\sin\phi\,, \quad 0\right) \qquad (6)$$

where p is a non-dimensional variable. While the wavenumber is low, the three lower branches, which start from the origin, correspond to flexural waves (the lowest branch with the smallest gradient), extensional waves (the middle branch), and shear waves (the branch with the largest gradient) waves, respectively. As the wavenumber increases, all three wave types get mixed.

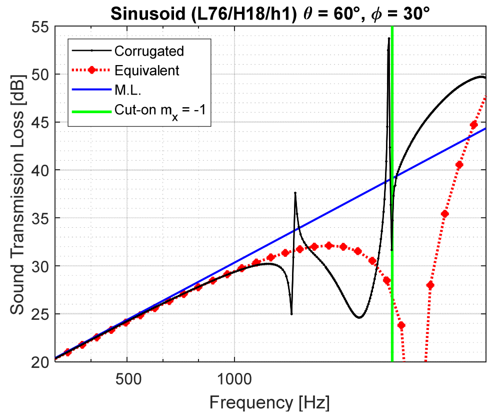


Figure 1: STL of the sinusoidally corrugated shell (black), of the equivalent homogeneous anisotropic panel (red dotted), and mass law (blue). The lowest cut-on frequency of diffracted waves is marked by the vertical green line.

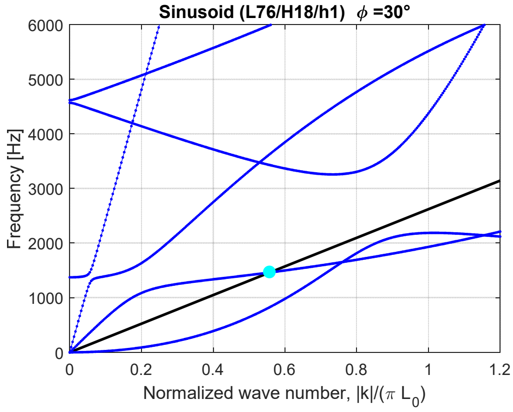


Figure 2: Band structure of the sinusoidally corrugated shell (blue) and dispersion of the acoustic trace wave (black).

Figure 2 also contains the dispersion relation of the acoustic trace wave given in Eq. (4). It intersects with a dispersion curve of a Bloch wave at 1.46 kHz. In Fig. 1, the first sharp STL dip followed by a sharp peak is observed at around 1.48 kHz, which is very close to the intersection frequency. The minor frequency difference is due to fluid loading, which is not accounted for in the band structure, since it is calculated for the shell in vacuum. The second STL dip at around 2.23 kHz is rather gentle and wide. There is no corresponding intersection point in Fig. 2, but the trace-wave dispersion approaches a Bloch branch. They are closest around 2.03 kHz. Obviously, that Bloch wave is efficiently excited, although there is only an 'approximate coincidence'.

Not all the dips of the transmission loss curve can be readily explained by the band structure. The third dip in Fig. 1 appears at 2.74 kHz, where none of the branches intersect or approach the acoustic trace line. The transmitted sound pressure field at 2.75 kHz in Fig. 3 indicates that in addition to the direct transmitted wave a diffracted wave began to propagate. The green vertical line in Fig. 1 denotes the cut-on frequency of the lowest diffracted wave, below which there is only direct transmission [1]. The cut-on frequency of any diffracted wave is fully determined by the periodicity and the incident wave. In case of the incident wave with $\theta = 60°$ and $\phi = 30°$ the lowest cut-on frequency of the sinusoidal shell with L_0 = 76 mm is found at 2.75 kHz, which is very close to the third dip frequency. However, this fact is not sufficient to understand why the acoustic energy could be efficiently transmitted by the diffracted wave near the cut-on frequency. Further analysis is required here.

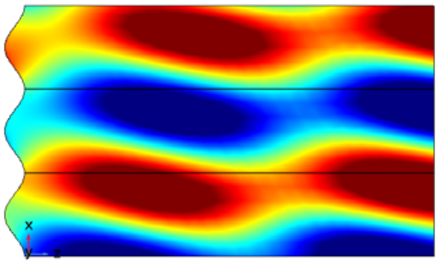


Figure 3: Transmitted sound pressure field at 2.75 kHz.

Problem with PML

A Perfectly Matched Layer is an artificial absorbing medium that is commonly used to truncate the discretization region to finite size [4]. With the occurrence of propagating diffracted waves a problem with the PML was encountered. At cut-on the diffracted waves leave the partition at grazing angle [1]. As the attenuation rate is proportional to $|\mathbf{k}_r|\cos\theta_r$, where k_r and θ_r are respectively the wavenumber and polar angle of the radiating wave [4], the PML encounters a problem near the cut-on frequency. Figure 4 shows the STL of the sinusoidal shell around $f_{\mathrm{cut-on}}$ for different PML settings. When the default settings of COMSOL are applied (black), another pair of a peak and a dip appears. By more appropriate PML settings, a smooth STL curve (blue) could be obtained.

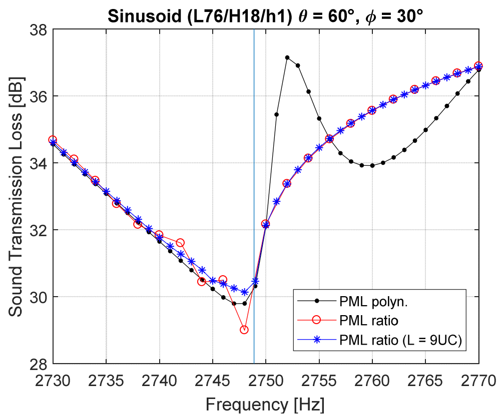


Figure 4: STL of the sinusoidally corrugated shell around the lowest cut-on frequency of diffracted waves for different PML settings: default (black), rational function (red), and thicker layer (blue).

Metamaterial-like double-leaf partition

Physical setting

Metamaterials are artificial, engineered structures having properties that are not found in nature. One metamaterial type has repetitive patterns, whose scale is smaller than the wavelength of the resultant peculiar (but useful, of course) properties. Figure 5 illustrates a partition made out of two metamaterial leaves with different thicknesses of 40 mm (sending side) and 30 mm (receiving side), separated by an air layer of 50 mm. Each leaf has a lattice structure with a 400 mm wide hard frame (particleboard) and 500 mm square softer panels (polyurethane foam board) in between.

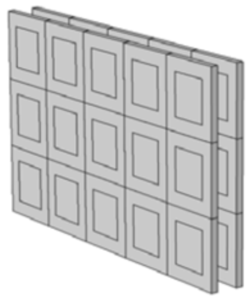


Figure 5: Metamaterial-like double-leaf partition.

Results

The STL of the metamaterial double-leaf partition was computed under a plane wave excitation with $\theta = 60°$ and $\phi = 23°$ (red line in Fig. 6). For comparison, the STL of a homogeneous double-leaf partition having averaged material properties (green line) and the mass law (blue line) are plotted.

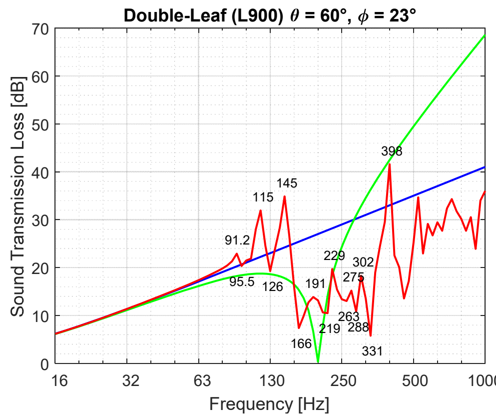


Figure 6: STL of the double-leaf partition with metamaterial like plates (red), with homogeneous plates (green), and mass law (blue).

The STL of the metamaterial partition has numerous peaks and dips. The homogeneous double-leaf partition shows a large dip due to the mass-air-mass resonance around 200 Hz. This is partially avoided by the metamaterial double-leaf partition. However, the higher loss at high frequencies, which is the primary benefit of the homogeneous double-leaf partition, is lost. At the peaks, the frame and the panels vibrate antiphase, and thus the radiation from the frame is largely cancelled by that from the panel. Actually, on average the volume swept by the frame and the panel takes minima at the peaks.

Similar to the case of the sinusoidal shell, the STL dips in Fig. 6 can be explained by consulting the band structure of the metamaterial double-leaf partition for $\phi = 23°$ (Fig. 7). The intersections of the Bloch-wave branches with the trace-wave dispersion, highlighted with circles, which happen to be at 93 Hz, 123 Hz and 170 Hz, correspond to the first three STL dips at 96 Hz, 126 Hz and 166 Hz, respectively.

To confirm the above dip explanation, the partition vibration forced by the acoustic wave at 166 Hz is compared with the corresponding Bloch wave in Fig. 8. The two pictures are alike. The two corresponding panels of the left leaves (facing the sending room) and right leaves (facing the receiving room) vibrate in-phase. Both left and right frames vibrate in a bending mode, although their amplitudes are much smaller than those of the panels. The anti-nodal patterns are different between the left and right frames. However, those of the left and right frames forced by the acoustic wave are virtually the same as those of the Bloch wave.

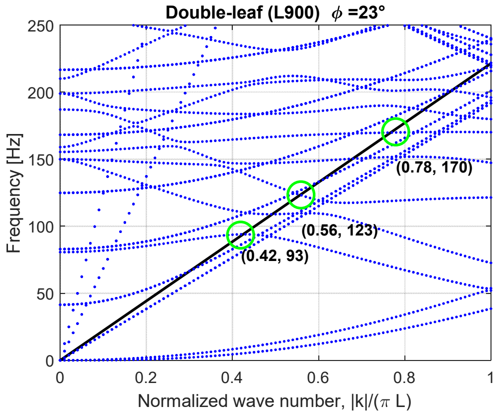
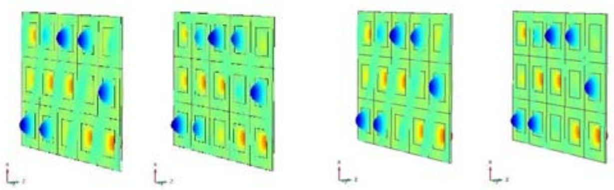


Figure 7: Band structure of the double-leaf structure (dotted lines) and dispersion of the trace wave (solid line) for azimuthal angle ϕ = 23°.

Not all intersections in Fig. 7 cause a minimum in the transmission loss. This is primarily due to the type of the Bloch wave at an intersection point. Obviously, shear and extensional motions do not radiate sound. Therefore, intersections involving Bloch waves dominated by extensional or shear motion do not cause noticeable transmission effects. But if the Bloch wave is mainly of bending type with displacements perpendicular to the partition, the acoustic wave can efficiently excite the structure.



(a) Forced vibration *(b) Bloch wave*

Figure 8: Comparison between the partition vibration at 166 Hz forced by an acoustic trace wave *(a)* and the Bloch wave having similar frequency (170 Hz) and wavelength *(b)*. For better visibility, the two leaves are intentionally drawn further apart.

Conclusion

For two periodic structures of infinite extent, a sinusoidally corrugated shell and a metamaterial-like double-leaf partition, the band structure and the sound transmission loss was computed numerically using the finite element method. Thanks to Bloch's theorem and perfectly matched layers the discretization can be confined to a unit cell of the periodic structure including fluid-filled regions on the sending and receiving sides. The COMSOL software is able to handle infinite structures with rectangular periodicity. In order to obtain band-structure diagrams, which visualize the dispersion ω(**k**) of Bloch waves, the wave vector **k** is prescribed for the numerical computation of the frequency ω via eigenvalue analysis. Varying the wave vector results in dispersion curves, which form the band structure.

Understanding the transmission loss curves is - at least for light adjacent fluids - facilitated by entering the dispersion of the trace wave of an incident plane wave in the band structure of the partition in vacuum. If – at the same frequency – the Bloch wave length agrees with the trace wave length of the incident sound, i.e. if the Bloch wave dispersion curve intersects the straight dispersion line of the trace wave, one speaks of coincidence. Like with homogeneous partitions a 'coincidence effect' on the transmission loss is observed, but only if the Bloch wave is efficiently excited by the incident wave and radiates efficiently on the receiving side.

Which Bloch waves are excited most by an incident wave at a particular frequency can be inferred from a comparison of the forced motion of the partition with the shapes of Bloch waves with appropriate wave vector. This works also away from coincidences.

In this manner the band structure can be used for a deeper understanding of the transmission loss characteristics and, moreover, also for optimization: Such a straightforward analysis may serve as a guidance for arriving at a modified periodic structure with desired acoustic properties and – at the same time - may help avoiding unnecessary experiments or computations.

However, not every intersection leads to strong transmission. Besides, coincidences cannot explain all transmission loss minima. There are other causes, such as the onset of propagation of diffracted waves. Enhanced exploitation of the band structure information calls for additional fundamental studies.

Acknowledgement

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