Acoustic scattering from a suspended ceiling with perforated facing

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Abstract

This paper focuses on the scattering effect of a finite perforated plate suspended from a rigid backing. In the considered frequency range up to 5 kHz, the period of the perforation is much shorter than the wavelength of the incident wave. Therefore, a perforated plate behaves as a flat homogeneous plate with uniform porosity. Thus, only the specular reflected wave is transmitted from the perforated plate to the far field, provided the plate is infinite. As a result, scattering effects are caused by the finiteness of the structure. As the finite length of the specimen increases, the scattering coefficient tends towards zero. Nevertheless, the perforated plate modifies the directional distribution of the reflected sound pressure by its absorption. If the quarter wavelength of the sound wave is equal to the suspension height of the perforated plate, the absorption of the perforated plate is maximized. If the absorber dissipates energy from the specular reflection direction, the scattering coefficient rises. Finally, the peak and dip frequencies of the scattering coefficient correspond well with those of the absorption coefficient.

Introduction

The reverberation time is a classic but still important measure in the field of room acoustics. The absorption coefficient of the surface elements is considered as the largest factor influencing the reverberation time. Scattering also affects the reverberation time, as scattering is related to the diffuseness of the sound field in the room.

Suspended ceilings with perforated facings have been commonly used as sound absorbers in rooms for many years, because they can produce high absorption at low frequencies. This absorption has been studied in detail [1], while less attention has been paid to the contribution of the periodic perforation to the scattering of the acoustic sound field [2]. One of the reasons for this is that the measurement of the scattering coefficient of a highly absorbing material becomes inaccurate [3]. Furthermore, when absorption is high, the direction of the remaining little energy is rather unimportant. Nevertheless, improving the scattering coefficient of the absorber promotes the diffuseness of the sound field and reduces the reverberation time, particularly in the frequency range where absorption is low.

In this paper, the scattering coefficient is computed numerically by using the Boundary Element (BE) and Finite Element (FE) hybrid method [3] with COMSOL. In order to evaluate the scattering effects due to the periodic non-flat surface and the finiteness of the structure separately, both infinite and finite models are considered.

Scattering Coefficient

The scattering coefficient refers to the ability of a surface to remove energy from the specular reflection direction. It is defined [3, Sec. 4.5] by the ratio of the diffusely reflected sound power, I_{diff}, to the totally (diffusely + specularly) reflected sound power, I_t, as

$$s = \frac{I_{diff}}{I_t} = 1 - \frac{I_{spec}}{I_t}. \tag{1}$$

The measurement procedure for the random incidence scattering coefficient is defined in [4] using a turntable in a reverberation chamber. The scattering coefficient under plane wave incidence can be measured in an anechoic chamber using a turntable [3, Sec. 4.5.2]. However, it is numerically expensive to simulate the physical arrangement using a turntable.

Alternatively, Mommertz [5] proposed to compute the scattering coefficient using the directional distribution of the scattered pressure of the sample and the reference plane plate in free field. The scattering coefficient is expressed by

$$s = 1 - \frac{\left|\overline{p_s p_{ref}^*}\right|^2}{\overline{\left|p_s\right|^2} \cdot \overline{\left|p_{ref}\right|^2}}, \tag{2}$$

where p_s denotes the scattered sound pressure of the sample in the far field and p_{ref} denotes that of the reference plane plate, which has the same surface area as the sample. The front surfaces of both specimens are located at the same coordinates. A bar above the symbol (Macron) denotes the spatial mean over the receiver array, and an asterisk denotes the complex conjugate. The validity of the Mommertz method, in accordance with ISO17497, has been confirmed in [6].

Numerical Models

Infinite Model

The suspended ceiling studied consisted of a perforated plate with 12 mm thickness backed by an air cavity of 188 mm, giving a total suspension height of 200 mm. Although a perforated plate often has a thickness of 12.5 mm, the effect of 0.5 mm difference on the scattering coefficient is negligible. The circular perforations with 8 mm diameter are periodically distributed with a spatial period L_0 = 18 mm in both directions.

COMSOL is able to handle infinite geometries with rectangular periodicity (rectangular unit cell and no offset between neighbouring unit cells). The acoustic pressures at parallel boundaries of the unit cell can be related to each other. They differ only in their phases [7, Ch. 7]: These boundary conditions are called “Floquet Periodicity” in COMSOL. With the additional items “Background Pressure Field“ and “Perfectly Matched Layer (PML)“ for the infinite extension perpendicular to the partition, a COMSOL model for the scattering of an incident plane wave is readily obtained by extending the unit cell. In this way, the computation time for predicting the sound scattering of infinite periodic structures can be significantly reduced as compared to the finite model discussed in the subsequent section.

As mentioned above, the Mommertz method requires the scattered sound field of a plane plate of the same dimensions as the sample for reference. For the infinite plate under the excitation of a plane wave p_{inc}, the reflected sound field p_{ref} can be analytically derived as:

$$\begin{aligned} p_{inc} &= P_0 \exp\left\{-i\left(k_x x + k_y y + k_z\left(z - z_0\right)\right)\right\} \\ p_{ref} &= P_0 \exp\left\{-i\left(k_x x + k_y y - k_z\left(z - z_0\right)\right)\right\} \end{aligned} \tag{3}$$

where P_0 is the sound pressure amplitude of the incident wave and k_x, k_y, and k_z are the wavenumbers along the x, y, and z-axes. z_0 is the z-coordinate of the front surface of the sample and the reference plate.

Finite Model

In case of finite structures in free space, the Boundary-Element-Method (BEM) is applied to model the free field, while the Finite-Element-Model (FEM) is applied for the air cavity between the perforated plate and the rigid backing. As illustrated in Fig. 1, the perforated plate is mounted on the front face of a box, composed of impervious rigid surfaces [3]. The width/depth ratio of the box must be high enough, otherwise the scattering from the edges dominates, not that from the surface roughness. According to [4], the diameter of a turntable should be at least 16 times bigger than the sample depth. Accordingly, the box has a height of 0.2 m and a length of 3.24 m, in which 360 perforation periods are included. The background pressure field in the infinite model is replaced by a monopol acoustic source located at R_m = (R cosθ, R sinθ), where R denotes the distance between the centre of the sample surface to the monopole. R should be sufficiently far, such that an incident plane wave can be assumed on the surface. θ is the polar incident angle (see Table 1). An anechoic termination of the sound field is not required for BEM.

An array of receivers on a semi-circular curve, located symmetrically over the sample, detects the total sound pressure, p_t, which is a combination of the background sound field p_{bg} and the scattered sound field p_s. The background sound field can be computed by removing the reflecting structure. The scattered sound waves from the sample consist of propagating and non-propagating (evanescent) waves. Considering the fact that the evanescent waves decay exponentially with distance, the receiver array should be located far from the sample surface: the distance needs to be much larger than the sample size of 3.24 m.

In both finite and infinite models, the perforated plate is assumed to be rigid and impervious. The discretized model follows the rule of six elements per minimum wavelength at the maximum frequency of 5 kHz. The vicinity of the perforation is segmented by finer elements because of near field effects. As the perforation diameter is relatively large compared to the viscous layer thickness, viscous boundary layer losses and thermal losses are neglected. The geometry and the material properties of the sample setup are summarized in Table 1.

Table 1: Geometric and material parameters

Parameter		**Value**
Air	Speed of sound	343 [m/s]
	Density	1.21 [kg/m^3]
Perforation (circular)	Hole diameter	8 [mm]
	Period	18 [mm]
	Thickness	12 [mm]
Fleece	Thickness (not counted)	0.2 [mm]
	Specific flow resistance	265 [N sec/m^3]
	Surface density	0.05 [kg/m^2]
Box	Height, H_b	0.200 [m]
	Length L_b	3.24 [m]
Excitation	Monopole Strength, P_{rms}	1 [W/m]
	Location, Distance, R	100 [m]
	Location, Polar angle, θ	45 [deg]

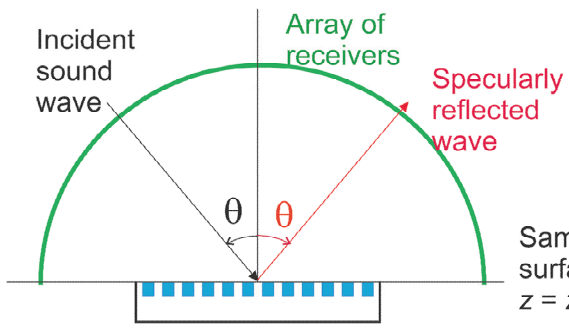

Figure 1: Schematics of the finite model of the perforated plate on the box.

Results

Infinite Perforated Plate

In the considered frequency range between 50 and 5000 Hz, the scattering coefficient of the infinite perforated plate backed by an air cavity results in values of about 10^{-5} for all oblique angles of incidence. This result indicates that the perforated plate reflects the incident sound wave only specularly, because the perforation period is much shorter than the considered wavelength. The period of the perforation is L_0 = 18 mm, which is nearly 4 times shorter than the minimum acoustic wavelength of 68 mm at the maximum frequency of 5 kHz. While the period is much shorter, the directions of the reflected waves follow Snell’s law, and the diffusely reflected sound waves decay exponentially (evanescent waves).

If the period exceeds one wavelength, scattered waves are directed in various directions ("diffracted waves"). The condition for the **m**-th diffracted wave to be scattered into the far field is given by:

$$\omega^2 > c_0^2 \left(\mathbf{G}_\mathbf{m} + \mathbf{k} \right)^2, \qquad (4)$$

where c_0 denotes the speed of sound, **k** is the wavenumber vector, and $\mathbf{G}_m$ is the $\boldsymbol{m}$-th reciprocal lattice vector of the periodic geometry [7, Ch. 7]. Equation (4) indicates a cut-on frequency, below which only the diffraction order of $\mathbf{m} = 0$ (specularly reflected wave) radiates into the far field. At normal incidence, the diffracted wave of lowest order is radiated from the perforated plate at $f_{cut\text{-}on} = 19.1$ kHz, which is far above the frequencies of interest.

Finite Perforated Plate

Figure 2 compares the scattering coefficient of two finite structures: a rigid box without surface plate (black line), below referred to as open box, and the perforated plate mounted on a rigid box (red line). Below 1 kHz, both curves show a very similar tendency. The scattering coefficient gently increases with small oscillations and reaches the first broad maximum with approximately s = 0.37 at around 400 Hz. The oscillations are due to the interaction between the scattered waves generated by the parallel edges of the box. When the box width is an integer multiple of the wavelength, the scattering coefficient is low, and when the box width is half a wavelength longer, the coefficient shows a peak. Above 1 kHz, the red curve starts oscillating strongly around the black curve with three sharp peaks at around 1.6 kHz, 2.8 kHz, and 4 kHz. These peaks coincide well with the resonances of the distributed Helmholtz resonator, which characterizes the perforated plate backed by a cavity [8]. Under an oblique plane wave excitation of 45 degrees, the resonant frequencies of the distributed Helmholtz resonator are given by 0.41, 1.5, 2.7, and 3.9 kHz [9, Ch. H.2].

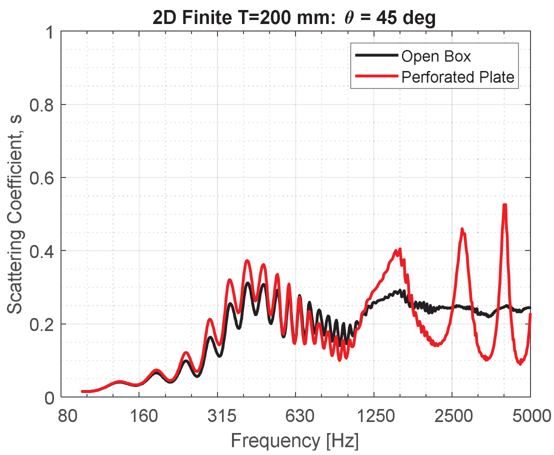

Figure 2: Scattering coefficient of the open box (black line) and of the perforated plate (red line).

Figure 3 shows the polar diagram of the scattered sound pressure on the semi-circular receiver at 1.6 kHz, which is the first sharp peak frequency of the scattering coefficient in Fig. 2. Both polar diagrams show very similar lobes, only small deviations are visible along the direction of ±45 degrees: Along the specularly reflected direction of -45 degrees, the red lobe is slightly smaller than the black lobe, while the red lobe is bigger along the incident angle of 45 degrees.

If no damping material is introduced into the system and the acoustic dissipation is excluded by the viscous and heat-conduction losses in the numerical model, the perforated plate backed by a cavity hardly dissipates sound energy. Therefore both polar diagrams show similar amplitudes. At a resonance of the resonator, the specularly reflected wave is well cancelled by the reflected wave from the bottom of the box [10]. As dissipation is not considered, the sound energy shifts to other directions, particularly to the incident angle, as shown in Fig. 3. As the sound energy is removed from the specular reflection direction, the scattering coefficient increases.

The results indicate that the reflected sound field is diffused due to (i) the finiteness of the specimen and (ii) the resonance of the Helmholtz resonator, but not due to the perforated facing.

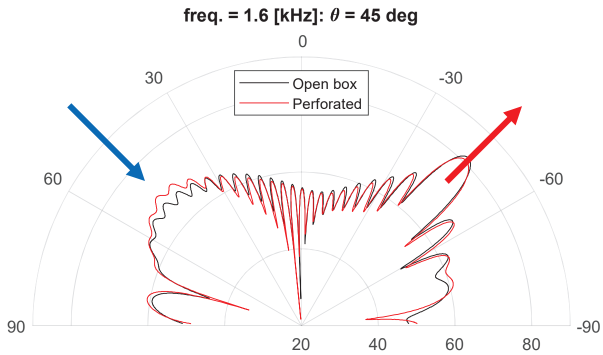

Figure 3: Polar diagram of the reflected sound field from the open box (black line) and from the perforated plate (red line) at 1.6 kHz. The arrows indicate the directions of the oblique incidence (blue) and the specular reflection (red).

Perforated Ceiling with Absorber

In practice, the rear surface of the perforated plate is covered with a thin layer of acoustic fleece. As the fleece is glued to the perforated plate, the mass of the fleece is ignored, and the fleece is modelled as an impedance layer with $Z_s = R_s$, where R_s denotes the specific flow resistance (see Table 1).

Figure 4 compares the scattering coefficient of a perforated plate with (blue line) and without (red line) the fleece at a plane wave incidence of 45 degrees. With the fleece, the peaks of the scattering coefficient are much more pronounced and the peak frequencies are slightly shifted toward higher frequencies. The scattering peaks coincide with those of the absorption coefficients (grey line).

Figure 5 shows the polar diagram of the scattered sound pressure on the receiver at 1.6 kHz, which is close to the frequency of the first sharp peak of the scattering coefficient in Fig. 4. The blue line (with the fleece) in general shows smaller lobes in all directions because of the absorption of the fleece layer, in particular in the specularly reflected direction at -45 degrees. If the height of the air cavity between the acoustic fleece and the rigid backing is a quarter of the wavelength, the particle velocity of the sound wave at the fleece is maximized, and thus the fleece efficiently absorbs sound energy of the specular reflection. As the reflected sound field becomes more uniform, the scattering coefficient shows the peaks at frequencies similar to the absorption coefficient.

The absorption coefficient is obtained by COMSOL using the infinite model (see sec. '*Infinite Perforated Plate*') of the periodic perforated plate covered with an acoustic fleece. The peak frequency is controlled by the air layer thickness. Therefore, the peak frequency slightly increases by adding the absorber on the rear surface, as this marginally reduces the air layer thickness, while keeping the construction at constant height.

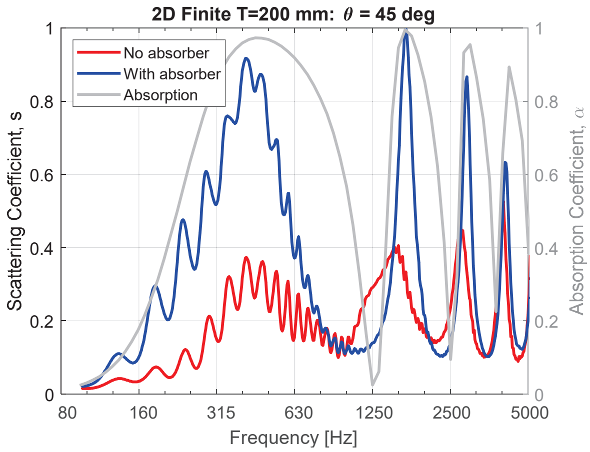

Figure 4: Scattering coefficients of the perforated plate without (red line) and with fleece (blue line), and the absorption coefficient of the suspended ceiling with the fleece (grey line).

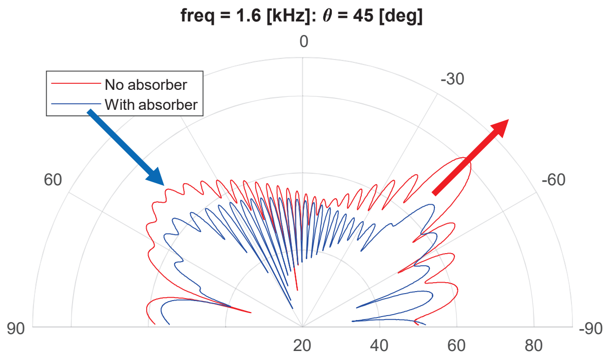

Figure 5: Polar diagram of the reflected sound field from the perforated plate without (red line) and with (blue line) the fleece at 1.6 kHz. The arrows indicate the directions of the oblique incidence (blue) and of the specular reflection (red).

Conclusions

This paper focused on the numerical computation of the scattering coefficient of a perforated plate backed by an air cavity, which could represent a suspended ceiling construction. As the period of the perforation is much shorter than the wavelength of the incident sound wave in the considered frequency range, the periodicity of the surface can be simplified to a flat surface with homogeneous porosity. Finally, the infinite model with Floquet periodicity showed near zero scattering.

The scattering coefficient of the finite perforated plate, modelled by a box with the perforated plate on the front side, is similar to that of the open box with a few extra peaks and dips. Considering this similarity and zero scattering of the infinite model, it can be concluded that the incident sound waves are mostly scattered by the boundary of the box, while the inhomogeneous surface of the perforated facing does not scatter the incident waves. The extra peaks of the scattering coefficient of the finite perforated plate correspond to the resonant frequencies of arrays of Helmholtz resonators, which are formed by the perforated plate and the air cavity behind it.

If an acoustic fleece is attached to the rear surface of the perforated plate as an absorber, the peaks of the scattering coefficient are more pronounced, as the incident sound energy is dissipated more efficiently by the absorber. The peak frequencies of the scattering coefficient corresponded well with the peak frequencies of the absorption coefficient.

A high absorption of the construction results in a small amount of the impinging sound energy being reflected at the surface. Therefore, even if the remaining little energy is well scattered, its effect on the diffuseness of the sound field in the room is limited. In order to improve the diffuse field of the room, high scattering coefficients at frequencies where absorption is low are mandatory to allow the reflected sound wave to propagate into the room. This is not the case with the considered construction. Introducing an additional periodicity of the perforation pattern with a larger period could be a solution, as the cut-on frequency of the diffracted wave will then be shifted to lower frequencies.

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