Composing absorption coefficients of sandwiched materials using measured four pole transfer matrices.

Rene Boonen

NABLA Technisches Beratungsbüro, Hauptstraße 23, 54619 Großkampenberg, Email: rene.boonen@kuleuven.be

Introduction

A transmission tube has been developed to measure the four pole matrix of a sample of absorbing material based on the ISO 10534-2 transfer function method. The electrical analogy of a sample in a transmission tube can be considered as a T-equivalent circuit of three impedances, two in series and one in parallel. As consequence, three measurements with three different closing impedances needs to be carried out. From these measurements, a set of equations will result from which the three impedances of the T-equivalent circuit can be determined in real and imaginary parts in terms of frequency. Once these impedances are known, the transfer matrix of the sample can be set up. These transfer matrices can be kept in a library. For a sandwich material, the sandwich transfer matrix will be determined by multiplication of the transfer matrices of each component in the appearing sequence. Then, the absorption coefficient is determined from the sandwich transfer matrix. Benchmark absorption measurements have been carried out on sandwiched materials in the impedance tube. A good agreement has been established between transmission tube and impedance tube measurements in a range of 40Hz-4kHz.

Setup of the transmission tube

Figure 1 presents a scheme of the developed transmission tube [1]. The tube is equipped with two microphone heads, one between the loudspeaker and the porous material sample Z_S and one between the sample Z_S and the closing impedance Z_A. The measurement head with microphones 1 and 2 measures the impedance according to the ISO 10534-2 standard behind the first reference section, consisting of the sample impedance, the impedance of the second measurement head and the closing impedance. The measurement head with microphones 3 and 4 measures the impedance behind the second reference section, consisting of only the closing impedance Z_A. Figure 2 presents the equivalent electrical circuit measured by the first measurement head. The sample is represented by the T-equivalent circuit consisting of the three impedances Z_1, Z_2 and Z_3. From electrical circuit theory, it is known that any passive linear two port circuit can be represented by a T-equivalent circuit. The transmission line T represents the duct between the sample and the closing impedance Z_A.

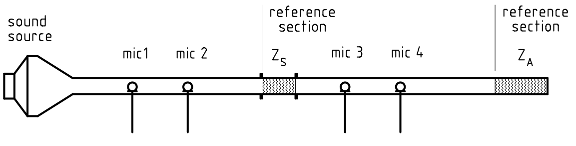

Figure 1: Transmission tube with two microphone heads, sample Z_S and closing impedance Z_A.

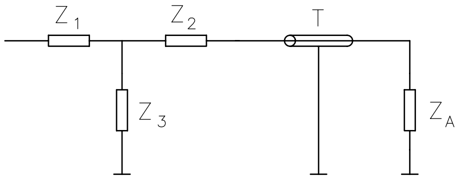

Figure 2: Electrical equivalent circuit of the sample, the part of the transmission tube with the second measurement head T and the closing impedance Z_A.

To determine the three impedances Z_1, Z_2 and Z_3, three impedance measurements at each measurement head have to be carried out with three different closing impedances using the ISO 10534-2 two microphone transfer function method. From these measured impedances, a set of three algebraic equations will be set up from which Z_1, Z_2 and Z_3 will be solved in amplitude and phase. Once these three impedances are known, any four pole matrix such as the impedance matrix, the admittance matrix, the transfer matrix or the scattering matrix can be obtained.

The transfer matrix $\mathbf{H}_S = \begin{bmatrix} A_S & B_S \\ C_S & D_S \end{bmatrix}$ of the sample can be determined from the three impedances Z_1, Z_2, Z_3 and the characteristic impedance Z_0 of the tube:

$$\mathbf{H}_S = \begin{bmatrix} 1+\dfrac{Z_1}{Z_3} & Z_0\,Z_1 + Z_0\,Z_2 + \dfrac{Z_0\,Z_1\,Z_2}{Z_3} \\ \dfrac{1}{Z_3\,Z_0} & 1+\dfrac{Z_2}{Z_3} \end{bmatrix} \tag{1}$$

These transfer matrices for each sample can be kept in a library. These transfer matrices can be used to predict the transmission loss, the acoustic reflection and absorption coefficients of different assemblies of absorption layers without prior measurements with the impedance tube.

The transfer matrix $\mathbf{H}$ of a sandwich material will be obtained by multiplying the transfer matrices of each component material in the sequence of appearance of each component in the sandwich.

The transmission loss TL of the sandwich can be calculated immediately from transfer matrix $\mathbf{H} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$$TL = 20\log[\tfrac{1}{4}(A+\frac{B}{Z_0}+CZ_0+D)\cdot\mathrm{conj}(A+\frac{B}{Z_0}+CZ_0+D)] \tag{2}$$

To determine the reflection and absorption coefficient of the sandwich, the first step is to obtain the transfer matrix $\mathbf{T}$ of the sample against a hard wall by multiplying the measured transfer matrix $\mathbf{H}$ by the transfer matrix of a hard wall:

$$\mathbf{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_H} & 1 \end{bmatrix} \tag{3}$$

wherein Z_H is the hard wall impedance, which is set to 1000 times the characteristic impedance Z_0.

Next, the impedance Z_T of the sample against the hard wall is determined from the the elements $\mathbf{T}(1,1)$ and $\mathbf{T}(2,1)$ of the transfer matrix $\mathbf{T}$:

$$Z_T = \frac{\mathbf{T}(1,1)}{\mathbf{T}(2,1)} \tag{4}$$

Finally, the reflection and absorption coefficient Γ and α are then determined as:

$$\Gamma = \frac{\frac{Z_T}{Z_0} - 1}{\frac{Z_T}{Z_0} + 1} \quad \text{and} \quad \alpha = 1 - \Gamma\,\mathrm{conj}(\Gamma) \tag{5}$$

Experimental results using the transmission tube.

Figure 3 presents the newly developed transmission tube. At the left end situates the loudspeaker, then the first measurement head with two microphones, then the sample holder, then the second measurement head with the closing impedance tube at the right side. The closing impedance tube has a valve system to create the three closing impedances without disassembling the setup. The transmission tube has a frequency range from 40 Hz until 4 kHz.

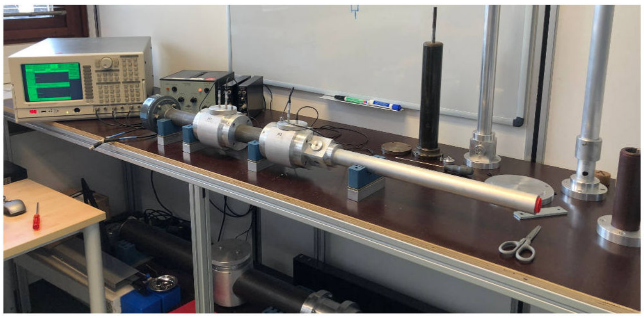

Figure 3: Transmission tube with internal diameter of 45mm with two measurement heads and closing impedance. Total length is 1.4 m

The test samples are presented in figure 4. The left sample is a melamine foam of 50mm thickness, the center one is an open polyurethane foam of 25 mm thickness and the right one is a closed polyurethane foam of 18 mm thickness. All the samples have 45mm diameter and fit slightly tight in the sample holder. The figures 5 and 6 presents the measurement results of the three impedances Z_1, Z_2 and Z_3 of the T-equivalent circuit.

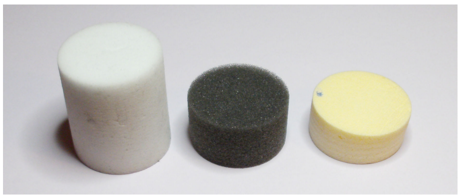

Figure 4: Samples of porous materials: left: melamine foam of 50 mm thickness; center: open polyurethane foam of 25 mm thickness; right: closed polyurethane foam of 18mm thickness.

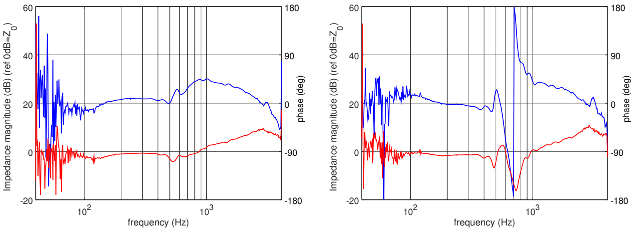

Figure 5: Measured series impedances Z_1 (left) and Z_2 (right) from the T-equivalent circuit. (magnitude in red line with reference 0 dB is Z_0, phase in blue line).

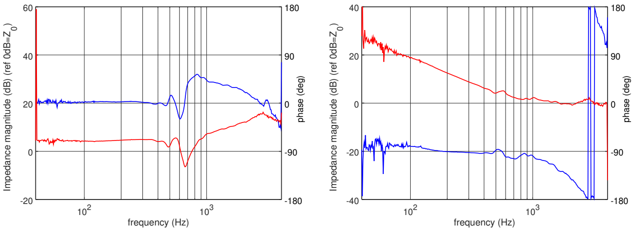

Figure 6: Total series impedance Z_1+Z_2 (left) and measured parallel impedance Z_3 (right) from the T-equivalent circuit. (magnitude in red line with reference 0dB is Z_0, phase in blue line).

Figure 6 (left) presents the sum of the two series impedances Z_1 and Z_2. This sum $Z_1 + Z_2$ is resistive in nature. The magnitude is mainly horizontal and the phase remains zero which indicates its resistive nature. At 700 Hz, a sample resonance occur and above this resonance, the magnitude increases and the phase tends towards +90 °, indicating that the sample behaves more as an acoustic inertia. Around 3 kHz, a second resonance occurs. The impedance Z_3 presented in figure 6 (right) behaves as an acoustic volume. The magnitude decreases with a slope of -20 dB/decade and the phase equals about -90°. The resonance at 3 kHz also appears in this characteristic.

From these impedances Z_1, Z_2 and Z_3, the transfer matrix $\mathbf{H}$ is calculated using expression (1). Then, the acoustic reflection and absorption coefficients are determined using expression (5). The results are presented in figure 7.

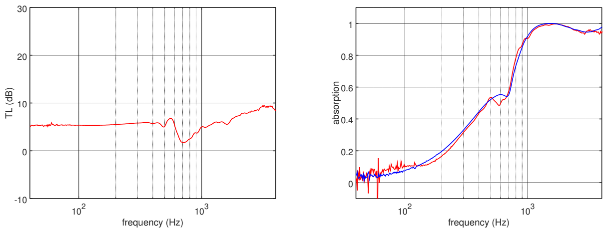

Figure 7: Transmission loss (left) en absorption coefficient (right) of the melamine sample obtained from the transfer matrix (red line) compared to the direct measurement in the impedance tube (blue line).

The transmission loss combines reflection and absorption effects. The sample resonance around 700 Hz effects as well the transmission loss as the absorption coefficient. The absorption coefficient obtained from the transfer matrix (red line) coincides well with the absorption coefficient measured directly in the impedance tube (blue line).

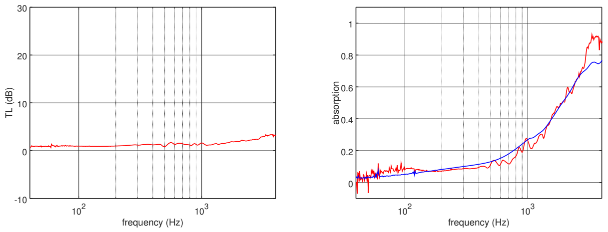

Figure 8: Transmission loss (left) en absorption coefficient (right) of the open polyurethane sample obtained from the transfer matrix (red line) compared to the direct measurement in the impedance tube (blue line).

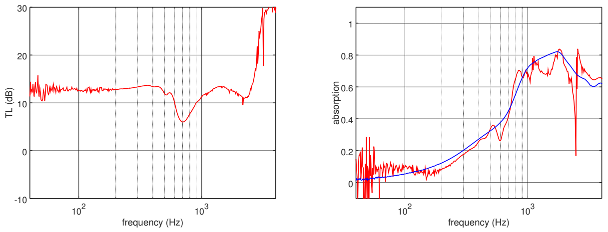

Figure 9: Transmission loss (left) en absorption coefficient (right) of the closed polyurethane sample obtained from the transfer matrix (red line) compared to the direct measurement in the impedance tube (blue line).

In the same way, the transmission loss and the absorption coefficient are obtained for the open polyurethane sample and are presented in figure 8. The transmission loss is very small, which indicates a small reflection coefficient at the front of the sample and a small absorption through the sample. Also for this sample, the absorption coefficient obtained from the transfer matrix (red line) coincides well with the absorption coefficient measured directly in the impedance tube (blue line).

Finally, the transmission loss and the absorption coefficient obtained for the closed polyurethane sample are presented in figure 9. The closed sample has a high reflection coefficient at the front of the sample, resulting in a high transmission loss.

The properties such as the reflection, the absorption coefficient and the transmission loss for sandwich structures composed of the samples measured above can be obtained from the transfer matrix of the sandwich. This transfer matrix is obtained by simply multiplying the transfer matrices of the components in the same sequence as the components appear in the sandwich.

Figure 10 presents a sandwich composed of the melamine sample with the open polyurethane sample. The sound is incident on the melamine sample. The transfer matrix $\mathbf{T}_{MPo}$ of the sandwich is obtained as:

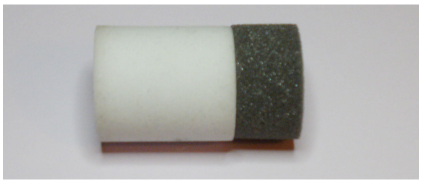

Figure 10: Sandwich composed of the melamine sample with the open polyurethane sample. The sound is incident on the melamine sample.

$$\mathbf{T}_{MPo} = \begin{bmatrix} A_M & B_M \\ C_M & D_M \end{bmatrix} \begin{bmatrix} A_{Po} & B_{Po} \\ C_{Po} & D_{Po} \end{bmatrix} \tag{6}$$

wherein the elements with index M belongs to the melamine sample and Po to the open polyurethane sample. Figure 11 presents the transmission loss (left) and the absorption coefficient (right). The transmission loss obtained from the product of the transfer matrices (6) is plotted in red. The transmission loss directly measured by putting the sandwich of samples in the tube is plotted in blue. In the same way, the absorption coefficient obtained from the transfer matrix is plotted in red and the absorption coefficient by direct measurement in the classical tube is plotted in blue. Both plots show a remarkable agreement.

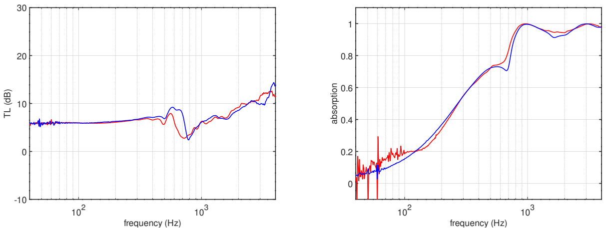

Figure 11: Transmission loss (left) en absorption coefficient (right) of the sample presented in figure 10 obtained from the transfer matrix (red line) compared to the direct measurement in the impedance tube (blue line).

If the open polyurethane sample and the melamine sample are exchanged, i.e. the sound is now incident to the open polyurethane sample, the transfer matrix of the sample is now obtained by expression (7) wherein the sequence of the sample transfer matrices is reversed.

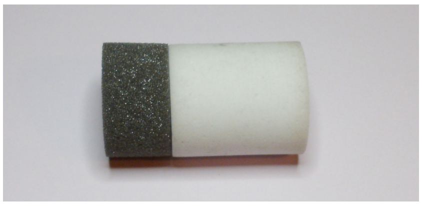

Figure 12: Sandwich composed of the melamine sample with the open polyurethane sample. The sound is incident on the polyurethane sample.

$$\mathbf{T}_{PoM} = \begin{bmatrix} A_{Po} & B_{Po} \\ C_{Po} & D_{Po} \end{bmatrix} \begin{bmatrix} A_M & B_M \\ C_M & D_M \end{bmatrix} \qquad (7)$$

Figure 13 presents the transmission loss (left) and the absorption coefficient (right). The transmission loss obtained from the product of the transfer matrices (7) is plotted in red. The transmission loss directly measured by putting the sandwich of samples in the tube is plotted in blue. In the same way, the absorption coefficient obtained from the transfer matrix is plotted in red and the absorption coefficient by direct measurement in the classical tube is plotted in blue. Also these plots show a remarkable agreement. When comparing figure 13 with figure 11, it is observed that the effect of the resonance of the melamine foam about 700 Hz, occurring in the series impedance of the melamine foam visible in figure 6, is still dominantly present in the absorption curve presented in figure 11 but has been masked in figure 13. When the sound is incident to the open polyurethane foam, it has a masking effect to the melamine foam resonance, which is not the case when the sound is incident to the melamine foam.

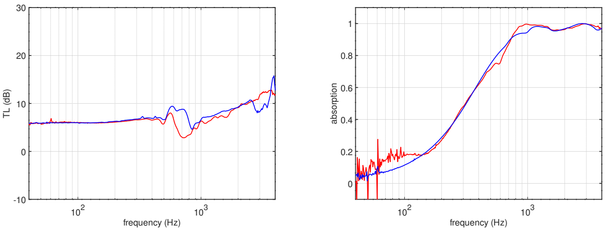

Figure 13: Transmission loss (left) en absorption coefficient (right) of the sample presented in figure 12 obtained from the transfer matrix (red line) compared to the direct measurement in the impedance tube (blue line).

The latter case is a combination of the three samples. Figure 14 presents the composition of the sandwich consisting of the melamine, the closed polyurethane and the open polyurethane sample in this sequence. The sound is incident on the melamine sample. The transfer matrix $\mathbf{T}_{MPcPo}$ of the sandwich is obtained as:

$$\mathbf{T}_{MPcPo} = \begin{bmatrix} A_M & B_M \\ C_M & D_M \end{bmatrix} \begin{bmatrix} A_{Pc} & B_{Pc} \\ C_{Pc} & D_{Pc} \end{bmatrix} \begin{bmatrix} A_{Po} & B_{Po} \\ C_{Po} & D_{Po} \end{bmatrix} \qquad (8)$$

wherein the elements with index M belongs to the melamine sample, Pc to the closed polyurethane sample, and Po to the open polyurethane sample.

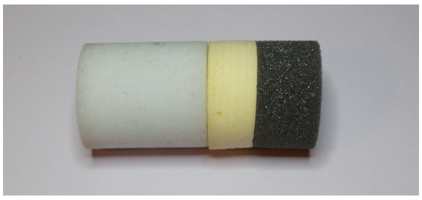

Figure 14: Sandwich composed of the melamine sample, the closed polyurethane sample and the open polyurethane sample. The sound is incident on the melamine sample.

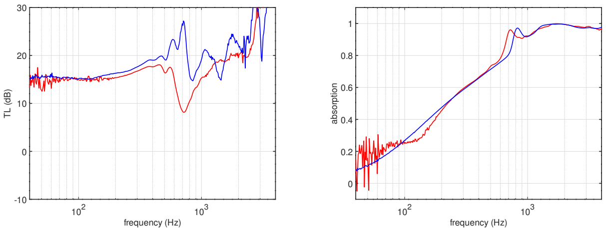

Figure 15: Transmission loss (left) en absorption coefficient (right) of the sample presented in figure 14 obtained from the transfer matrix (red line) compared to the direct measurement in the impedance tube (blue line).

Figure 15 presents the transmission loss (left) and the absorption coefficient (right). The transmission loss obtained from the product of the transfer matrices (8) is plotted in red. The transmission loss directly measured by putting the sandwich of samples in the tube is plotted in blue. In the same way, the absorption coefficient obtained from the transfer matrix is plotted in red and the absorption coefficient by direct measurement in the classical tube is plotted in blue. Again, both plots show a remarkable agreement.

These benchmarks prove that is possible to reconstruct reflection and absorption coefficients in several assemblies by manipulating transfer matrices of samples measured in this newly developed transmission tube.

Conclusion.

A new method for determination four pole parameters by impedance measurements has been presented. The sample is represented by a T-equivalent circuit of three impedances. These three impedances will be identified from three impedance measurements with three different closing impedances. The measurements are conducted in such a way that the source impedance does not appear in the measurements. No prior calibrations of closing impedances are required. It has become possible to collect material transfer matrices in a library, from which reflection and absorption coefficients for several assemblies can be predicted without prior measurements of the assembled samples.

References

[1] Boonen R., "*Measurement of the Four Pole Matrix of a Sample in a Transmission Tube*", paper nr. 463, DAGA2019, Rostock, 2019.