

# Measurement of Rotational Degrees of Freedom in Vibroacoustic Transfer Functions

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## Introduction

Auralisation requires the determination of three different transfer function types: airborne, structure-borne and the vibroacoustic transfer functions which describe the structure-fluid interactions. In a vehicle these transfer functions consist of multiple transfer paths within the structure and the fluid (fig.1). For measuring these transfer paths, direct or reciprocal measurement setups usually contain impact hammers, shakers or volume velocity sources on the excitation site and triaxial accelerometers or microphone placement in the interior on the receiver site.

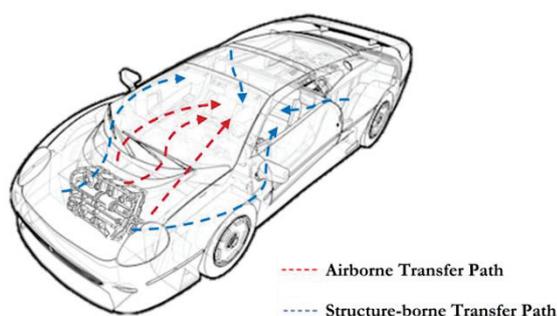


Fig. 1: Different types of transfer paths in a vehicle.

Problematically, these measurements are restricted to three translational degrees of freedom, whereas the motion of a rigid body would be fully described with six degrees of freedom. Thus, the influence of rotations on a resulting sound field cannot be considered. In order to deal with uncertainties due to the neglected rotational degrees of freedom in vibroacoustic measurement setups, a reciprocal measuring concept using a MEMS gyroscope was developed. In a case study, this method was used to determine the relevance of rotational degrees of freedom to the airborne sound radiation in vibroacoustic transfer functions of an electric vehicle drivetrain. Auralisation can be used to predict the sound field at a receiver point based on given excitation signals and the according transfer functions. Practically, this is used to simulate vehicle interior noise based on drivetrain test bench data or within building acoustics based on given sound reduction indexes of walls [1].

## 6 DoF Source Characterisation

The motion of a solid is locally described in three translational (TDOF) and three rotational degrees of freedom (RDOF) and yields a 6N-by-6N matrix for transfer function

measurements at N contact points. In order to characterise the movement with measurable quantities, the mechanical admittance (or also called *mobility*) is used. The mobility matrix  $\mathbf{Y}$  is defined as

$$\mathbf{Y} = \frac{\mathbf{v}(\omega)}{\mathbf{F}(\omega)} \quad (1)$$

where  $\mathbf{v}(\omega)$  is the local frequency-dependent velocity response of a solid to a frequency-dependent force excitation  $\mathbf{F}(\omega)$ . This resulting mobility matrix can be schematically subdivided by cross-couplings between all six DoF [2]:

$$\text{Mobility Matrix} = \begin{bmatrix} \frac{T}{F} & \frac{T}{M} \\ \frac{R}{F} & \frac{R}{M} \\ \frac{F}{F} & \frac{M}{M} \end{bmatrix} \quad (2)$$

where

T = Translational Response (x,y,z)

F = Force Excitation ( $F_x, F_y, F_z$ )

R = Rotational Response ( $R_x, R_y, R_z$ )

M = Moment Excitation ( $M_x, M_y, M_z$ )

Due to the lack of rotational accelerometers as well as standardised moment exciters in experimental analysis, only one fourth of the matrix entries in equation (2) can be properly measured with translational accelerometers whereas three fourths are usually neglected.

## Reciprocity Theorem

The basic idea of this work lies in the extension of the widely known formulation of the vibroacoustic reciprocity theorem [3] in translational degrees of freedom

$$\left. \frac{p}{F} \right|_{Q=0} = \left. \frac{v}{Q} \right|_{F=0} \quad (3)$$

where F is a force, v a velocity of a rigid body, p a sound pressure and Q a volume velocity, to rotational degrees of freedom (4):

$$\left. \frac{p}{M} \right|_{Q=0} = \left. \frac{\omega}{Q} \right|_{M=0} \quad (4)$$

where M is a moment and  $\omega$  an angular velocity.

## Measurement Setup

The measurement of rotational degrees of freedom is far more complicated than compared to its translational

counterparts. State-of-the-art measurement methods are either based on laser vibrometry [4] or the signal processing of three triaxial accelerometers [5]. The first one has drawbacks due to its large experimental setup, the second due to its limited frequency range up to 300 Hz [6] and the need of mechanical auxiliary structures [7]. Due to equation (4) another possibility is used here by directly measuring angular velocities  $\omega$  instead of deriving them from translational data (as in the two above mentioned methods). For this purpose, a MEMS gyroscope within the Bosch BNO055 is used and its data stream is read out via I<sup>2</sup>C interface. The device incorporates an accelerometer as well as a gyroscope and can measure movements in all six degrees of freedom. The accelerometer signal is sampled with a 14 Bit ADC. Therewith, the physical sensitivity by given full-scale range of  $\pm 2g$  can be calculated to:

$$S_{acc} = \frac{4g}{2^{14} \text{ LSB}} = 244.14 \frac{\mu g}{\text{LSB}} \quad (5)$$

which is smallest acceleration that can be resolved by the MEMS accelerometer. The frequency range is limited at the upper frequency of 1000 Hz. Analogously to the calculation of the highest acceleration resolution in equation (5) the physical sensitivity of the gyroscope can be mathematically derived from the rate range in the most sensitive mode with  $R=125^\circ/s$ . Different to the accelerometer within the BNO055, the gyroscope data is digitized with a 16 Bit ADC:

$$S_{gyro} = \frac{125^\circ/s}{2^{16} \text{ LSB}} = 33 \frac{\mu\text{rad}}{\text{s}} \quad (6)$$

which is the smallest angular velocity, that can be resolved by the MEMS gyroscope. The upper frequency limit of the gyroscope is at 523 Hz.

### Case Study: Electric Drivetrain

On an electric vehicle (fig.2), the vibroacoustic transfer paths were measured reciprocally in all 6DoFs.



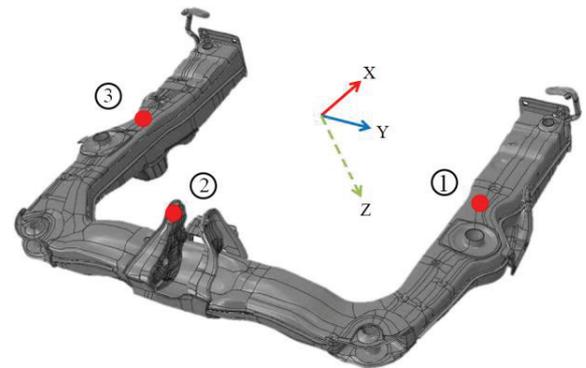
**Fig. 2:** Vehicle on whose drivetrain the case study was conducted

To get a better insight into the structure on which the sensors are attached, the engine subframe in connection with the electric engine and gear is shown in figure 3 on a test bench.



**Fig. 3:** Subframe with electric engine, gear and mounts

In the picture below, three red-marked positions on the subframe correspond to the contact points of the engine subframe with the body, with positions 1 and 3 arranged in the z-direction axially above the bearing position. At these points, therefore, the structure dynamic force transmission emanating from the engine must take place during operation.



**Fig. 4:** Three positions on the engine subframe where the responses were measured in 6 DoF.

As depicted in the picture below (fig.5), over a calibrated dodecahedron (which can be considered in the measured frequency range as an acoustic monopole and thus a source with defined volume acceleration  $\dot{Q}$ ) in the vehicle interior a 45 second exponential sweep in the 20 - 2000 Hz range was played as excitation. Thereby, the frequency response of the ITA dodecahedron is digitally equalized to a maximally flat amplitude response by internal FIR filtering. The system consists of a bandpass subwoofer, a midrange dodecahedron and can be enhanced by a tweeter dodecahedron.



**Fig. 5:** Positioning of the ITA dodecahedron as excitation monopole source with a defined volume flow  $Q$  on the driver's seat.

The responses were measured at three positions on the engine subframe (fig.4) to obtain the transfer functions in 6DoF according to equation (4). As reference sensor the triaxial accelerometer B&K Type 4506 was used. This device linearly works between 0.6 Hz lower corner frequency and 3000 Hz upper corner frequency at a sensitivity of  $10.3 \text{ mV} \frac{\text{s}^2}{\text{m}}$ . This signal can directly be compared to the MEMS accelerometer signal to become aware of the ability, quality and limitations of this sensor technology when the B&K4506 is referenced as the benchmark. Exemplarily shown below are the plots as responses in time domain in x-direction at one position.

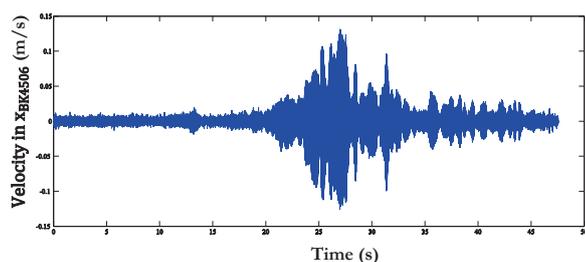


Fig. 6: Reference translational velocity response on engine subframe

At first, the measured velocity of the reference sensor (fig.6) can be compared to the velocity of the MEMS accelerometer (fig.7).

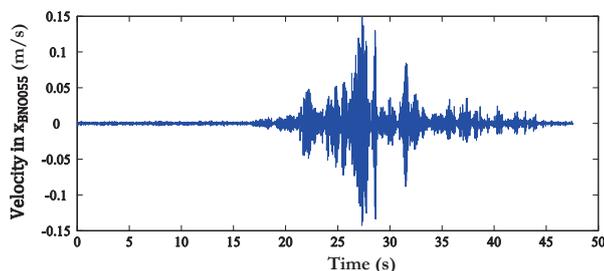


Fig. 7: MEMS translational velocity response on engine subframe

Both signals have a similar characteristic, whereas the MEMS sensor has a notably lower sampling rate and reduced frequency range compared to the reference sensor. For the final assessment of the relevance of rotational degrees of freedom, the MEMS gyroscope signal in the form of an angular velocity (see formula 4) should be compared (fig.8).

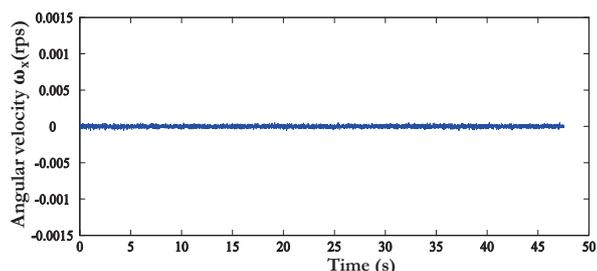


Fig. 8: MEMS angular velocity response on engine subframe

It turns out that the proportion of RDOF in the vibroacoustic transfer function is below the limit of provability. With regard to the RDOF, the case shown is exemplary and provides the same statement for all other measurement positions and spatial directions.

## Conclusion

The work presented shows that the vibroacoustic reciprocity theorem can be reformulated for rotational degrees of freedom. Even if the application of a new measurement approach is enabled, it suggests a negligibly small influence of RDOF within the case study. Based on the assumptions of researchers who are concerned with the structural dynamic transmission of engine mounts, rotational moments can – if excited by the engine – pass through the engine mounts to a considerable extent and get into the vehicle body. This raises the question, in which wave form of structure-borne sound these moments propagate: Even if the waves – as can be assumed from the results of the case study – do not lead to airborne sound radiation in the interior, which, according to the theory would be stimulated dominant by transversal waves, do not propagate as such wave type, nevertheless a significant proportion of longitudinal structure-borne waves would be conceivable. From the point of view of metrology, a further development of rotary measuring instruments should be observed, since the class of micrometer-sized sensors could have achieved a comparable level in the foreseeable future.

## Literatur

- [1] Vorländer, M. Auralization. 1st edition. Springer-Verlag, Berlin Heidelberg, 2008
- [2] Helderweirt, S. Application of Accelerometer-based RDOF Measurements for Engine Subframe Modelling. Proc. IMAC, 2001
- [3] Fahy, F. The Vibroacoustic Reciprocity Principle and Applications to Noise Control. *Acustica* 81, 544-558, S. Hirzel, Stuttgart, 1995
- [4] Bokelberg, E. A Six-degree-of-freedom Laser Vibrometer (Pt. I). *Journal of Sound and Vibration* 5, 643-667, 1994
- [5] Sattinger, S. A method for experimentally determining rotational mobilities. PhD thesis, Massachusetts Institute of Technology, 1978
- [6] Löfdahl, M. Mobility Measurement and Evaluation in Six DOFs Applied to the Hub of a Car. *Applied Acoustics* 83, 108-115, 2014
- [7] Yoshimura, T. FRF Estimation on Rotational DOFs by Rigid Block Attachment Approach. Dept. of Mech. Eng., Tokyo Met. Univ., 1999