Stochastic stability analysis of a general rotor-bearing system considering the misalignment effect

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Introduction

The dynamics characteristics of hydrodynamic bearing(HDB), which is related to the rotation speed, have an great influence on the dynamics response of rotating machines. When a certain rotation speed is reached, dynamics characteristics of HDB can lead to the instability of rotating system [1,2]. This fluid-induced instability will bring a severe damage to the rotating system. Therefore, the stability analysis is necessary for a rotating system with hydrodynamic bearing.
In the past, many approaches are presented to evaluate the stability threshold. For example, Marhomy [3] used the Routh-Hurwitz method to investigate the stability of a rotor-bearing system. Dyk [4] compared different bearing models, and determined the stability threshold by using Routh-Hurwitz method and numerical continuation method. Based on bifurcation theory, Amamou [5] discussed the stability problem of journal bearings by applying numerical continuation method. Smolík [6] used the nonlinear simulation to evaluate the stability threshold for a nonlinear rotor-bearing system. From previous research, it can be concluded that the characteristics of hydrodynamic bearing is the basis for the stability analysis. However, most of current researcches on stability analysis ignored the misalignment effect caused by factors such as wear and deflection of shaft, which have a great influence on the characteristics of bearing and stability threshold. To this end, the misalignment is considered in the stability analysis in this paper [7]. At the same time, there are some uncertain variables in the system, such as clearance due to wear and viscosity due to the changes of temperature. The characteristics of bearing and stability threshold are varied with these uncertain parameters. Therefore, the uncertainty analysis to evaluate the impact of these uncertainties is necessary. For uncertainty analysis, there are two commonly used methods. One is the sampling method represented by Monte Carlo method (MCM). This kind of method can quantify the uncertainty problem with a relatively accurate result, but it is time consuming. Another method is the non-sampling method represented by generalized polynomial chaos (gPC) expansion. This method has been widely used in uncertain problems since its high efficiency and considerable accuracy [8–10].
The main purpose of this paper is to investigate the stability problem of rotor-bearing system considering the misalignment effect, and evaluate the impact of inherent uncertainties. The arrangement of this paper is as follows: the process of determining the stability threshold is discussed in first part, then the construction of gPC expansion and the uncertainty analysis for stability threshold is introduced, finally, a numerical case is given in the last section.

Stability threshold

When considering the misalignment effect, the scheme of the hydrodynamic bearing can be presented as Fig. (1) The pressure of oil-film is governed by Reynold's equation

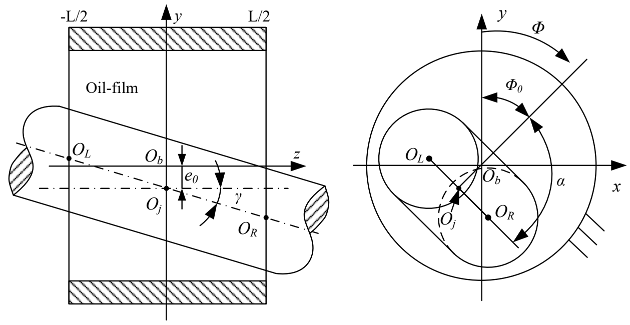

Abbildung 1: Scheme for the hydrodynamic bearing

$$\frac{1}{R^2}\frac{\partial}{\partial\Phi}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial\Phi}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial z}\right)=\frac{1}{2}\Omega\frac{\partial h}{\partial\Phi}+\frac{\partial h}{\partial t} \tag{1}$$

Where p is the oil-film pressure, h is the thickness of oil-film, R is the radius of bearing, μ is viscosity, Φ is the coordinate in circumferential direction, z is the coordinate in width direction, Ω denotes the rotating speed. Accordingly, two misaligned angles are introduced to describe the thickness of oil film [7], as shown in Fig. (1)

$$H=1+e_0\cos(\Phi-\Phi_0)+\tan\gamma\left(\frac{y}{c}-\frac{L}{2c}\right)\cos(\Phi-\alpha-\Phi_0) \tag{2}$$

Where e_0 is the eccentricity of mid-plane, Φ_0 is attitude angle, α and γ are misaligned angles. The pressure of oil-film can be calculated by finite difference method. After that, the bearing forces can be determined by integrating the pressure in the whole film

$$\begin{Bmatrix}F_x\\F_y\end{Bmatrix}=-\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi}p(\Phi,z)\begin{Bmatrix}\cos\Phi\\\sin\Phi\end{Bmatrix}Rd\Phi dz \tag{3}$$

When the static load is applied in y direction, the bearing force F_y equals to static load, and F_x equals to 0.

For a specific rotor-bearing system, the stability threshold is described by the cross point of the startup curve and borderline, as shown in Fig. (2). The startup curve is used to describe the equilibrium position at different rotating speed under a constant static load. While the borderline is used to describe the relationship between eccentricity and natural frequency of rotor-bearing system.

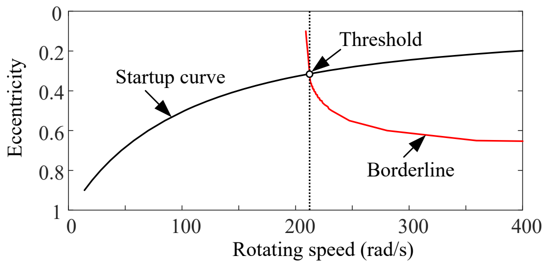

Abbildung 2: Stability threshold

With a constant static load which usually equals to gravity force, the startup curve can be obtained by using Eq. (1) and (3).

The motion of a Jeffcott rotor supported by two symmetrical hydrodynamic bearing can be described by

$$\begin{cases} m\ddot{x}_s + 2F_x = me\Omega^2\sin\Omega t \\ m\ddot{y}_s + 2F_y = me\Omega^2\cos\Omega t + W \end{cases} \tag{4}$$

Where $\ddot{x}_s$ and $\ddot{y}_s$ are acceleration components of shaft center, and W denotes the static load in y direction. And the bearing forces can be represented as the linear form shown in Eq. (5) by using Taylor series expansion.

$$\begin{cases} F_x = F_{x0} + k_{xx}x + k_{xy}y + d_{xx}\dot{x} + d_{xy}\dot{y} \\ F_y = F_{y0} + k_{yx}x + k_{yy}y + d_{yx}\dot{x} + d_{yy}\dot{y} \end{cases} \tag{5}$$

Where x and y are displacement components of journal center, and $\dot{x}$ and $\dot{y}$ are corresponding velocities. From this point, the coefficients of displacements and velocities can be calculated by combining the Eq. (1-5).

By substituting the Eq. (5) into Eq. (4), the characteristic equation can be obtained. Further, the eigenvalue can be solved, which is complex value. The real part of eigenvalue represent the situation of vibration. In this way, the borderline can be obtained since the dynamic coefficients of displacements and velocities in Eq. (5) are function of eccentricity. Then, with the startup curve and borderline, the threshold can be obtained.

Uncertainty analysis

The gPC expansion is applied to represent uncertain parameters as the combination of a finite number of orthogonal random polynomials with unknown deterministic coefficients, i.e. for random parameter X in a random sample space $\Gamma \to R$, we have

$$\boldsymbol{X} = \sum_{i=1}^{\infty} x_i \Psi_i(\boldsymbol{\xi}) \approx \sum_{i=1}^{N} x_i \Psi_i(\boldsymbol{\xi}) \tag{6}$$

in which $\boldsymbol{\xi}$ is the vector of the standard random variables, x_i are unknown coefficients, and $\Psi_i(\boldsymbol{\xi})$ denote orthogonal random polynomials . The optimal polynomials are determined by the distribution of random parameters [8]. The unknown coefficients can be calculated by using the Galerkin projection technique:

$$x_i = \frac{1}{\langle \Psi_i^2 \rangle} \int_\Gamma \boldsymbol{X}(\xi)\Psi_i(\boldsymbol{\xi}) f(\boldsymbol{\xi}) \mathrm{d}\boldsymbol{\xi} \tag{7}$$

Where $\langle \Psi_i^2 \rangle$ denotes the norm of the i-th random polynomial, $f(\boldsymbol{\xi})$ represents the joint probability density function (PDF) of the vector of random variables, and Γ is the random space.
In this paper, the clearance c and viscosity of lubricated oil μ are considered as random input parameter, which can be represented by the truncated gPC expansion as:

$$c(\xi) = \sum_{i_1=1}^{N} \alpha_{i_1} \Psi_{i_1}(\xi); \quad \mu(\xi) = \sum_{i_2=1}^{N} \alpha_{i_2} \Psi_{i_2}(\xi) \tag{8}$$

Owing to the uncertain input parameter, the stability threshold becomes uncertain. Thus, the random threshold can be expressed as:

$$\Omega_{th}(\xi) = \sum_{i_3=1}^{N} \beta_{i_3} \Psi_{i_3}(\xi) \tag{9}$$

Since the mentioned gPC expansion is a truncated expression, there is an error between the random parameters and gPC-represented one. The collocation method [11] is usually adopted to minimize the error by calculating the responses at some specific collocation points. And the process of solving these deterministic responses can be considered as a black-box. The collocation points is the combination of the roots of one higher order polynomial and zero if zero is not included in the roots. With several collocation samples, the coefficients of the truncated gPC expansion can be determined by the least square method.

Numerical study

The parameters of bearing in [12,13] is adopted to study the numerical case, as shown in Table. 1.

Tabelle 1: Parameters of bearing

Parameter	value
Bearing radius (m)	0.1
Bearing length (m)	0.2
Clearance (m)	0.15e-3
Viscosity (Ns/m^2)	8.45e-3
Rotating speed (RPM)	3000

The program to calculate pressure distribution and characteristics of bearing are verified by comparing with the results in [12, 13], as shown in Table 2 and Fig. (3). It can be seen that the current results agree well with the results in literature.
In this paper, the uncertain input parameters c and μ are assumed as the uniform and normal distribution, respectively. The details for random parameters can be seen from Table. 3.

Tabelle 2: Maximum pressure of oil film

Eccentricity	p_{max} / bar		
	Ref. [13]	Current work	Difference %
0.2	5.25	5.4	2.93
0.4	14.1	14.39	2.06
0.6	35.6	36.01	1.15
0.8	128	127.16	-0.66

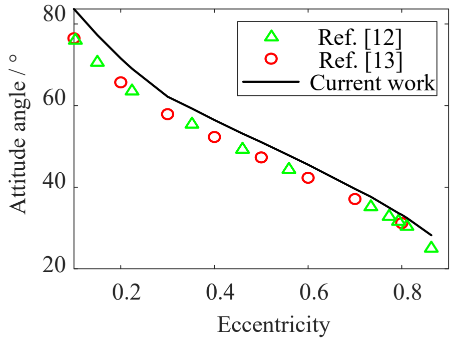

Abbildung 3: Attitude angle and comparison with references

Tabelle 3: Uncertain parameters and distribution

Parameter	distribution
Clearance (1e-3 m)	$c \sim U(0.1, 0.12)$
Viscosity (1e-3 Ns/m^2)	$\mu \sim N(8.45, 0.2)$

Then, the uncertain threshold is represented by 3 order gPC expansion. The PDF of the threshold is extracted in Fig. 4. Meanwhile, the result from gPC expansion are compared with that from MC simulation. The result shows a considerable accordance between these two methods.

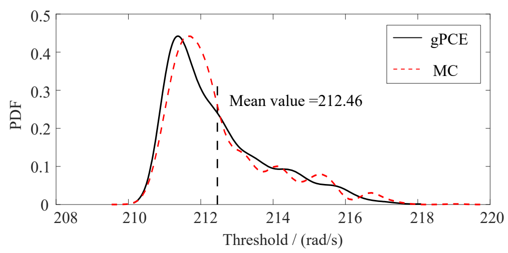

Abbildung 4: PDF of threshold by gPC and MC

Conclusions

In this paper, the stability threshold of a rotor-bearing system is investigated considering the misalignment effect and uncertainties in hydrodynamic bearing. The threshold is determined by the cross point of startup curve and borderline. By comparing with the results from literature, the accuracy of calculated results are verified. The gPC expansion is employed to study the uncertain impact from bearing. The results show that the gPC expansion can effectively describe the uncertainty parameters of system with a considerable accuracy.

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