The use of Fresnel theory to predict attenuator values of attenuators in room acoustics

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Abstract

In order to predict the performance of noise mitigation measures a number of rules for calculations apply. The calculation is fed with laboratory results of different elements that are part of the mitigation measures. In reality predictions and results are not fully equal. In an extensive evaluation of measures in practice trends were discovered that could also be derived from a finite element model. In acoustic engineering the application of a finite element model is not practical to combine with optimization in other tools where multiple degrees of freedom apply. In this paper the shielding equations will be altered to angle dispersion, so they could be combined with wave propagation and predict an angle of incident dependent leakage through splices.

Introduction

When waves have a specific frequency, the potential orthogonal interaction is dependent on the energy potential of the wave. At low frequencies these interactions have more time before the reverse driving force is active than at higher frequencies. As a result low frequencies can more easily spread over a shorter radius then high frequencies. This has been one of the basics in acoustics for over 100 years [2]. The potential to divert around objects by an acoustic wave is frequency dependent. This could be used to calculate the shielding effect [3]. From the shielding equation a statistical chance dispersion η as a function of the radius r based on the acoustic wavelength can be derived eg.

$$\eta(r) = \frac{-(2-r^2)-\left((2-r^2)^2-4\right)^{0,5}}{\frac{2}{f_{m*0.047}}} \qquad (1)$$

Where f_m is the mid-frequency. This equation may not add vale for the predictability of shielding. But it can be used in order to predict the chance of waves getting undisturbed through a splice. There are three governing scenarios of an undisturbed path through a slice (see figure 1)

Geometric equations

Note that the geometric equations on the splice radiation is not only applicable for acoustics.

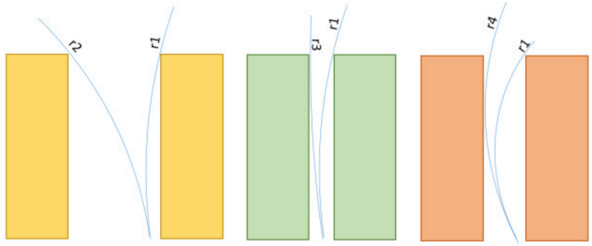

Figure 1: Three scenarios with four corresponding radii.

There are four radii in figure 1. The angle of incidence is always In such a way that the wave is coming in from under the right splitter. r_1 is the radius where the wave bends back to the direction of origin and skims on the outer edge of the object in this direction. r_2 describes a curvature in the opposite direction of r_1 that skims the outer edge of the object in the direction of propagation. r_3 is similar to r_2, but the curvature in the same direction as r_1. r_4 is similar to r_3, but skims the side of the object in the direction of propagation. The picture can be mirrored, so the angle is coming in from under the left splitter, and the r numbers are turned around.

(1)

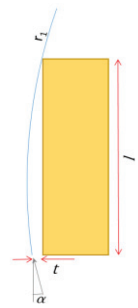

Figure 2: definitions of dimensions around r_1.

$$sin(\alpha) = A$$
$$sin(\beta) = B$$
$$cos(\alpha) = C$$
$$cos(\beta) = (1-B^2)^{0.5}$$
$$l = r_1(A+B)$$
$$t = r_1(C-(1-B^2)^{0.5})$$

With t, the distance from the edge of the splitter on the right side, l the length of the splitter, and β the skimming angle. Eventually leading up to

$$\left(\frac{1}{l^2}+\frac{1}{t^2}\right)B^2 + \frac{2}{l}\left(\frac{A}{l}-\frac{C}{t}\right)B + \left(\frac{A}{l}-\frac{C}{t}\right)^2 - \frac{1}{t^2} = 0 \qquad (2)$$

Thus solving B, solving β, solving r_1.

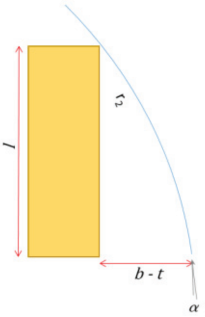

Figure 3: definitions of dimensions around r_2.

$$sin(\delta') = D'$$
$$cos(\delta') = (1 - D'^2)^{0.5}$$
$$l = r_2(D' - A)$$
$$b - t = r_2(C - (1 - D'^2)^{0.5})$$

With b, the thickness of the splice, and δ the skimming angle. Eventually leading up to

$$\left(\frac{1}{l^2} + \frac{1}{(b-t)^2}\right) D'^2 - \frac{2}{l}\left(\frac{A}{l} + \frac{C}{b-t}\right) D' + \left(\frac{A}{l} + \frac{C}{b-t}\right)^2 - \frac{1}{(b-t)^2} = 0 \qquad (3)$$

Thus solving D', solving δ', solving r_2.

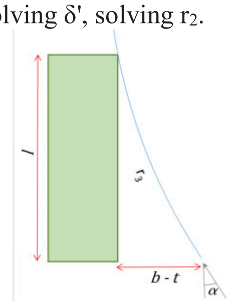

Figure 4: definitions of dimensions around r_3.

$$sin(\delta) = D$$
$$cos(\delta) = (1 - D^2)^{0.5}$$
$$l = r_3(A - D)$$
$$b - t = r_3((1 - D^2)^{0.5} - C)$$

Eventually leading up to

$$\left(\frac{1}{l^2} + \frac{1}{(b-t)^2}\right) D^2 - \frac{2}{l}\left(\frac{A}{l} + \frac{C}{b-t}\right) D + \left(\frac{A}{l} + \frac{C}{b-t}\right)^2 - \frac{1}{(b-t)^2} = 0 \qquad (4)$$

Thus solving D, solving δ, solving r_3.

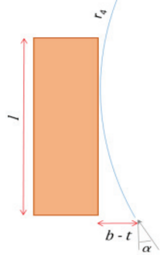

Figure 5: definitions of dimensions around r_4.

$$b - t = r_4(1 - C)$$
$$r_4 = \frac{b-t}{(1-C)} \qquad (5)$$

Numerical solutions

Now there are three scenarios, the applicable scenario needs to be picked automatically, the geometric boundaries result in equation 6.

$$IF(l * \tan(\alpha) > b - t)$$
$$IF(\delta < 0)$$
$$IF(r_1 > r_4) \rightarrow zero$$
$$else \rightarrow III$$
$$else \rightarrow II$$
$$else \rightarrow I \qquad (6)$$

Where *zero* means that there is no leakage assumed.

Still for each scenario the probability needs to be derived from both angles. For scenario I this leads to equation 7.

$$\eta(r) = \frac{(2-r_1^2)+\left((2-r_1^2)^2-4\right)^{0,5}+(2-r_2^2)+\left((2-r_2^2)^2-4\right)^{0,5}}{\frac{4}{fm*0.047}} \qquad (7)$$

For both scenario II and III equation 8 holds.

$$2\eta(r) = \frac{(2-r_1^2)+\left((2-r_1^2)^2-4\right)^{0,5}-(2-r_3^2)-\left((2-r_3^2)^2-4\right)^{0,5}}{\frac{2}{fm*0.047}} \qquad (8)$$

In both eq. 7 and eq. 8 the maximum value of η is 1.

In general the propagated wave could have a similar angle of incidence throughout a small splice. The numeric relation between the angle of incidence and leakage can be found by integration value t from $t = 0$ till $t = b$. A five parameter dependent numerical result is found, relating leakage, frequency, thickness of the splice, length of the splice and angle of incidence together. Some examples are given in figure 6-8.

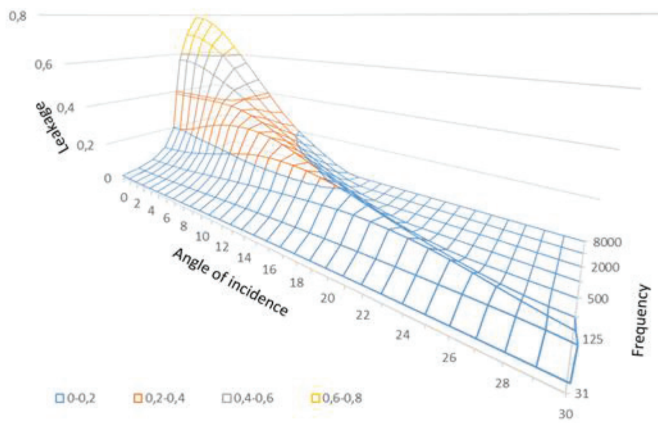

Figure 6: Leakage through 0.5 m thick 2.5 m long splice.

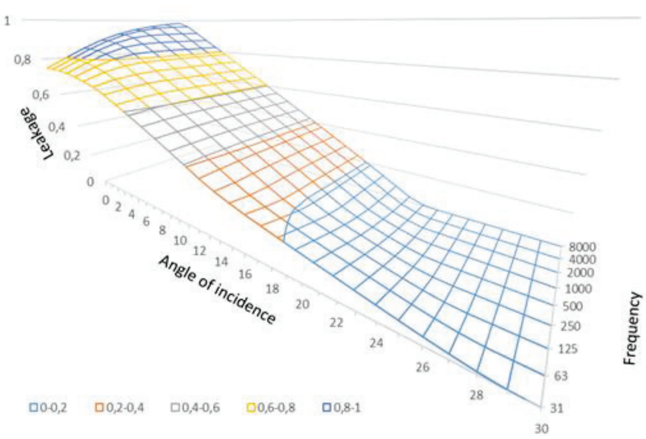

Figure 7: Leakage through 0.2 m thick 0.5 m long splice.

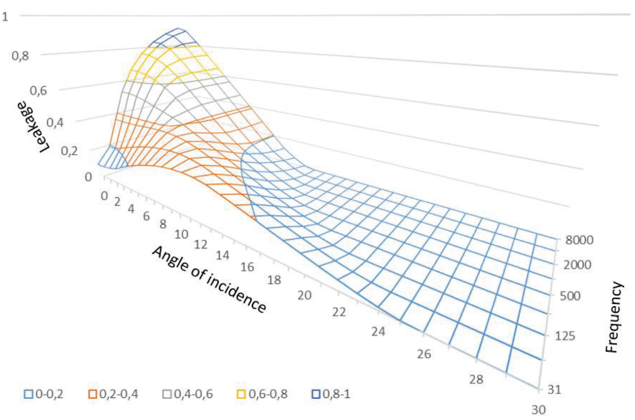

Figure 8: Leakage through 0.2 m thick 1.0 m long splice.

Despite the continuous switch between different scenarios for different angles, even within figure 6-8. All curves are smooth and continuous. In figure 8 low frequency leakage benefits from a small angle. This is a consequence of eq. 1 and the absence of a 4^{th} scenario that describes oscillation between two splitters. Note that figure 6-8 are no physical real leakage numbers. Both the thickness of the attenuator absorbing package and the proportion of direct noise in relation to reverberant noise will reduce the physical leakage *G*.

$$G = G_0 \frac{(b)}{b+a} \cdot \frac{10^{\frac{Lw_{direct}}{10}}}{10^{\frac{Lw_{direct}}{10}} + 10^{\frac{Lw_{reverberant}}{10}}} \qquad (9)$$

Where a is the thickness of the absorbing package (shielded) and L_w is the sound power acting on the attenuator, being either direct or reverberant sound. By using eq. 9 diffraction on the silencers edge is neglected, because leakage has more influence on the higher frequencies where diffraction is less.

The influence of the second important angle of incidence (in the direction of the height of the splices) increases the effective length, further limiting leakage (equations are not in this paper).

An acoustic wave propagates from a source. The shape of the source as well as the distance from the source determine the shape of the wave front of direct sound. In a laboratory, when the reference attenuation of an attenuator is quantified, diffusivity is (un)intentionally added to the wave front in order to increase predictability. After eq. 9. The leakage is the sum of the partial leakage over all entry points and under the corresponding angle. The leakage is considered just one path of sound to propagate through an attenuator. The attenuation value D' including leakage is:

$$D' = -10\log\left(10^{-\left(\frac{D}{10}\right)} + 10^{-\left(\frac{-10\log(G)}{10}\right)}\right) \qquad (10)$$

Where D is the attenuation value without leakage. One may argue that this is physically incorrect, because the leakage will only downgrade the attenuation value, while when the leakage is less than the reference value, the attenuation value will not increase using eq. 10. Note that this is solved when D is replaced with an attenuation value without leakage, and other terms such as flanking noise are added.

Comparison with real projects

Alara-Lukagro produces noise control solutions based on their own engineering calculations. In an evaluation of the calculations in practice [1] the room acoustics calculations were proven to be reasonably accurate, see figure 9. The black lines are the standard deviations of similar systems and measurement equipment. In solid blue the average overestimation of the sound pressure level in octave bands and in dashed blue the average combined with the standard deviation of 12 different projects.

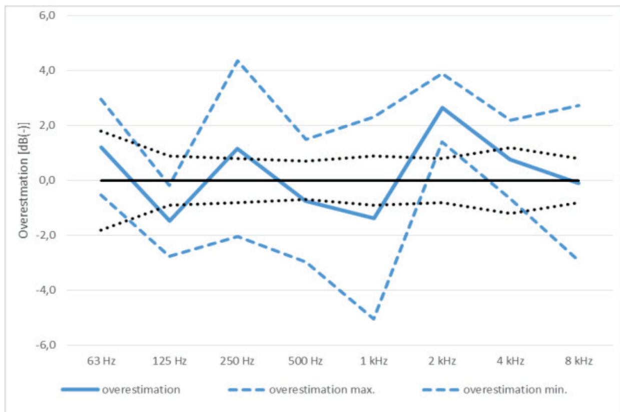

Figure 9: Overestimation of sound pressure levels based on sound power levels of present installations.

Unfortunately the attenuation value of splitter attenuators is not as accurately predictable as room acoustics. In figure 10 the deviation of 15 splitter attenuators is given. The average is spot on, but the deviation is around 5 dB, while three out of 15 had a much stronger deviation. The two blue lines suffer significantly from leakage, and will be used to evaluate eq. 1-10 in figure 11. The dashed green line on top has a favorable angle and therefore much less leakage than in laboratory conditions. In reality leakage is the main risk factor in acoustic engineering.

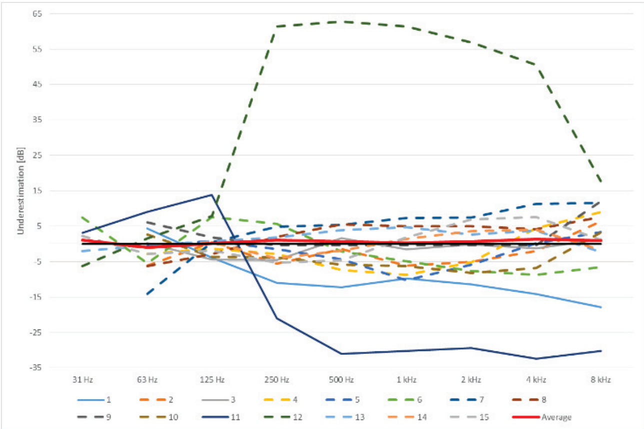

Figure 10: Underestimation of the attenuation in practice compared to data from laboratories/manufacturers

When the two blue lines in figure 10 are re-calculated, using leakage equations 1-10, the attenuation values are very different. While the attenuation values of the dotted lines in figure 10 undergo no significant changes using eq. 1-10. The implementation of eq. 1-10 leads to D', which is compared to the attenuation in practice in figure 11.

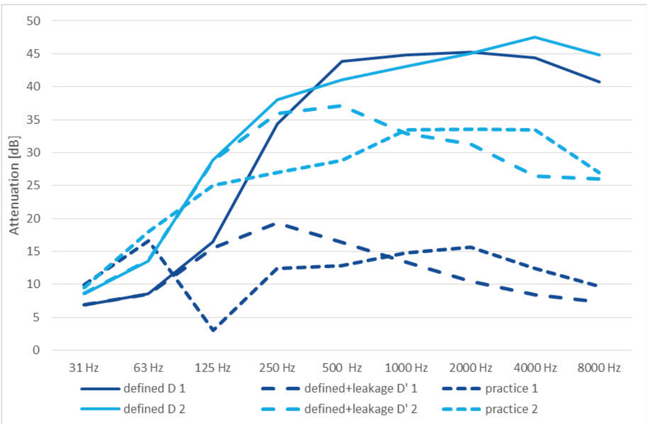

Figure 11: Theoretical attenuation values D compared to attenuation values including leakage D' and attenuation values found in projects with a high leakage potential

What is not in figure 11 is the overall dB(A) result, which was 10 dB off for both cases using D, but is less than 1 dB off, using D'. Other effects in literature, such as VDI 2081 [4] don't come near the same order of magnitude.

The leakage in reality is slightly less, for high frequencies, and more for mid-frequencies. The model has been rather pessimistic about the increase of effective length, and hasn't considered that mid-frequencies can bend towards the splices, which is in contradiction to eq. 9. Where the reflective hard surface of the splice edge is assumed to receive as much sound as the splice itself. As mentioned there is a fourth scenario benefitting low frequency leakage. Further extension of this theory should improve these two phenomena. In case 1 a standing wave is acting strongly on the 63 Hz to 125 Hz region.

Conclusions and recommendations

Leakage from the not reverberant part of the source is the most important factor in the difference between the attenuation value in reality and in a laboratory. From a probability equation that relates the radius to chance dispersion and geometric calculations a numerical model can be made, which can well predict acoustic leakage without underestimating the attenuation value when leakage has no significant contribution. This powerful engineering tool could be improved in accuracy, taking into account the effects of the acoustic wave between the source and the attenuator, or improving the equation that relates the radius to chance, or making the effective length as a function of the orthogonal angle of incidence more frequency dependent.

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Literature

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