Sound Propagation in Microchannels

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Introduction

With the increasing multifunctionality of modern headphones, hearing aids or smartphones, the available space for the individual components is getting more and more limited. For this reason, technological approaches to component miniaturization are required. The semiconductor industry offers an attractive approach with MEMS technology (**M**icro-**E**lectro-**M**echanical-**S**ystems), in which the micro-mechanical structure and the electronic circuits of a component, e.g. microphones, are combined to form a system. In terms of miniaturization, acoustic MEMS actuators have to work efficiently to achieve the acoustic requirements despite smaller dimensions. Hence, for proper transducer design the physical characteristics of the sound propagation and the coupling of various acoustic loads such as the ear canal have to be considered. The so-called thermoviscous effects dominate the propagation of sound waves in narrow structures. Frictional forces occur which cause acoustic attenuation. These have to be considered in the design process of MEMS loudspeakers and adjacent sound guides.

Thermoviscous Acoustics

Sound waves lose energy as they travel through ducts. The interaction of sound waves with boundary surfaces causes attenuation effects in which the sound energy of the wave field dissipates into heat. Viscosity (internal friction), heat conduction, molecular absorption and boundary layer effects are the main loss mechanisms which cause acoustic absorption. When sound propagates in narrow channels, most energy losses are boundary layer effects, known as thermoviscous effects [1, p. 298].

If a time-harmonic sound wave propagates along a wall, tangential forces in the form of friction occur at the transition between medium and wall surface. The particles of the medium inside the fluid oscillate, but the wall surface shows no movement parallel to the wall. Due to the viscosity, the amplitude of the sound velocity decreases towards the wall. This boundary condition is called "no-slip" condition, in which the velocity field at the wall is $\mathbf{u} = 0$. At the same time, a heat transfer between medium and wall takes place by thermal conduction. Since the thermal conductivity of solids is much higher than in air, isothermal boundary conditions are assumed, with the temperature at the wall $T = \text{const}$ [2].

The viscous and thermal transition characterizes the boundary layers in acoustics and is frequency dependent. According to [1, p. 323] the thickness of the viscous boundary layer δ_{visc} can be approximated by:

$$\delta_{\text{visc}} = \sqrt{\frac{2\nu}{\omega}}, \tag{1}$$

where ν is the kinematic viscosity and ω the angular frequency. The thickness of the thermal boundary layer δ_{therm} can be approximated [1, p. 324] by:

$$\delta_{\text{therm}} = \frac{\delta_{\text{visc}}}{\sqrt{\text{Pr}}}, \tag{2}$$

with Pr being the Prandtl number. Since air under normal conditions has $\text{Pr} \approx 0.7$, the viscous and the thermal boundary layer thicknesses are of the same order. Figure 1 shows both boundary layer thicknesses compared to the acoustic wavelength λ.

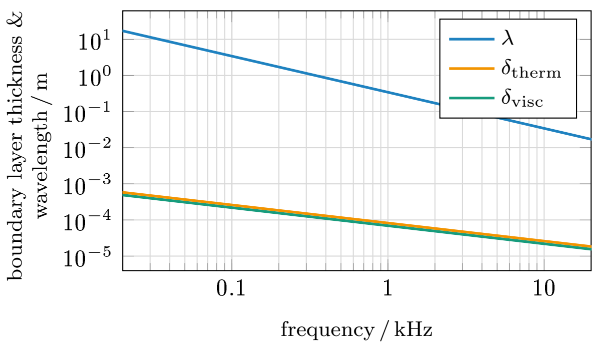


Figure 1: Viscous δ_{visc} and thermal δ_{therm} boundary layer thicknesses as well as acoustic wavelength λ as functions of frequency.

Thus the viscous boundary layer is about 220 µm thick for 100 Hz and about 22 µm thick for 10 kHz. It is clear that the boundary layer thickness is much smaller than the acoustic wavelength λ.

To demonstrate thermoviscous effects in narrow sound channels, numerical simulations were performed in COMSOL Multiphysics®. The linearized Navier-Stokes equations with no-slip and isothermal boundary conditions are used [7, Ch. 2, pp. 19–42]. Only acoustic wave propagation without mean flow is considered. Another assumption is that the flow is laminar, which is typical for sound propagation in microchannels due to the low particle velocity. At the beginning of the channel, a plane harmonic wave with a sound pressure amplitude of $p_{\text{in}} = 1\,\text{Pa}$ from 20 Hz to 20 kHz is excited. The channel walls are defined as ideal sound-reflecting and smooth surfaces.

A simulation model of a narrow circular sound channel with infinite length investigates the attenuation of sound pressure level (SPL) due to thermoviscous boundary layer effects. For this purpose, two measuring points at a distance of $l = 1\,\mathrm{cm}$ were defined in the sound channel. Figure 2 shows the result of the simulation performed for circular channels of different diameters d.

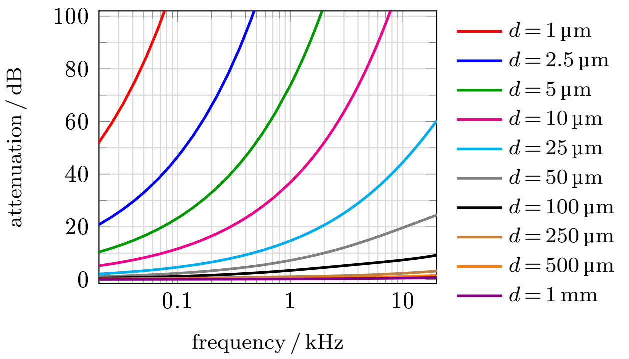


Figure 2: Simulated attenuation of the sound pressure level (SPL) per 1 cm in the circular channel as functions of frequency for different values of diameters d.

The attenuation for channels with diameters of $d > 100\,\mu\mathrm{m}$ is below $10\,\mathrm{dB}/1\,\mathrm{cm}$ while for $d < 50\,\mu\mathrm{m}$ the attenuation increases significantly. This is because for $d < 25\,\mu\mathrm{m}$ and smaller the acoustic boundary layer spans the sound channel completely over the entire audible spectrum. Very much energy is lost here due to friction. In the laminar case considered, these losses are independent of the initial pressure p_{in} of the plane wave.

If this circular sound channel with $l = 1\,\mathrm{cm}$ is bent in the middle by $90\,°$, the simulation shows a negligible difference in attenuation compared to the straight circular channel of the same size. The losses are approximately the same as in Figure 2 with deviations below 0.1 dB. Depending on the wave and channel length, reflections may occur with larger channel diameters. However, there is a local increase in velocity at the bend. This does not affect the attenuation, because the channel dimensions are small compared to the acoustic wave length.

The acoustic particle velocity profile at particular location in the sound channel changes with the channel diameter d. The reason is the relationship between the channel radius r and the viscous boundary layer thickness δ_{visc}, which can be described with the dimensionless Womersley number Wo for the circular channel as defined in [4]:

$$Wo = r \cdot \sqrt{\frac{\omega}{\nu}} = \frac{r \cdot \sqrt{2}}{\delta_{\mathrm{visc}}}. \tag{3}$$

The velocity profile also varies over time at a given point in the channel. When excited with a sinusoidal pressure gradient, the velocity oscillates with the same frequency as the driving pressure gradient. For the normalized circular channel radius, the velocity profiles in Figure 3 are obtained for two different Womersley numbers at different times (phases) during a single cycle of an applied sinusoidal pressure gradient.

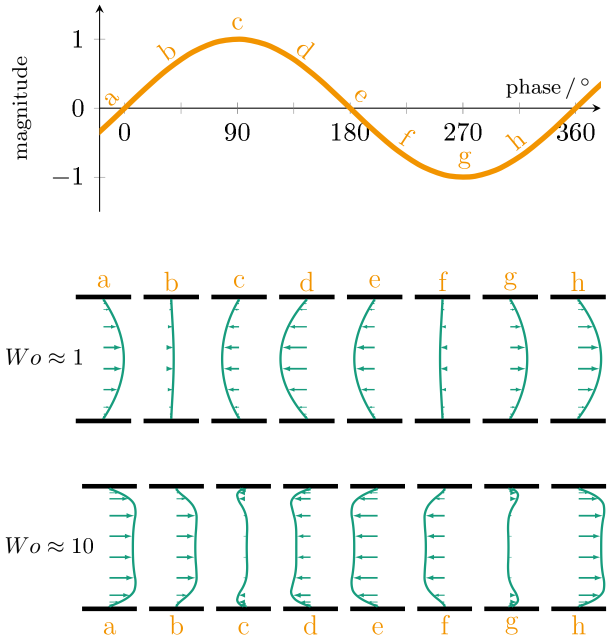
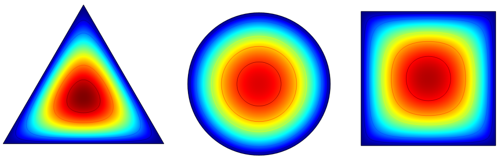


Figure 3: Velocity profile of a sinusoidal oscillating flow in the cross section of a circular channel at different times (phases) for different Womersley numbers Wo.

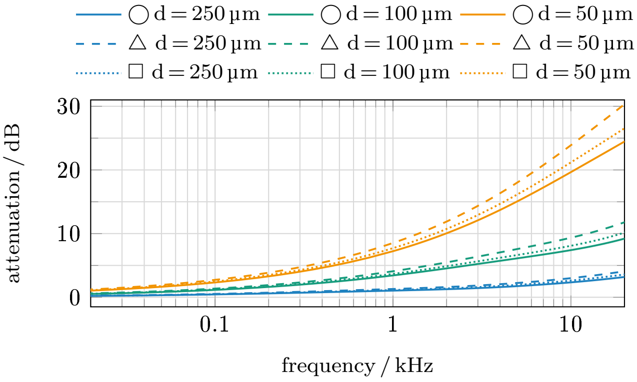
If the Womersley number is $Wo < 1$, then the acoustic boundary layer fills out all the sound channel, which is completely distinct, and the velocity profile is parabolic with a maximum in the middle of the sound channel. On the other hand, if $Wo > 1$, the maximum of the velocity profile moves more and more towards the wall. The flow is phase shifted in time relative to the driving pressure gradient. In this case, the influence of the boundary layer decreases with increasing Womersley number. The simulations showed that, depending on the topology and acoustic load, thermoviscous losses for Womersley numbers of $Wo \approx 1$–20 can be relevant. For $Wo < 1$ they are not negligible.

Shape of Microchannels

The shape of a sound channel also influences the attenuation of sound waves as they propagate through the channel. Simulations and comparisons were performed with the cross sectional shapes of an equilateral triangle, square and circle (compare Figure 4a). The diameter refers to the circular channel and is matched to the corresponding shapes so that all cross sectional shapes have the same surface area. More velocity profiles of different channel cross sectional shapes are evaluated in [8, p. 102]. Figure 4b illustrates that triangular and square shapes have more attenuation than circular channels of the same cross sectional area. The reason is the larger circumference, which causes a larger friction surface. The cross sectional shape has far less influence than the diameter.



(a)



(b)

Figure 4: Simulated velocity profile of narrow sound channels with different cross sectional shapes with $Wo \ll 1$ in (a) and corresponding attenuation of sound pressure level after $l = 1\,\mathrm{cm}$ in (b). The symbols △ ☐ ◯ indicate the corresponding cross sectional shape.

Cross-section Change

A sudden alteration in the cross-section of a narrow sound channel can lead to large abrupt acoustic impedance changes. For a steady flow this can happen due to possible boundary layer separation with local flow reversal [3, p. 104]. Separation is caused by the reduction of acoustical particle velocity within the boundary layer, combined with a positive pressure gradient (a so-called adverse pressure gradient, since it opposes the flow). This process can result further in a vortex formation [6], known as a laminar separation bubble (LSB).

Something similar can occur for an acoustic flow as shown in Figure 5. Due to the oscillating character of the flow, combined with the retarded behavior of fluid particles within the boundary layer for $Wo > 1$, the adverse pressure gradient will be periodically created such that the location of separation and the location of transition move back and forth. The periodically appearing LSB causes the acoustic energy is to be transformed into kinetic energy of the vortex and dissipate into friction [5]. This effect is dependent on the topology of the sound channel and the respective phase position of the sound wave.

To illustrate the phenomena of a sudden alteration in the cross-section, simulations were performed with a 1 cm long circular channel coupled into the free field. Figure 6 shows the velocity profile with free field radiation of a narrow circular channel. In Figure 6a the pressure gradient is favorable. If the pressure gradient is adverse, as in Figure 6b, it leads to a velocity inversion within the boundary layer.

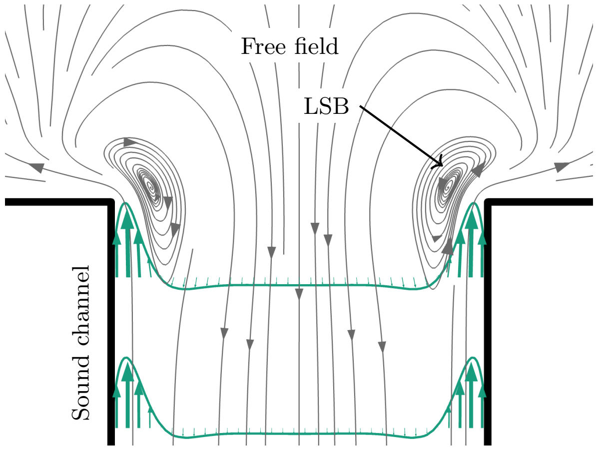
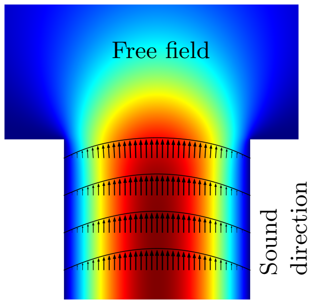
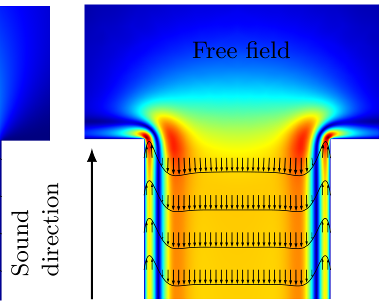


Figure 5: Streamlines (gray) with a laminar separation bubble (LSB) through an abrupt cross-sectional change of a narrow sound channel and corresponding velocity profile (green).



(a) $Wo = 1.1$



(b) $Wo = 17.7$

Figure 6: Simulated acoustic particle velocity profile in a sound channel radiating into the free field at different Womersley numbers Wo.

When looking at the transfer impedance $\mathcal{Z}_{\mathrm{trans}}$ between the channel input and the cross-section transition (channel output) by:

$$\mathcal{Z}_{\mathrm{trans}} = \frac{\Delta p}{Q}, \tag{4}$$

using the pressure drop Δp and the volume velocity Q, a maximum at about 8 kHz appears as shown in Figure 7a. The channel length $l = 1\,\mathrm{cm}$ is approximately a quarter of the wavelength λ of 8 kHz. At this point there is a velocity minimum at the cross-section transition and the pressure gradient changes from a high (in the channel) to a low (in the free field) pressure gradient. This makes the transfer impedance $\mathcal{Z}_{\mathrm{trans}}$ increase with a real and imaginary part. If now there is a velocity maximum at the cross-section transition, a resonance in the sound channel at about 16 kHz occurs. In this case the acoustic wavelength λ is half as long as the sound channel. If the channel diameter d is reduced, this resonance is attenuated as illustrated in Figure 7b until it is completely attenuated at about $d = 50\,\mathrm{\mu m}$. This is because of the thermoviscous losses, which slow down the velocity in the sound channel as the channel diameter d decreases. The attenuation of SPL in Figure 7b is only valid for the inside of the channel.

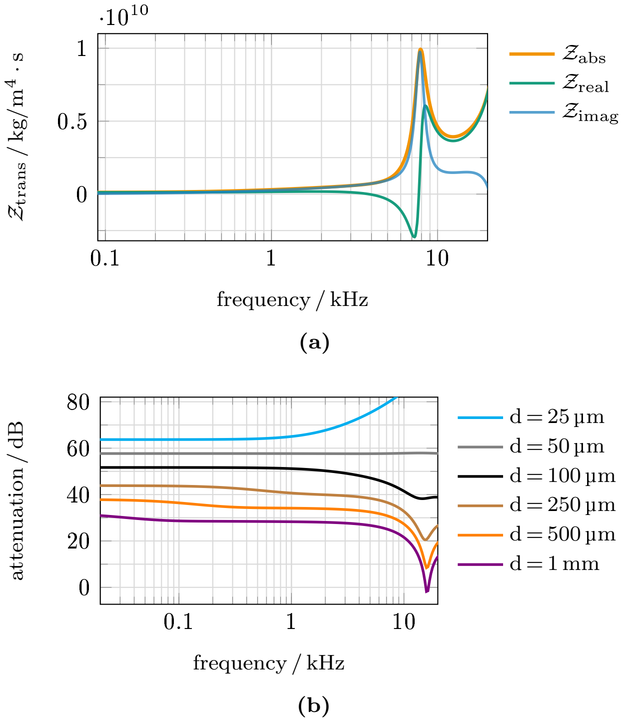


Figure 7: Transfer impedance $\mathcal{Z}_{trans}$ of a circular channel radiating into the free field of $l = 1$ cm length and $d = 0.5$ mm diameter with absolute $\mathcal{Z}_{abs}$, real $\mathcal{Z}_{real}$ and imaginary $\mathcal{Z}_{imag}$ part (a) and attenuation of SPL inside of the channel with different diameter d depending on the frequency (b).

Small Sound Channels with Ear Simulator

More practical is the application of an ear simulator. To investigate how much SPL of a narrow sound channel reaches the ear simulator, a simulation model of a GRAS RA0401 ear simulator was used. A circular channel with different diameters d and the length of $l = 1$ cm is located at the entrance of the ear simulator and a plane wave with $p_{in} = 1$ Pa is excited at the entrance of the channel. The SPL was measured at the microphone position of the ear simulator and is shown in Figure 8.

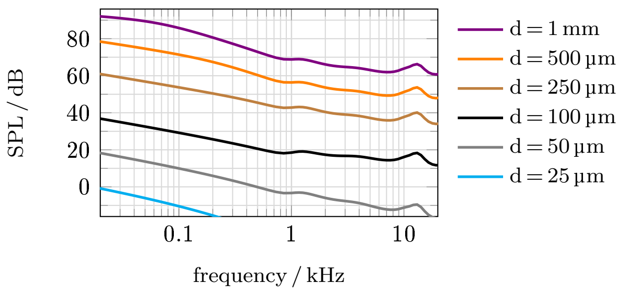


Figure 8: Simulated sound pressure level (SPL) at the microphone position of an ear simulator with coupled circular channel of length $l = 1$ cm with different diameter d.

With a channel diameter of $d = 1$ mm, lower frequencies are attenuated less compared to small diameters, while from a diameter of $d = 25$ µm and below no SPL is detected at the measuring point. Below this diameter, the sound energy remains in the channel, because the acoustic impedance of the ear simulator is too high.

Conclusion

The theoretical principles of thermoviscous effects in microchannels have been verified and illustrated by FEM simulations. The thermoviscous losses in topologies at micrometer level confront transducers with additional challenges. Acoustic impedances make a significant impact on the results. Depending on the acoustic load, e.g. a sudden alteration in the cross-section, the acoustic response of sound transducers changes significantly. The knowledge gained must be taken into account when developing novel and miniaturized transducers. In the future, the importance of thermoviscous effects will be demonstrated in practical examples. The phenomena of cross-sectional changes of narrow sound channels including the LSB will be discussed in more detail.

References

[1] Blackstock, D. T.: Fundamentals of Physical Acoustics. Wiley-Interscience, New York, 2000

[2] Theory of Thermoviscous Acoustics: Thermal and Viscous Losses, URL: `https://www.comsol.de/blogs/theory-of-thermoviscous-acoustics-thermal-and-viscous-losses/`

[3] Sigloch, H.: Technische Fluiddynamik. Springer, 2011

[4] Loudon, C.; Tordesillas A.: The Use of the Dimensionless Womersley Number to Characterize the Unsteady Nature of Internal Flow. J. theor. Biol. (1998) 191, 63–78

[5] Peters, M.C.A.M.; Hirschberg, A.; Reijnen, A. J.; Wijnands, A. P. J.: Damping and Reflection coefficient measurements for an open pipe at low Mach and low Helmholtz numbers. J. Fluid Mechanics 256 (1993), 499-534

[6] Fox, J.A.; Hugh A.E.: Localization of atheroma: a theory based on boundary layer separation. Heart 28 (1966), 388-399

[7] Bruus, H.: Theoretical Microfluidics. Oxford University Press, Oxford, 2010

[8] Nijhof, M.: Viscothermal Wave Propagation. PhD thesis, University of Twente, Netherlands, 2010