Uncertainty Analysis in Dynamic Response of Composite Structures in Thermal Environment

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Introduction

Uncertainty analysis of the engineering structure in various manufacturing sector such as aerospace, automobile etc. is essential for safe design of the same. Nowadays, composite material is used as a basic structural material to these sector due to the various advantages such as, high structural stiffness, high strength-to-mass ratio. These structures, however, are often experienced rise of temperature during high-speed maneuvering, and composite material can be advantageously used due to low coefficient of thermal expansion (CTE) and high thermal stability. Owing to the high strength and low CTE, graphite-epoxy, carbon-epoxy, and IM7-PEEK composites are generally used as a basic structural materials to these structures in various thermal environments. Various researchers reported dynamic response of the graphite-epoxy composite plate [1, 2] and shell [3, 4] at different incremental thermal environments. Due to the variability of the constituent materials, the stiffness parameters of the composite structure are became uncertain, which imparts uncertainty to the dynamic response of the structural component [6, 5]. Moreover, due to the random temperature increment dynamic response of the composite plate also became uncertain [7]. During fabrication process, folded-plate structures are often preferable than single plate type structure. The dynamic response for one and two-folds composite structures using folded-plate formulation [8] has already reported in literature. However, uncertainty analysis of the folded-plate structure in various thermal environment has yet not been reported in the literature. In the present study uncertainty analysis of the single-fold IM7-PEEK composite structure the in thermal environment is presented using generalized polynomial chaos (gPC) expansion method [9].

Mathematical formulations

The mathematical modeling of the single-fold composite structure in thermal environment is done here by finite element (FE) method considering proper folded-plate transformation. Elastic properties of the IM7-PEEK composite at various temperature is assumed as uncertain and uncertainty response in modal frequency of the single-fold composite structure is represented by collocation based gPC expansion method.

FE formulation

The first order shear deformation theory (FSDT) for analysis thin laminated composite plate in thermal environment is used for FE analysis of the folded-plate as shown in Figure 1. According to the FSDT, generalized displacement field in terms of mid-plane displacement is described by $\{d\} = \{u \quad v \quad w \quad \theta_\mathrm{x} \quad \theta_\mathrm{y}\}^\mathrm{T}$. The constitutive equation of the composite plate subjected to the thermal environment is stated as

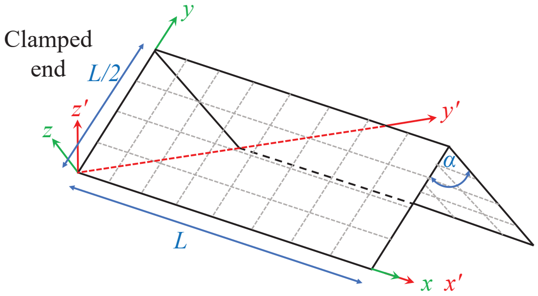

Figure 1: Geometry of the single-fold composite structure with crank angle α.

$$\{F^r\} = [D]\{\varepsilon^*\} - \{F^N\}, \tag{1}$$

in which D is the stress-strain relationship matrix, ε^* is generalized mid-plane strain vector, and thermal resultant force and moment vector is denoted by F^N. For FE analysis the structural domain has been subdivided by 2-dimensional 8-node finite element. The elemental stiffness matrix $[K_\mathrm{e}]$, elemental geometric stiffness matrix $[K_\mathrm{Ge}]$, and elemental mass matrix $[M_\mathrm{e}]$ of the flat composite plate can be written in the form of [1]

$$\begin{aligned}
[K_\mathrm{e}] &= \int_{\mathrm{A_e}} [B]^\mathrm{T}[D][B]\mathrm{d}A_e, \\
[K_\mathrm{Ge}] &= \int_{\mathrm{A_e}} [G]^\mathrm{T}[S_\mathrm{r}][G]\mathrm{d}A_e, \\
[M_\mathrm{e}] &= \int_{\mathrm{A_e}} [N]^\mathrm{T}[\bar{M}][N]\mathrm{d}A_e.
\end{aligned} \tag{2}$$

Herein, strain-displacement matrix, load-strain relationship matrix, and thermal residual stress resultant-displacement matrix are denoted by $[B]$, $[D]$, and $[S_\mathrm{r}]$, respectively. The corresponding inertia matrix and shape function matrix for 8-node element is described by $[\bar{M}]$ and $[N]$, respectively. For FE formulation of the single-fold plate, as shown in Figure 1, an orthogonal transformation matrix $[T]$ [8] is proposed to relate the global displacement d' of the composite folded-plate with the local displacement d of the composite plate as $\{d\} = [T]\{d'\}$. Therefore, final form of transformed elemental stiffness $[K'_e]$, geometric stiffness $[K'_\mathrm{Ge}]$, and mass $[M'_e]$ matrices are written after transformation as

$$\begin{aligned}
[K'_e] &= [T]^\mathrm{T}[K_e][T], \\
[K'_\mathrm{Ge}] &= [T]^\mathrm{T}[K_\mathrm{Ge}][T], \\
[M'_e] &= [T]^\mathrm{T}[M_e][T].
\end{aligned} \tag{3}$$

Prior to applying the transformation, the elemental matrices are expanded by inserting θ_{z} drilling degree of freedom at each node of the element. To avoid numerical instability, on-diagonal terms of the stiffness matrix $[K'_e]$ are filled with very small positive values, where as off-diagonal terms are remained zeros. Henceforth, transformed elemental matrices $[K'_e]$, $[K'_{\mathrm{Ge}}]$, and $[M'_e]$ are assembled to obtained corresponding global stiffness matrices $[K']$, $[K'_{\mathrm{G}}]$, and $[M']$ of the folded-plate structure. The free vibration analysis of the folded-plate structure in thermal environment is consisted of solution of

$$\{[K'] + [K'_{\mathrm{G}}]\}\{d'\} + [M']\{\ddot{d'}\} = \{0\}. \tag{4}$$

Stochastic FE formulation

The temperature-dependent elastic moduli of the IM7-PEEK composite folded-plate are considered as an input random variable, and represented by $\boldsymbol{\xi} = \{\xi_1, \xi_2, ...\xi_{\mathrm{n}}\}$. This uncertainty in elastic moduli at a specific temperature affects $[K']$ and $[K'_{\mathrm{G}}]$ matrices, and corresponding modal responses in terms of modal eigenfrequency. The stochastic FE (SFE) formulation at a specific temperature can be written with reference to Eq.(4) as

$$\{[K'(\boldsymbol{\xi})] + [K'_{\mathrm{G}}(\boldsymbol{\xi})]\}\{d'(\boldsymbol{\xi})\} + [M']\{\ddot{d'}(\boldsymbol{\xi})\} = \{0\}. \tag{5}$$

The random elastic moduli can be represented by truncated gPC expansion method as

$$\{E_{11} \;\; E_{22} \;\; G_{12}\}^{\mathrm{T}} = \sum_{i=0}^{N} \{e_{11} \;\; e_{22} \;\; g_{12}\}_{\mathrm{i}}^{\mathrm{T}} \Psi_{\mathrm{i}}(\boldsymbol{\xi}), \tag{6}$$

where, $\{e_{11} \;\; e_{22} \;\; g_{12}\}^{\mathrm{T}}$ is matrix of coefficients for random elastic moduli, and Ψ_{i} is orthogonal basis function with 3-dimensional random vector $\boldsymbol{\xi} = \{\xi_1, \; \xi_2, \; \xi_3\}$. Galerkin projection technique is adopted to evaluate coefficients of the random elastic moduli. Accordingly, uncertain modal eigenfrequency $f_{\mathrm{m}}(\boldsymbol{\xi})$ are approximated using truncated gPC expansion as

$$f_{\mathrm{m}}(\boldsymbol{\xi}) = \sum_{i=0}^{N} a_{\mathrm{mi}} \Psi_{\mathrm{i}}(\boldsymbol{\xi}), \tag{7}$$

wherein $[a]_{\mathrm{mi}}$ are unknown deterministic coefficients for gPC expansion for m^{th} eigenfrequency f_{m}. The collocation based least square minimization technique is adopted here by forcing the error to be zero at predefined collocation points to evaluate $[a]_{\mathrm{mi}}$. The responses at the predefined collocation points are evaluated by realizing the deterministic system responses. The numbers of collocation points should be equal to at least numbers of unknown coefficients. To reduce the computational time $N+1$ combination of collocation points are selected as suggested in [10]. Herein, a 2nd order gPC expansion method is adopted to represent uncertain input parameters. Therfore, $N = 10$ numbers are unknown coefficients are determined by realizing 10 numbers of system response at the predefined collocation points. Once a_{mi} are known, mean μ_{f_m} and variance $\sigma^2_{f_m}$ of the modal eigenfrequency can be determined as

$$\mu_{f_m} = a_{\mathrm{m0}}, \qquad \sigma^2_{f_m} = \sum_{i=1}^{N} a^2_{\mathrm{mi}} \left\langle \Psi_{\mathrm{i}}(\boldsymbol{\xi})^2 \right\rangle. \tag{8}$$

Numerical examples

In this section, uncertainty analysis of the single-fold IM7-PEEK composite plate in thermal environment is done by using 2nd order gPC expansion method. Stochastic free vibration analysis of single-fold cantilever folded-plate structure of (0°/90°/90°/0°) laminate has been carried out at temperature of 25°C (room temperature), 75°C, and 125°C due to corresponding temperature dependent random elastic parameters. The geometry of the cantilever folded plate of $L = 1200$ m has been shown in Figure 1. For FE analysis each plate of the folded plate structure with crank angle α has been discretized into 8×4. In order to validate present folded plate formulation, frequencies are compared with the results obtained by Niyogi et al [8] and subsequently developed ANSYS simulation in Table 2. The dimensions of the single-folded plates and corresponding material properties are given in [8]. The results are closely matched. For the SFE analysis, temperature dependent random elastic properties of the IM7-PEEK composite plate are adopted and are given in Table 1. Here, elastic moduli such as E_{11}, E_{22}, and G_{12} of the composite lamina are considered as random input parameters, and follow the Gaussian distribution. The random input parameters are represented by 2nd order gPC expansion method by determining coefficient matrix by Galerkin projection technique. The modal eigenfrequency at various temperatures are considered as unknown random output parameters, and approximated by 2nd order gPC expansion as suggested in Eq. 7. A set of collocation points $\{\xi_1^{(\mathrm{r})}, \; \xi_2^{(\mathrm{r})}, \; \xi_3^{(\mathrm{r})}\}$, $\mathrm{r} = 1, 2, ...27$ are generated from the roots of 3rd order Hermite polynomials, i.e. 0, $-\sqrt{3}$, and $\sqrt{3}$. This leads to generation of $3^3 = 27$ sets of collocation points, from which 10 set of collocation points are selected to evaluate 10 unknown coefficient, a_{mi}. The selection of collocation points is such that the information matrix became full-rank matrix. The eigen value solution of Eq. 4 is performed on pre-selected collocation points to realize the modal eigenfrequency at each collocation points. The responses at the collocation points are used to estimate the unknown coefficients a_{mi} by least-square minimization technique. Subsequently, mean and standard deviation (SD) of the random eigenfrequency can be evaluated following the Eq. 8 at each temperature. The mean and SD of first three eigenfrequency for single-fold composite plate with crank angles 90° and 120° are presented in Table 3. From Table 3, a reduction of mean eigenfrequency with the increase of temperature is observed for both crank angles. However, stochastic free vibration analysis at a temperature of 125°C for crank angle 90° is not possible, as the folded-plate structure has already reached critical bucking temperature prior to 125° C. The probability distribution functions (PDFs) of first three eigenfrequencies for crank angle 90° at a temperature 25° C and 75° C are presented in Figures 2 and 3. A sharp reduction of first mean eigenfrequencies are observed, whereas, corresponding SDs are increasing with the increment of the temperature. The 2nd and 3rd mean natural frequencies are reducing with the increment of the temperature in much slower rate than

Table 1: Mean elastic properties of IM7-PEEK lamina at different temperatures, cf. [11], $G_{13} = G_{12}$, $G_{23} = 0.5G_{12}$.

Elastic properties	Temperature		
	25 °C	75 °C	125 °C
E_{11} (μ, σ) [GPa]	(160.9, 32.18)	(159.7, 31.94)	(158.6, 31.71)
E_{22} (μ, σ) [GPa]	(9.7, 1.93)	(8.9, 1.79)	(8.2, 1.64)
G_{12} (μ, σ) [GPa]	(7.7, 1.54)	(6.6, 1.34)	(5.4, 1.09)
μ_{12}	0.289	0.282	0.275
α_1, α_2 [\°C]	-1.26×10^{-7}, -3.14×10^{-5}		
ρ [kg/m^3]	1578		

Table 2: Comparison of eigenfrequency (Hz) of cantilever single-fold structure with crank angles 90° and 120°.

Crank angle	Mode	$30°/-30°/-30°/30°$			
	m	Niyogi et al [8] λ_m	Present λ_m	Present f_m (Hz)	ANSYS f_m (Hz)
90°	1	0.0394	0.0389	28.99	28.98
	2	0.0719	0.0801	59.64	59.62
	3	0.1488	0.1368	101.93	101.87
120°	1	0.0394	0.0389	29.02	29.00
	2	0.0703	0.0766	57.09	57.02
	3	0.1488	0.1366	101.75	101.68

Non-dimensional frequency, $\lambda_i = \omega_i L\sqrt{\rho(1-\nu^2)}$ and $\omega_i = 2\pi f_i$.

first mean eigenfrequencies, and the variation of the corresponding SDs are not much significant with the temperature. Similarly, the mean first natural frequencies are decreasing with the increment of the temperature for the folded plate with crank angle 120°, however the corresponding SDs are increasing with temperatures. The PDFs of first three eigenfrequencies for single fold-plate with crank angle 120° are presented in Figures 4, 5, and 6 at temperatures 25° C, 75° C, and 125° C, respectively.

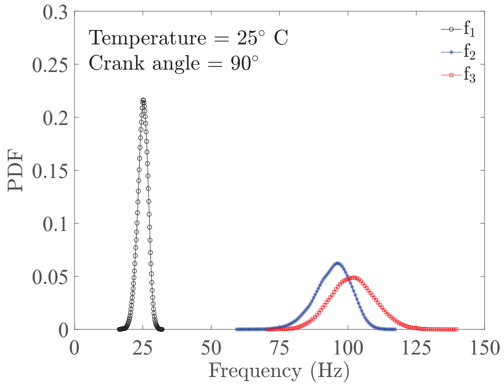

Figure 2: PDF of first three natural frequencies (Hz) using 2nd order gPC expansion for (0°/90°/90°/0°) composite folded-plate with 90° crank angle at 25° C.

Conclusion

The stochastic finite element method is adopted to study the stochastic dynamic response of the single-fold IM7-PEEK composite structure in various thermal environment. The temperature dependent elastic properties of the laminate are considered as random input parameters. The gPC expansion method has been used as a surrogate to evaluate the random eigenfrequency of the structure at various temperature level. The collocation based gPC expansion method is efficiently used to determining unknown coefficient of the frequency response by deterministically solving FE model at predefined collocation points. The results have shown that uncertainty of random parameters have strong impact in the frequency response of the folded plate structure at various temperature. The mean eigenfrequencies are reducing with the increment of the temperature. Moreover, crank angle of the folded-plate structure have a influence on the stability of the structure at the higher temperature. The single-fold plate with crank angle 90° shows instability at

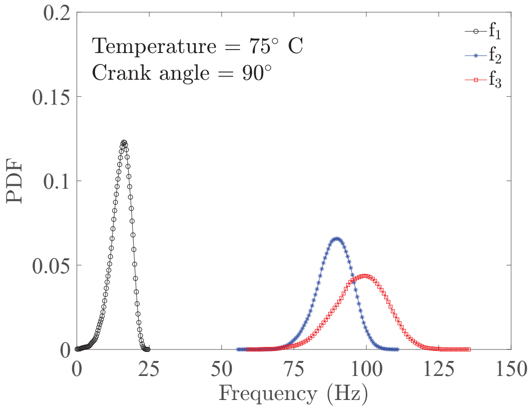

Figure 3: PDF of first three natural frequencies (Hz) using 2nd order gPC expansion for (0°/90°/90°/0°) composite folded-plate with 90° crank angle at 75° C.

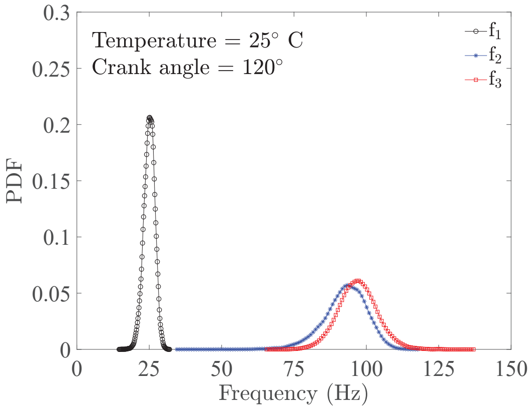

Figure 4: PDF of first three natural frequencies (Hz) using 2nd order gPC expansion for (0°/90°/90°/0°) composite folded-plate with 120° crank angle at 25° C.

Table 3: Mean and standard deviation (SD) of eigenfrequency (Hz) of (0°/90°/90°/0°) laminated cantilever single-fold structure at various temperatures with crank angles 90° and 120°.

Crank angle	Mode m	Temperature					
		25 °C		75 °C		125 °C	
		μ_{f_m}	σ_{f_m}	μ_{f_m}	σ_{f_m}	μ_{f_m}	σ_{f_m}
90°	1	25.0	1.9	15.2	3.3	-	-
	2	94.6	6.7	89.1	5.9	-	-
	3	101.9	8.1	98.0	9	-	-
120°	1	25.0	1.9	20.3	2.1	14.7	2.8
	2	92.7	7.5	89.2	6.8	84.6	6.0
	3	97.0	6.6	93.1	6.6	89.6	7.2

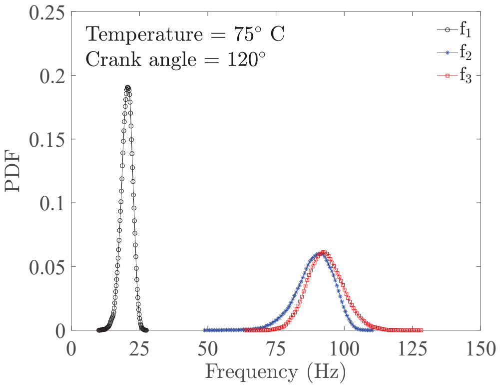

Figure 5: PDF of first three natural frequencies (Hz) using 2nd order gPC expansion for (0°/90°/90°/0°) composite folded-plate with 120° crank angle at 75° C.

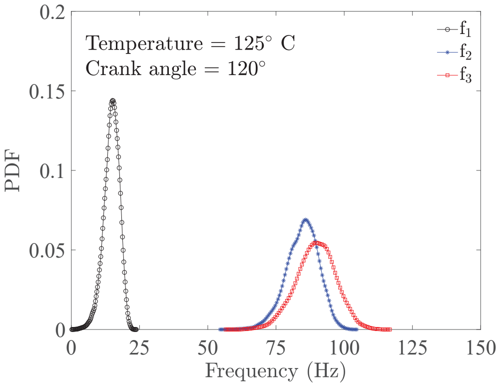

Figure 6: PDF of first three natural frequencies (Hz) using 2nd order gPC expansion for (0°/90°/90°/0°) composite folded-plate with 120° crank angle at 125° C.

125° C temperature. It is also observed that, dispersion of the first natural frequencies are increasing with the increment of the temperature. Therefore, this study shows the necessity of the uncertainty analysis of the folded plate structure with various crank angles when they are exposed to the thermal environment during it's life circle.

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