Further considerations on theoretical applications on the sound power substitution method

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Introduction

To overcome low frequency deviations in sound power determination based on different methods, the establishment of traceability in airborne sound has been proposed [1]. For the dissemination of the unit watt the use of aerodynamic reference sound sources as transfer standards has been experimentally investigated and presented [2]. The limitations of the substitution method, which is the basic tool for the traceability, was the basis for an initial theoretical study [3].

Further investigation was performed and the results are presented in this contribution. The geometries of the models are explained and the substitution is performed not only for free field conditions but also for models including reflecting boundaries of different absorption, which was also a research topic. The determination of the sound intensity level over the measurement surface is another topic. The sound power levels after the substitution method were compared to the free field levels and corresponding results are shown.

Model geometry

The model of a point source over a reflecting boundary as described by Mechel [4] was the basic model for this contribution as it has been presented in DAGA 2018 [3]. The solution of the spherical wave approach follows Suh's analysis [5]. The geometry of the model is an important setting since it is related to the source-receiver distance and the reflection angle. Each parameter of the geometry was carefully defined in order to parametrise all distances related to the source position.

The substitution method includes two sources. One of a known sound power and another whose sound power is to be determined. The study focuses on the limitations of the method in two major areas: the order of the two sources and the acoustic environment. For this reason, three models were implemented: one for free field, one including a highly reflecting floor and one for a highly absorbing side wall. In all cases, the source of the known sound power was always stationary, while the other was vertically (along the positive z-axis) or horizontally (along the negative x-axis) translated. As it has already been described [3], the translation steps were defined as ratios over the measurement radius (namely 0.1, 0.2, 0.3, 0.4, 0.5). For the calculation of the reflections spherical and plane wave approach were both considered [3]. Table 1 summarises the model configurations used for this study. Figure 1 visualises horizontal and vertical translation of a monopole for the investigation of the influence of a reflecting floor.

Table 1: Model configuration table

Known source	Unknown source	Configuration
Monopole	Monopole	• Free field • Highly reflecting floor • Highly absorbing side wall • Vertical translation of unknown source • Horizontal translation of unknown source • Plane and spherical wave approach
Dipole	Dipole	
Monopole	Dipole	

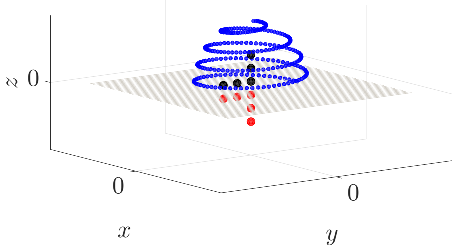

Figure 1: Source positioning for vertical and horizontal translation over reflecting floor.

In the mathematical equations, all distances included in figure 1, such as the distance from the floor or from the axes origin, are of great importance, because they are related to the distance between source and receiver and the reflection angle. Figure 2 presents the distances taken into consideration in the models.

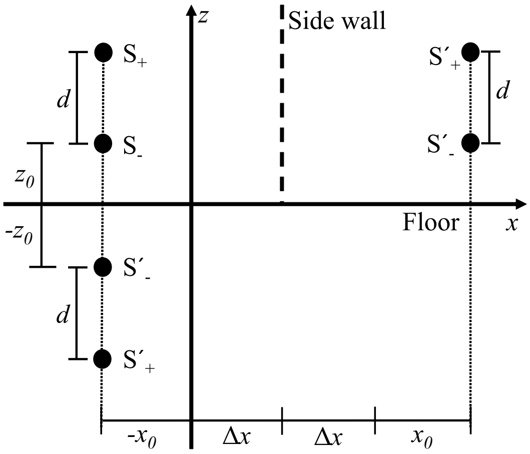

Figure 2: Geometry explanation for the configuration of reflections.

Sound intensity calculation

A novelty of the contribution is the use of sound intensity for the substitution method apart from sound pressure. The radial component of the sound intensity is given by [6]

$$I_{\mathrm{r}} = \frac{1}{2}\mathrm{Re}\left[p\left(r_{\mathrm{S}}\right) \cdot u_{\mathrm{r}}^{*}\left(r_{\mathrm{S}}\right)\right] \tag{1}$$

The radial particle velocity is derived by [6]

$$u_{\mathrm{r}}\left(r_{\mathrm{S}}\right) = -\frac{1}{j\,\omega_{\mathrm{ang}}\,\rho}\frac{\partial p\left(r_{\mathrm{S}}\right)}{\partial r_{\mathrm{S}}} \tag{2}$$

Equation (2) can be approximated by [7]

$$u_{\mathrm{r}}\left(r_{\mathrm{S}}\right) = -\frac{1}{j\,\omega_{\mathrm{ang}}\,\rho}\frac{p\left(r_{\mathrm{S}}+\Delta r/2\right) - p\left(r_{\mathrm{S}}-\Delta r/2\right)}{\Delta r} \tag{3}$$

The distance Δr between the two points where the sound pressure is calculated must be smaller than the wavelength.

The time and surface averaged sound intensity level for the case of a hemispherical measurement surface was calculated for both plane and spherical wave approach, and using also both equations (2) and (3) and the results are shown in figure 3.

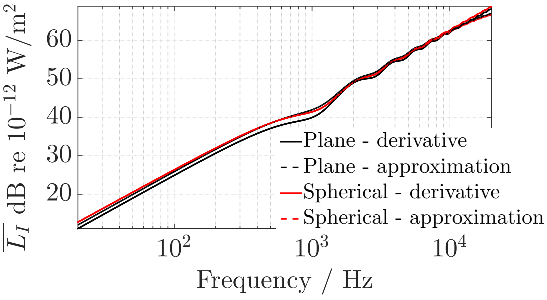

Figure 3: Time and surface averaged sound intensity level for various calculation methods.

As it may be seen, the use of equation (3) influences the high frequency end of the levels. For the rest of the frequencies, the sound intensity level does not significantly change. In terms of computational cost, equation (3) requires the determination of the sound pressure at two different measurement radii. For the spherical wave approach, this means the calculation of the Bessel function of equation (4) of reference [3] two times, which significantly increases the computational time. Based on these, the sound intensity was determined using equation (2).

For low kr the surface averaged sound intensity level is larger than the corresponding sound pressure level. An initial investigation on the factors that could presumably influence this behaviour was performed, without reaching conclusive results. Therefore, the substitution method was further applied, leaving this topic open for future investigation.

Absorption coefficient

The absorption coefficient is, according to Kuttruff [8], related to the specific acoustic impedance normalised by the air characteristic impedance ξ and to the angle θ in case of oblique incidence by

$$\alpha\left(\theta\right) = \frac{4\mathrm{Re}\left\{\xi\right\}\cos\theta}{\left(\left|\xi\right|\cos\theta\right)^{2} + 2\mathrm{Re}\left\{\xi\right\}\cos\theta + 1} \tag{4}$$

In a previous work [9], three different values were used to cover low, moderate and high absorption. In the present study, the selection of impedance ξ becomes more complicated due to the number of receivers over the measurement surface. The impedance value was set after experimental trial. The selected values provided as correct as possible sound intensity levels for both monopoles and dipoles.

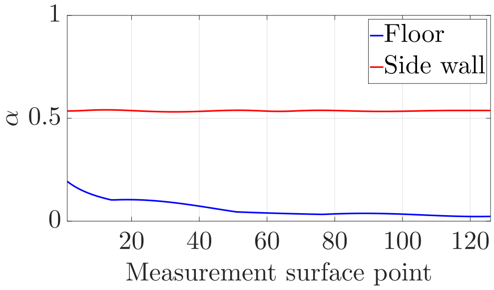

Figure 4: Absorption coefficient for reflecting floor and absorbing side wall.

The absorption coefficient changes slightly when one of the two substitution sources is translated. Figure 4 shows the maximum value of the absorption coefficient. For the case of the highly reflecting floor, the impedance was set to 15+50j and for the absorbing side wall to 0.6+1.4j.

Substitution in free field

Initially, the substitution method was applied in free field conditions and the sound power level of the translated source was compared to its free field sound power level. Figure 5 shows the deviation of the sound power level after the application of the substitution method for vertical translation of the unknown source.

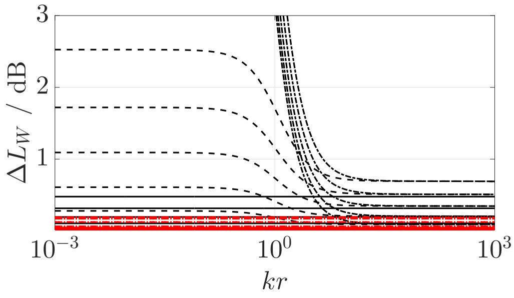

Figure 5: Deviation from the free field sound power level for the free field model. Results based on sound pressure (black) and sound intensity (red) for vertical translation. Monopole-monopole (continuous), dipole-dipole (dashed), monopole-dipole (dotted).

The most profound observation of figure 5 is that the use of sound intensity yields sound power levels very close to the free field values independently from the source order and the translation distance. This justifies the inclusion of sound intensity to the substitution method. Based on the sound pressure results, if both sources are of the same order (monopole-monopole, dipole-dipole), the deviation of the free field sound power level is up to 2.5 dB at low kr. If the sources are of different order, the free field deviation is larger than 3 dB for $kr < 0$.

Substitution including a reflecting plane

As expected, the presence of a reflecting plane strongly affects the substitution method. Figure 6 shows the deviation from the free field sound power level for the case of horizontal translation over a highly reflecting floor for the largest translation.

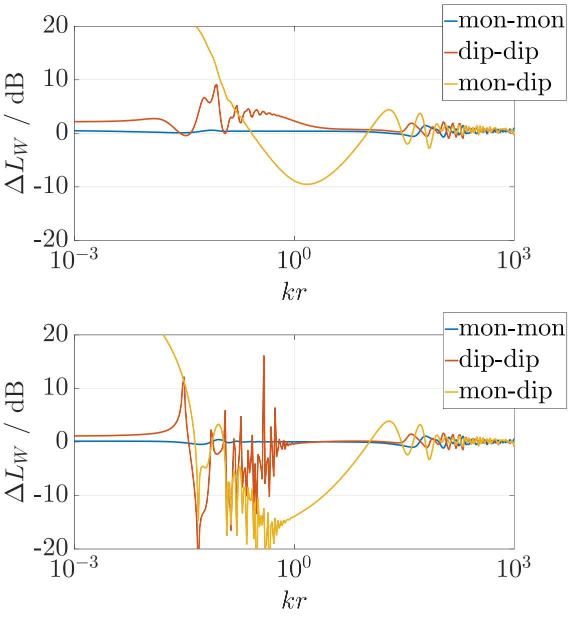

Figure 6: Deviation from the free field sound power level for the highly reflecting floor model. Results based on sound pressure (top) and sound intensity (bottom) for maximum horizontal translation and various source configurations.

If the sources are both monopoles, the result is very close to the free field sound power level. In case both sources are dipoles, it may be observed that below $kr = 1$, the use of sound pressure overestimates the sound power. Sound intensity provides closer values to free field but is strongly affected by the dipole poles distance. The use of different order sources for the substitution method, is related to results that deviate to a significant extent from the free field sound power for $kr < 10$. Further investigation is required to widen the substitution method in this case.

Conclusions

The substitution method has been theoretically investigated in continuity to a previous work [3]. In the present study, a detailed description of the model geometries was given. The time and surface averaged sound intensity level was calculated using two different mathematical expressions. The calculation of the particle velocity by the derivative of sound pressure requires lower computational time. The determination of the absorption coefficient of the reflecting plane was experimentally determined showing dependence on the receiver position. The implementation of the substitution method by using sound intensity, was validated by free field calculations. In the presence of a reflecting plane, the substitution method using sound intensity provides results close to free field, when both sources are of the same order.

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