Energy FEM and BEM for High Frequency Acoustics

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Introduction

The solution of an acoustical problem in the high frequency range requires a very fine discretization and leads to systems that are not efficiently computable by conventional methods, as the Finite Element Method (FEM) or the Boundary Element Method (BEM). The oscillating sound pressure has to be modeled by 6-10 elements per wavelength. An enhancement of the conventional BEM towards higher frequencies can be achieved by the use of H-matrices or the Fast Multipole Method [1], but also these methods have a high frequency limit. An alternative to the use of the oscillating sound pressure is the use of the energy density that depends only on the RMS value of the sound pressure. The distribution is much smoother and less frequency dependent. This replacement and some high frequency assumptions lead to very the efficient Energy FEM (EFEM) [2] and Energy BEM (EBEM) [3, 4, 5]. An advantage of these methods is the reuse of the conventional discretizations. A mesh is generated for FEM or BEM in the low frequency range and the same mesh is also valid for the high frequency range, where the EFEM/ EBEM is applied. This is a main difference to other high frequency methods, as for instance the Statistical Energy Analysis (SEA). The EFEM is applicable to structures as well as to fluids and the EBEM has its advantages in infinite domains.

The assumptions for the energy methods are related to the high frequency range. The main assumptions are:

- sources are incoherent,
- a diffuse field is present,
- the kinetic energy is equal to the potential energy,
- a plane wave approximation with no nearfield is used and
- an averaging in time and space is applied.

Energy Boundary Element Method

The governing relations of the EBEM are described in detail by [4, 5] and an application is presented by [6]. The derivation is based on the relationship between the power density p and the intensity I

$$\nabla \cdot \mathbf{I} + p_{diss} = p_{in} \quad . \tag{1}$$

If the dissipation inside the domain is neglected and an indirect BEM is applied with the new source strength σ, the equations for the energy density e and the intensity I read

$$e(\mathbf{M}) = \int_{\partial\Omega} \sigma(\mathbf{P}) G(\mathbf{P}, \mathbf{M}) \mathrm{d}\mathbf{P}, \tag{2}$$

$$\mathbf{I}(\mathbf{M}) = \int_{\partial\Omega} \sigma(\mathbf{P}) \mathbf{H}(\mathbf{P}, \mathbf{M}) \mathrm{d}\mathbf{P} \quad , \tag{3}$$

where $\mathbf{M}$ is the evaluation point inside the domain and $\mathbf{P}$ is on the surface. The corresponding fundamental solutions in terms of the speed of sound c_g are

$$G(\mathbf{S}, \mathbf{M}) = \frac{1}{4\pi c_g} \frac{1}{r^2} \quad , \tag{4}$$

$$\mathbf{H}(\mathbf{S}, \mathbf{M}) = \frac{\mathbf{u}_{\mathrm{SM}}}{4\pi} \frac{1}{r^2} \quad . \tag{5}$$

To achieve the final boundary integral equation, the intensity is integrated over the surface and has to be equal to the external power of the domain. Subsequently, a collocation method is used to achieve a system of equations for the unknown source strength σ

$$\underline{K} \sigma = \mathbf{P}_{\mathrm{ext}} \tag{6}$$

with

$$\underline{K}_{ij} = \int_{S_i} \int_{S_j} f(\mathbf{u}_{\mathrm{P_i P_j}}, \mathbf{n}_{\mathrm{P_j}}) \mathbf{H}(\mathbf{P}_{\mathrm{j}}, \mathbf{P}_{\mathrm{i}}) \cdot \mathbf{n}_{\mathrm{P_i}} \mathrm{d}\mathbf{S}_{\mathbf{j}} \mathrm{d}\mathbf{S}_{\mathbf{i}} \ . \tag{7}$$

In figure 1 the solution for a radiating sphere is presented for two different frequencies [6]. The EBEM result is compared to the conventional BEM result and to the analytical solution. In the nearfield the energy solutions differs, but achieves a good matching for the far field. Especially for the high frequency case of $f = 3000$ Hz a discretization of 384 elements is sufficient to approximate the solution. For the conventional BEM this number of elements is not sufficient. If the number of elements is increased in accordance to the rule of thumb, which leads to 38000 elements, the correct solution can be reproduced. For the BEM, an increase by a factor of 100 in the elements leads to computation complexity factor of 10000. Besides the advantages in computation efficiency, the correct decay of the SPL shall be remarked.

Energy Finite Element Method

The EFEM is based on the high frequency energy assumptions, as in the EBEM, and the underlying equations are mostly the same. The main difference is in the discretization of the equation and the corresponding

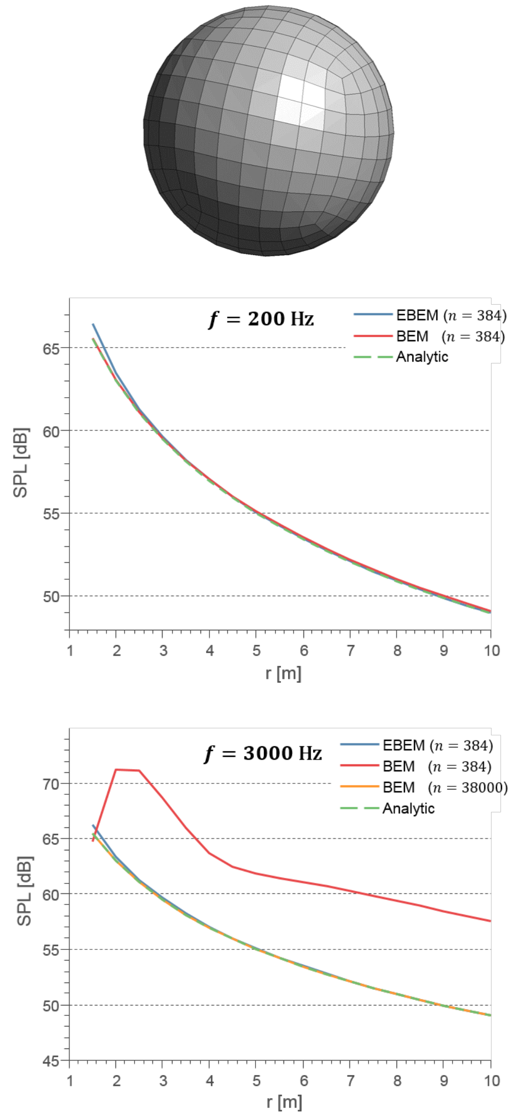

Figure 1: Radiating sphere (R=1m): Discretization with 384 elements, EBEM solution for the frequencies $f = 200$ Hz and $f = 3000$ Hz in comparison to analytical and conventional BEM results [6]

propagation of the waves in terms of the intensity description. The starting point is the energy conservation law

$$p_{in} = \nabla q + p_{diss}, \tag{8}$$

where the inserted power density p_{in} is equal to the energy fluxes q and the dissipated power density p_{diss}. The dissipated power can be time averaged and set to

$$p_{diss} = \omega\eta e \tag{9}$$

with the energy density e. This is the sum of the kinetic and potential energy due to a time averaging over one period. Additionally, the damping loss factor η is introduced. The diffuse field assumption leads to plane waves that can be represented as

$$\nabla q = \frac{c_g^2}{\eta\omega}\Delta e. \tag{10}$$

In this formula the group speed of the wave types c_g is used. For a structural simulation, this corresponds to the different wave types, e.g. longitudinal, shear or flexural waves. Combining the previous relations leads to an equation that is analog to the heat equation (Laplace)

$$p_{in} = \frac{c_g^2}{\eta\omega}\Delta e + \omega\eta e\ . \tag{11}$$

This equation can be discretized by the FEM approach. The corresponding element interaction matrices are set up for each wave type with its corresponding wave speed. An exchange of energy between the different wave types is possible at the joints. These coupling elements have to be introduced, e.g. if the material or the geometry changes. A description is given in [7]. For the case of a fluid to structure interaction, an area coupling is required and additionally, the radiation efficiency needs to be known.

Numerical Examples

The methods are compared in terms of an engineering application. The sound field within a generic car cabin shall be computed. The mesh is based on an element length of $l = 24$ mm and shown in figure 2. The energy methods are compared to a conventional BEM, which is valid up to 2000 Hz under the assumption of 6-10 elements per wavelength.

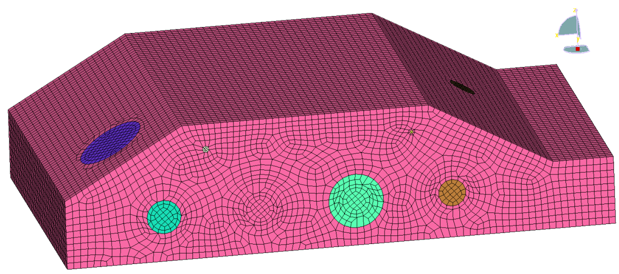

Figure 2: Mesh of a generic car cabin with an element length of $l = 24$ mm

For high frequency acoustics the mid speakers and tweeters are the interesting loudspeakers. Therefore, diameters that correspond to these types are chosen and placed in the car cabin with following numbers

- center: 1. mid,
- front door: 2. tweeter,
- front door: 3. mid,
- rear door: 5. tweeter.

For these speakers the so called Thiele-Small parameters are available that transform an electrical power to a corresponding membrane velocity [8]. This corresponding membrane velocity is used to scale the transfer function from membrane velocity to sound pressure at the hearing position. In figure 3 the almost frequency independent solution of the energy method is shown for the different loudspeakers. The frequency dependency of the EFEM is based on the low damping inside the fluid. The EBEM is completely frequency independent, since no damping is applied in the current case.

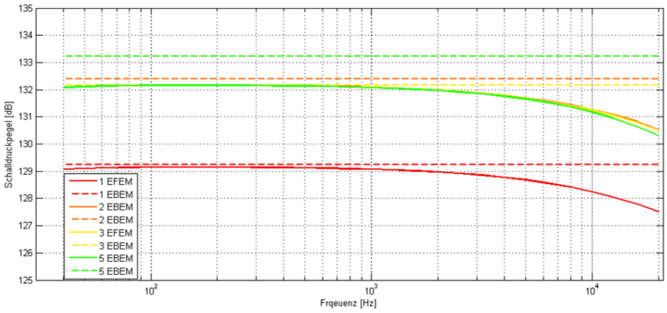

Figure 3: SPL, EBEM and EFEM in the complete frequency range for a unit power input 1W

One crucial point for the energy method is the input power. Since no phase relation is present and kinetic energy is equal to the potential energy, an approximation of the power is required. This corresponds to the term radiation efficiency, which is the ratio from real acoustic power to the free field acoustic power based on the squared velocity. For the energy solutions an approximation based on the Rayleigh integral is used to compute the input power. For comparison, the full 3D solution from the BEM is used. Here the radiation efficiency is computed exactly by the computed sound pressure based on the prescribed velocity. In figure 4 and figure 5, the curves for the loudspeakers 1 and 5 are depicted.

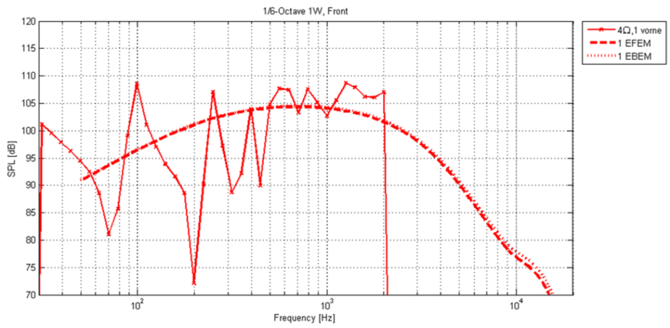

Figure 4: SPL, energy solutions in comparison to the conventional BEM for mid loudspeaker 1

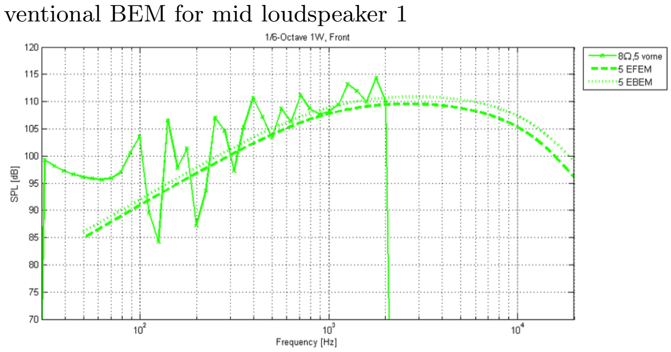

Figure 5: SPL, energy solutions in comparison to the conventional BEM for tweeter loudspeaker 5

The results show the good estimation of the exact solution by the energy approximation. In the low frequency range the modal behavior with its large peaks is visible, which shows less variation as the frequency increases.

The energy solution of both formulations are comparable and for this case the different propagation assumptions tend to have no significant influence. With this procedure a solution in the complete frequency range is possible at a low computational effort. The current example shows the good behavior for fluid domains.

The next step is the computation of the structure, which can be coupled to the fluid domain. For the structural analysis the EFEM is more suitable, since it can model inhomogeneous objects very good. For instance, the different parts in a car, a ship or an airplane can be modeled by the EFEM. Nowadays, at least low frequency discretizations are available in most development divisions. This model can just be reused to achieve a solution, also for the high frequency range. The joint elements are automatically introduced. A solution for an airplane compartment is used for the verification. This model can still be computed by the conventional FEM for comparison. In figure 6 the mesh, the excitation and both solutions are visualized. The figure shows the good, qualitative approximations of the FEM solution.

Conclusion

In this contribution the two high frequency methods Energy BEM and Energy FEM are presented. The assumptions are stated and the formulations are briefly given. The advantages are the low computations times and the simple reuse of the low frequency discretizations. The numerical examples show the correct behavior the methods. The correct propagation into an infinite domain is shown for the EBEM case of a radiating sphere. Both methods show a very cood correlation to each other and to the conventional BEM for the engineering problem of a sound field inside a car cabin. In addition to the fluid domain results, the EFEM shows a good approximation in a structural analysis of an airplane compartment. The crucial points are the input power and the radiation efficiency, where either measurements are required, good approximations have to be made or analytical submodels have to be introduced.

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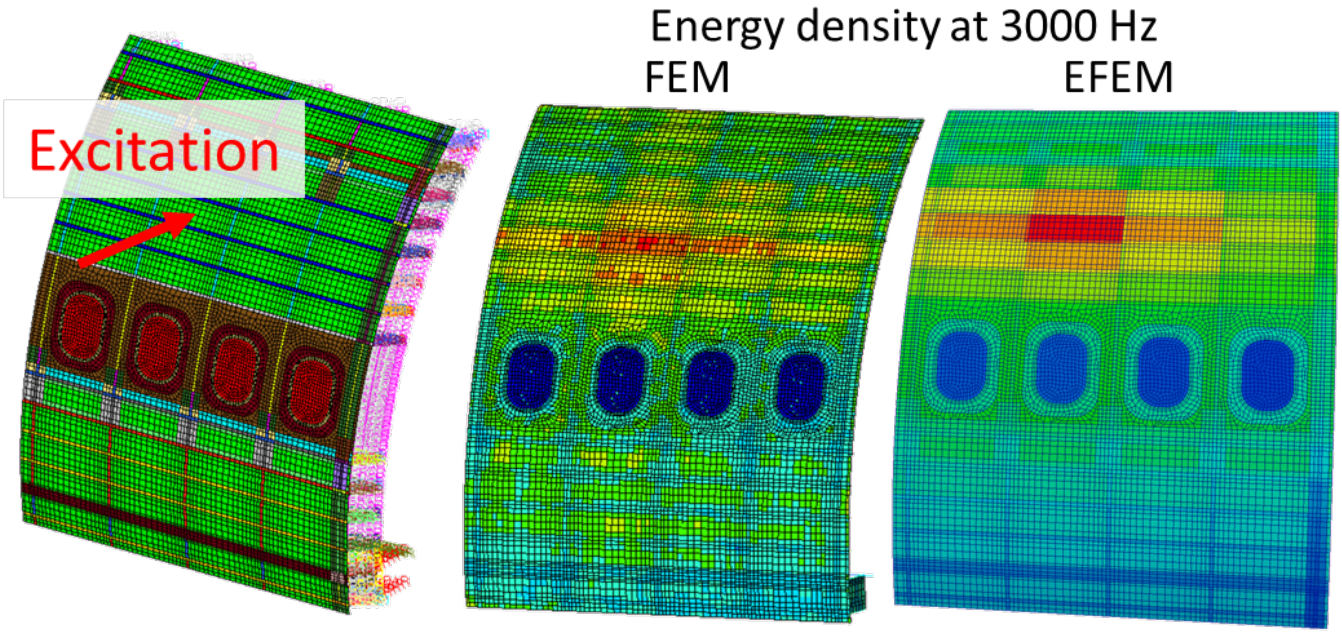

Figure 6: FlightLab: Structural anaylsis of an airplane, FEM-EFEM comparison

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