Frequency Limits of Locally Resonant Acoustic Metamaterials

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Abstract

The advent of Acoustic Metamaterials (AMM) has opened great potential in highly effective noise control. Thanks to their engineered micro-structure, AMMs can present extraordinary characteristics, which are normally not observed in the nature. In Locally Resonant Acoustic Metamaterials (LRAM), negative values of effective mass density are achieved through local resonances within the material structure. The negative effective mass hinders wave propagation through the LRAM structure, proposing LRAMs as highly effective acoustic insulation. However, this effect is present only at a narrow frequency range, called bandgap. As a result, LRAMs are required to be designed for each specific application. Accordingly, the performance limits of LRAMs should be considered during the design process. In this study, the frequency range for LRAM operation is analyzed analytically and examined numerically, in particular for a membrane type one.

Introduction

According to the mass density law, the sound transmission loss increases proportional to the mass of the sound isolating layer, which may contradict the lightweight compact designs in modern industries, especially at low frequency range. Using LRAMs is a lightweight compact innovative solution for highly effective noise control, where local resonances within the engineered structure of the material resist the sound energy propagation at specific frequency ranges called bandgap. In this regard, LRAMs have to be tailored for each application to adjust the narrow bandgap frequency. In aim for a better design, it is essential to have a good knowledge on required considerations and limits of LRAMs. This is even more important, when different LRAM unit cells (UC) are combined to cover a wider frequency range.

An important feature of LRAMs is the subwavelength characteristic, which means that a LRAM can modify propagation of waves with wavelengths much larger than its UC [6]. In the subwavelength region, a LRAM is able to attenuate wave energy exponentially [4]. The subwavelength characteristic is also the prerequisite for applying the effective medium theory on LRAMs [1, 3, 7], which facilitates the interpretation of LRAM behavior without the need for micro-structural analysis of the system. The subwavelength characteristic also ensures an independent operation of the LRAMs, regardless of the shape and the boundary conditions [1, 2]. This means that once a LRAM is tailored to a frequency, it can be mounted on every point of a structure.

Though the subwavelength characteristic enables LRAMs to attenuate deep frequencies under 1 kHz, it brings upper frequency limit on the bandgap. At higher frequencies the wavelength gets smaller and closer to size of the LRAM. This upper frequency limit determines how fine should a UC be constructed. Regarding the importance of the subwavelength constraint, it is important to know its margins. In the literature, mostly a lattice constant of orders of magnitude smaller than the wavelength is assumed for the LRAM to ensure the fulfillment of the subwavelength constraint[4, 6]. However, to the knowledge of the author, no study has been performed to explore the margins of the subwavelength constraint and examine the behavior of a LRAM by changing its dimensions with respect to the wavelength. In this study, firstly the motion of a LRAM and the role of the subwavelength constraint on it is explained analytically. Later, using numerical methods, the response of a plate is studied with varying the size of the LRAM on it with respect to the wavelength.

Subwavelength Size and Normal Displacement of LRAM

Although different configurations have been devised for LRAMs, they work principally similarly. In figure 1(a), a membrane type LRAM is shown schematically, where a disk is embedded in an elastic membrane. The whole is then fixed with a frame, which can also represent the main structure, whose vibrations are to be mitigated. When an incident wave meets the frame, the disk acts against the force from the wave and neutralize the system motion at a specific frequency range. The response of the supposed LRAM, namely the disk and the membrane layer, to an incident wave can be characterized by its normal displacement S_x, which can be decomposed into two components:

$$S_x = \langle S \rangle + \delta S_x \tag{1}$$

The term $\langle S \rangle$ defines the surface-averaged normal displacement and the term δS_x refers to the remaining high spatial frequency components, corresponding the lateral variations of the displacements along the membrane. If the wavelength of the incident wave is assumed as λ, the wave number of the incident wave can be written as $k = 2\pi/\lambda$. As the incident wave arrives the membrane, the wave vector can be divided into its tangential and normal components $k_{\parallel}$ and $k_{\perp}$, respectively. The following equation is valid according to the dispersion relation [4]:

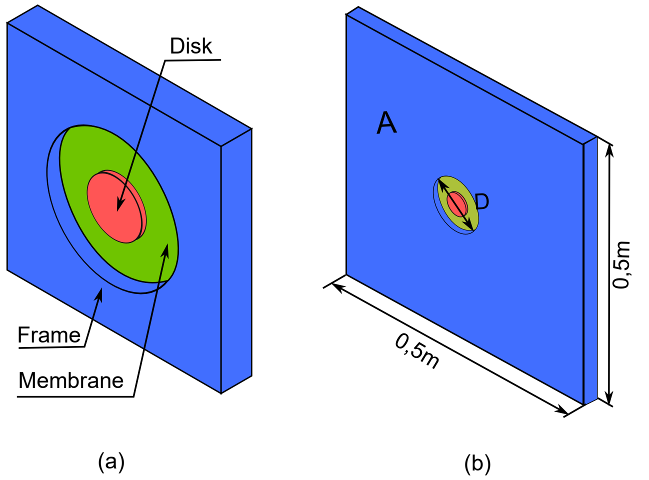


Figure 1: (a) Schematic view of a membrane LRAM unit cell (b) Application of a membrane LRAM with diameter D on a plate

$$k^2 = k_{\parallel}^2 + k_{\perp}^2 \quad (2)$$

Equation (2) can be rewritten as:

$$k_{\parallel}^2 + k_{\perp}^2 = (\frac{2\pi}{\lambda})^2 \quad (3)$$

The previously mentioned term δS_x can be described by $k_{\parallel}$, whose magnitude obeys the inequality $k_{\parallel} \geq 2\pi/R$ [4]. R refers to the radius of the membrane. Considering the subwavelength constraint ($\lambda \geq D$, $D = 2R$), it can be concluded that $k_{\parallel} \geq 2\pi/\lambda$. Regarding equation (3), it means that $k_{\perp}$ is imaginary. In other words, δS_x couples only to evanescent waves, which are non-propagating and decay exponentially. In contrast, $\langle S \rangle$ couples with propagating waves and describes the piston-like motion of the LRAM. Hence, in the far field, LRAM can be regarded as a one-dimensional system with $\langle S \rangle$[4].

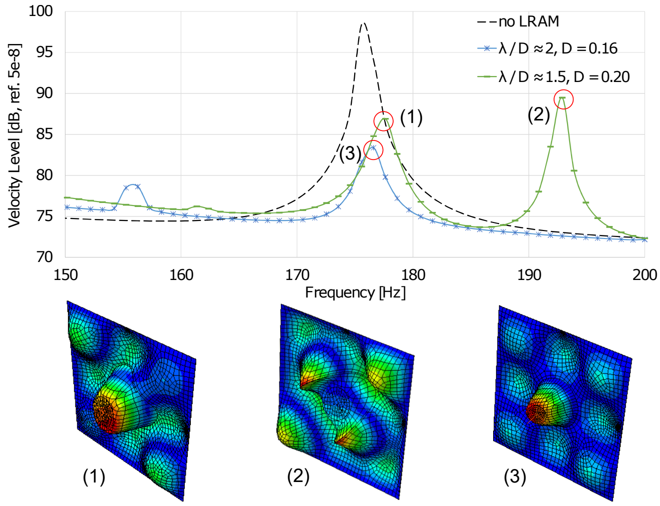
The subwavelength constraint is not only a keypoint for analytical interpretation of the exponential wave decay by LRAMs, but also results in a great simplicity in analysis of the system. Nevertheless, the above analytical interpretation does not explain whether the borders of the subwavelength constraint nor the behavior of the LRAM in the neighborhood of this border.

The Numerical Study on the Effect of the Subwavelength Size

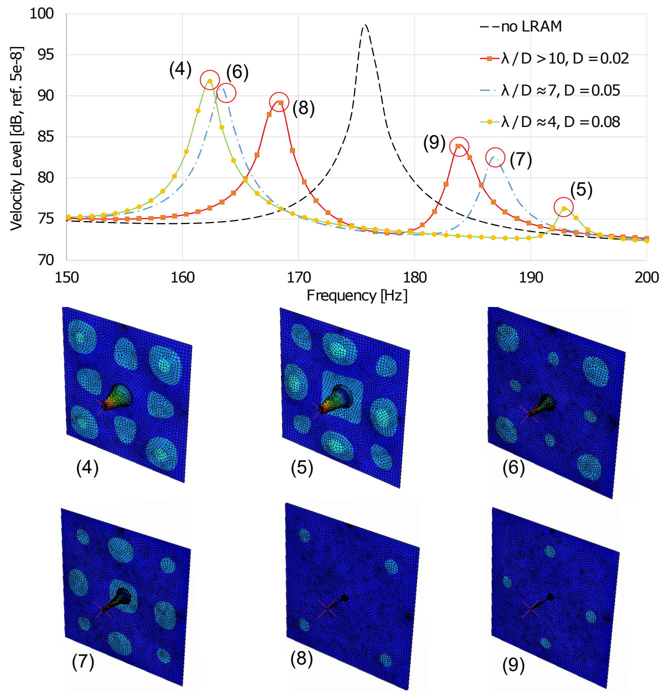
Though the importance of the subwavelength constraint has been discussed in the literature, to the knowledge of the writer the borders of the subwavelength region has not been studied yet. Mostly, a minimum value of 10 has been assumed for the ratio of the incident wavelength to the lattice constant of the LRAM (λ/D) to fulfill the subwavelength constraint. However, it is interesting to know how does the response of a LRAM change, when this ratio is changed and specially, when the dimensions of the LRAM get closer to the size of the wavelength and tend to the edges of the subwavelength region. In this study, numerical investigations are performed by using the Finite Element Method (FEM) and with the help of Abaqus 2018 software in search for the answer of this question. In this regard, a LRAM UC is placed in the middle point of an aluminum plate with the size of 0.5 m × 0.5 m (figure 1(b)). The plate is subjected to a plane wave on its surface A. The consequent bending wave inside the plate excites the LRAM. In this study, the vibration of the plate at its 3 × 3 mode is observed. At the 3 × 3 mode, the middle point of the plate, where the LRAM is positioned, has the maximum deflection. The investigations on higher modes like 5 × 5 or 7 × 7 require finer discretization of the model. On the other hand, the interpretation of the system response at higher frequencies may be difficult and uncertain, since more modes exist close to each other at higher frequencies, which can potentially interact with the LRAM. The frequency of the LRAM is adjusted to the natural frequency of the plate. It is expected that the resonance of the LRAM counteracts the resonance of the plate. Using a LRAM, the resonance of the plate is substituted with two new resonances. Between these two frequencies, the wave energy propagation is hindered due to anti-resonance effect. In order to understand the effect of the LRAM, the effective velocity of the plate on its back surface is calculated and compared for the cases with and without LRAM. The study is repeated for different sizes of the LRAM, to analyze the performance of the LRAM, when the ratio of λ/D is changed. This helps to understand the behavior of the LRAM with respect to the subwavelength constraint.

The results of the study are shown in figure 2. The natural frequency of the original plate without the LRAM at its 3 × 3 mode is about 175 Hz. Generally, it can be observed that the new resonances resulted after applying LRAM have lower peak points as the original resonance of the plate without LRAM. However, by using a large LRAM, with a ratio of $\lambda/D \approx 1.5$, the mode shapes of the plate are influenced dramatically, which is due to the considerable size of the LRAM with respect to the wavelength. As it can been observed in the mode shapes (1) and (2) in figure 2a, the LRAM resonates in phase with the waves along the plate. As a result, the wave is not mitigated in the target frequency range and the performance of the LRAM is unsatisfactory. A similar behavior can be also noticed for the case using a LRAM with lattice constant of $D = 16$ cm and the ratio $\lambda/D \approx 2$. Similarly, the LRAM moves in phase with the wave, which is shown in mode shape (3) in figure 2a.

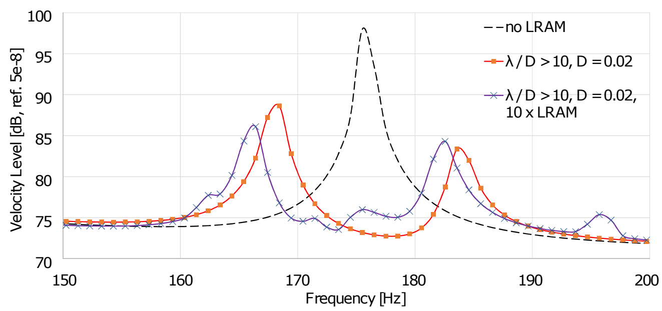
By further reducing the size of the LRAM, a strong wave mitigation at the target frequency can be first observed, when its lattice constant gets slightly smaller than $\lambda/4$. Comparing the related shape modes (4) and (5) with the shape modes (1) to (3) from larger LRAM, the difference in behavior of the system can be noticed more clearly. While the LRAM has an in-phase resonance with the plate in shape mode (4), it has a 180 degree out of phase movement with respect to the plate resonance in mode (5). Between these two resonances, an anti-resonance



(a) The velocity level of the plate in figure 1(b) without LRAM as well as with LRAMs of different sizes, corresponding to $\lambda/D \approx 1.5$ and $\lambda/D \approx 2$. The mode shaps 1, 2 and 3 represent the corresponding resonances in the diagram.



(b) The velocity level of the plate in figure 1(b) without LRAM as well as with LRAMs of different sizes, corresponding to $\lambda/D \approx 4$, $\lambda/D \approx 7$ and $\lambda/D > 10$. The mode shaps 4 to 9 represent the corresponding resonances in the diagram.



(c) The velocity level of the plate in figure 1(b) without LRAM, with one LRAM with $\lambda/D > 10$ in the middle point and with 10 randomly scattered LRAMs with $\lambda/D > 10$.

Figure 2: The vibrations of the plate shown in figure 1(b) in presence of LRAMs with different sizes as well as without LRAM

effect is generated, which attenuates the wave energy. In figure 2b, the response of LRAMs with ratios $\lambda/D \approx 7$ and $\lambda/D > 10$ and the corresponding shape modes are illustrated, which have a similar behavior to the LRAM with $\lambda/D \approx 4$. The in phase and out of phase motion of the LRAM in modes (4) and (5) with respect to the plate motion causes an anti-resonance effect and strong wave attenuation at the target frequency. Based on the velocity level curves in figure 2, a smaller LRAM causes closer resonances and hence, the bandgap gets narrower. However, a smaller LRAM is less influenced by the mode shapes of the plate and therefore, performs more reliably.

As mentioned previously, the subwavelength characteristic enables LRAMs to function independently from boundary conditions. That means that a tailored LRAM will have the same resonance frequency, wherever it is installed. In order to examine this characteristic, 10 LRAM unit cells with $\lambda/D > 10$ were mounted randomly in different positions on the supposed plate. All the LRAMs on the plate showed almost the same natural frequency with differences less than 1 Hz, though they were positioned in different places on the plate and therefore, undergoing different boundary conditions. The response of the system is shown in figure 2c. The effective velocity level curve indicates a notable wave mitigation in a neighborhood around 175 Hz.

It should be also mentioned that using a smaller LRAM does not only make the bandgap narrower, but also makes it more possible that the LRAM is placed on the wave node. At a wave node, the LRAM is not excited and as a consequence, it has theoretically no effect. This phenomena was examined in this study by tailoring the LRAM in figure 1(b) to the 4×4 mode of the plate. By the 4×4 mode, the middle point of the plate, where the LRAM is positioned, is at the wave node and at rest. The response of the system can be seen in figure 3. The LRAM is quite ignored by the wave along the plate and no change is observed in the resonance of the plate.

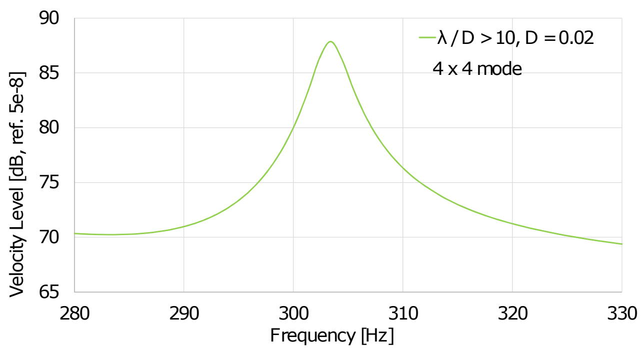


Figure 3: The application of LRAM at wave node

Generally, a modal analysis on the structure is advantageous to use LRAMs more effectively. It has been already shown in another study [5] that by placing LRAMs on a point with larger deflection, a greater extent of wave energy can be attenuated. A prior modal analysis help to position the LRAMs on the anti-nodes and attenuate the wave maximally. Otherwise, small

LRAM unit cells with $\lambda/D > 10$ can be scattered over the structure.

Conclusion

In order to benefit the considerable wave mitigation by Locally Resonant Acoustic Metamaterials(LRAM), they have to be designed and tailored to the specific target frequency range. One of the most important considerations in application of LRAMs is the subwavelength constraint, meaning the size of the LRAM should be greatly smaller than the incident wavelength. In spite of the importance of the subwavelength constraint, its margins and the behavior of the LRAM with respect to it has not been studied yet. In this study, the response of a plate was investigated numerically with varying the size of the LRAM on it with respect to the wavelength (λ/D). It was observed that the size of the LRAM should be maximally $\lambda/4$ to expect the anti-resonance effect at the target frequency. It was noticed that a smaller LRAM causes a narrower bandgap but a more reliable and independent function of the LRAM. It was also noticed, that when a LRAM is placed on a wave node, it has no effect on the system and is ignored by the wave. The study recommends a preceding modal analysis of the structure, to install LRAMs on point with the the maximal deflection and mitigate the wave energy more effectively. This study was performed mostly with regard to the 3×3 mode of the plate. Further studies on higher frequency modes can be beneficial to understand the performance of LRAMs better. Moreover, experimental studies are also required to validate the numerical results.

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