Modelling and Optimizing Acoustic Diffusers Using Finite-Difference Time-Domain Method

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Introduction

In room acoustics design, the use of acoustic diffusers for scattering sound reflections in an enclosed space is essential in terms of reducing artefacts and improving room acoustics quality. The scattering performance of a diffuser is largely determined by its surface geometry, and finding a geometrical pattern that produces maximum diffusion has been a significant concern of acousticians. Here, maximum diffusion means sound energy is scattered in all directions as evenly as possible. Compared to traditional sound scattering experiments which are often restricted by physical limitations in space and time, numerical room acoustics modelling techniques, such as the Finite-Difference Time-Domain (FDTD) method [2], offer an opportunity for fast and accurate measurement and optimization processes.

The objective of this study is:

- to investigate the scattering effect of sound diffusers through numerical room acoustics simulations,
- to look for optimal diffuser geometry based on existing diffuser design theories.

Quadratic Residue Diffusers (QRDs), after introduced by Schroeder[1] in the 1970s, have been widely accepted as 'optimum'. A QRD consists of wells of different depths, whose values are decided by the quadratic residue sequence:

$$s_n = n^2 \bmod N \tag{1}$$

where N is a prime number, n is the sequence index from 0 to $N-1$, $n^2 \bmod N$ is the least non-negative remainder. Such sequences are periodic with period N, and symmetric around $n = 0$ and $n = (N-1)/2$. For example, if $N = 7$, the first period of the sequence is $s_n = \{0, 1, 4, 2, 2, 4, 1\}$.

QRDs scatter sound waves through both edge diffraction and diffusive reflection on their lumpy surfaces. Distribution of sound energy external to the diffuser is determined by the interference of radiating waves from different wells. Well depths of a QRD with N wells per period are determined by:

$$d_n = \frac{\lambda_0}{2}\frac{s_n}{N} \tag{2}$$

λ_0 is the design wavelength corresponding to the design frequency f_0 ($\lambda_0 = \frac{c}{f_0}$). f_0 is set as a lower frequency bound of the QRD's operational bandwidth. In other words, the QRD produces optimum diffusion for wavelengths smaller than the maximum wavelength λ_0. λ_0 is bounded by the maximum well depth d_{max} of the QRD:

$$\lambda_0 = \frac{2Nd_{max}}{n_{max}} \tag{3}$$

For wavelengths over this limit, the diffuser behaves similarly to a plane surface as the wells would be too shallow to scatter the sound waves [7].

On the other hand, there is also an upper frequency limit f_{max}, whose corresponding wavelength λ_{min} is determined by the well width w:

$$\lambda_{min} \approx 2w \tag{4}$$

Sound waves with a wavelength lower than λ_{min} break down in the wells, and plane wave propagation does not dominate within the wells anymore [7].

QRDs are considered optimal since their invention because they produce good diffusion over wide bandwidths in a predictable manner. However, due to the numerical characteristics of the quadratic residue sequence, the most primary drawbacks of QRDs are the critical frequency phenomenon and the paradox of periodicity. Details can be found in [5] and [3]. In other words, the performance of QRDs is constrained by their geometrical design, so that they can not act as optimal in terms of effective bandwidth and dispersion uniformity as desired. Therefore, it is possible to improve their geometry through optimization processes.

Methodology

Sound diffusion simulation and measurement

Firstly, a two-dimensional room acoustics model for measuring sound diffusion is built in MATLAB using the FDTD method (Fig. 1).

For comparing and improving the design of a diffuser, it is essential to quantify the spatial distribution of reflections from it. *Polar responses* of a diffuser can be obtained by placing a number of receivers on a polar arc surrounding the diffuser. After the enclosing space is excited with a signal, the receivers record the reflections from the diffuser. Sound energy of the recorded signals is then calculated and plotted against receiver positions either in the Cartesian coordinates or the polar coordinates. Generally, polar responses vary with different frequency bands and incidence angles.

Polar responses offer an intuitive assessment of the scattering from a diffuser, but with the vast amount of information displayed, they mask the absolute quality of the diffuser. For precise comparison and prediction of sound diffusion, it is more desirable to have a single figure of merit that characterizes the diffusers' behaviour. Therefore, what is known as the *diffusion coefficient* is derived from polar responses.

The expression of a diffusion coefficient as defined in the AES standard, AES 4id 2001 [4], is:

$$d_\theta = \frac{(\sum_{i=1}^{m} 10^{\frac{L_i}{10}})^2 - \sum_{i=1}^{m}(10^{\frac{L_i}{10}})^2}{(m-1)\sum_{i=1}^{m}(10^{\frac{L_i}{10}})^2} \quad (5)$$

where θ is the angle of incidence, L_i is the sound pressure level of the impulse response at the i^{th} receiver in decibels, m is the total number of receivers. It is a value between 0 and 1, 0 for no diffusion and 1 for full diffusion.

For either experimental or numerical measurements of sound scattering, it is necessary to isolate the diffusive reflections from the direct sound from the source and reflections from room surfaces. As an anechoic condition can not be achieved with this FDTD model, two simulations are needed for obtaining one polar response. The first simulation runs with the diffuser placed at the center of the room and the second one runs with the diffuser removed. The reflected signal H from the diffuser are then extracted by subtracting the second impulse response h_2 from the first one h_1:

$$H = h_1 - h_2$$

In real-life measurements, time windowing is also required for separating the diffusive reflections from the direct sound, but numerical measurements saves the effort as the direct sound is also eliminated by this subtraction. Sound pressure values of H can then be used to calculate the diffusion coefficient.

Diffuser geometry optimization

Principles

Before considering optimization methods, it is necessary to define the disciplinary form of a numerically optimized diffuser, as well as the associated bounds on its geometry.

First of all, optimization principles are based on the control variates method. The geometry of the optimized diffuser should be QRD-like. Due to the fin-related drawback of QRDs as discussed by [3], as well as the limitation in spatial resolution of the FDTD model, a geometrical approximation of the QRDs without fins, i.e. a 'stepped QRD', is taken as the initial geometry to optimize (Fig. 2).

Secondly, the well width, maximum well depth and the overall length of the optimized diffuser should be the same as those of the corresponding QRD so that better

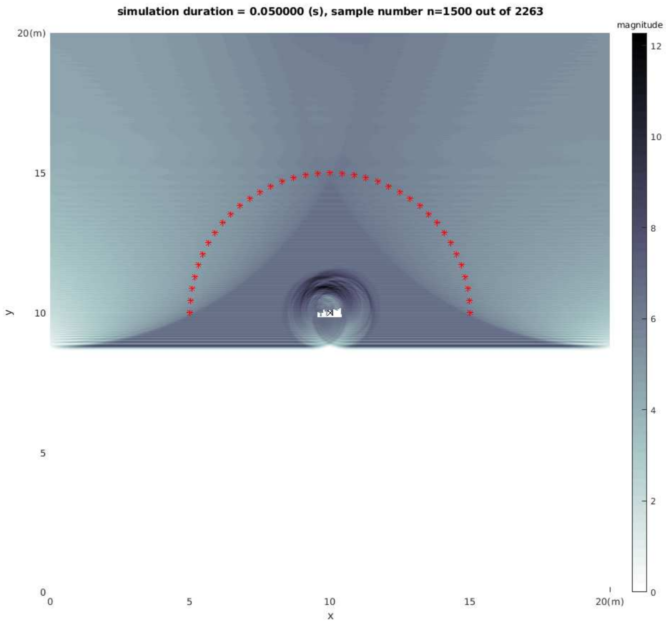


Figure 1: Snapshop of the FDTD room acoustics model in MATLAB, excitated by a linear arrangement of impulse sources to imitate a plane wave. The diffuser is placed at the center of the room. The red asterisks mark the receiver locations.

diffusion is considered to be owing to the specific combination of well depths. The design frequency is set to 500Hz. The well width is set at 5cm, so the theoretical f_{max} of the reference QRD is around 3400Hz. The sample rate for simulation is set to 45255Hz for accuracy reasons.

Finally, the optimized diffuser should not rely on periodic geometry, namely it would be a single-period diffuser.

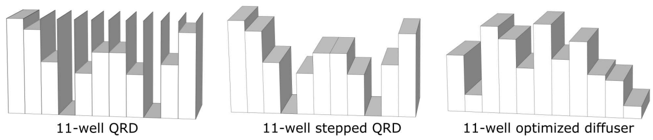


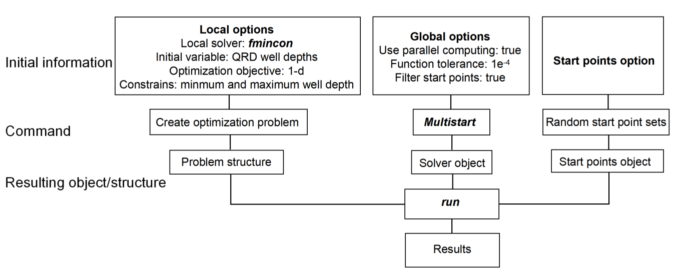
Figure 2: Illustration of diffuser types

This project only measures and optimizes normal-incidence diffusion coefficients. Therefore, the results are valuable to those whose major concern is to deal with first-order reflections in a venue where the sound source is most likely to be at the front.

For testing the reliability of measurements taken by the numerical room acoustics model, normal-incidence diffusion coefficients of a plane surface are compared to published data from [3], which were predicted by the 2-D Boundary Element Modelling method. The two agree with each other, thus the validity of the model is verified.

Procedure

The optimization criteria is to numerically alter the well depth sequence until a maximum diffusion coefficient, which represents largest degree of diffusion, is found. Using the 2-D FDTD simulation engine, numerical optimization is carried out for six different lengths (7, 11, 13, 17, 19, 23 wells) on the full frequency range (wideband) of an impulse excitation, and for four different lengths (7, 11, 13, 17 wells) on each 1/3-octave band from 500Hz to 4000Hz (the reflected signals are filtered by 1/3-octave filters before the calculation of sound pressure levels), according to the following workflow:



Results

Wideband-optimized diffusers

Table 1: Wideband diffusion coefficients

Number of wells	Diffuser type		
	stepped QRD	plane surface	wideband-optimized diffuser
7	0.68	0.51	0.81
11	0.64	0.50	0.84
13	0.67	0.50	0.83
17	0.66	0.48	0.74
19	0.61	0.48	0.70
23	0.59	0.48	0.67

According to Table 1, the wideband-optimized diffusers dramatically outperform the other two types of surfaces over the full frequency range of the excitation signal. A prominent feature is that longer surfaces produce less diffusion. In particular, the superiority of optimized diffusers with 7, 11 and 13 wells is more obvious, as the increase of diffusion coefficient compared to stepped QRDS is between 0.13 to 0.20, while the increase for the longer diffusers is 0.08 - 0.09. To investigate these diffusers' performance at individual frequency bands, frequency-diffusion coefficient curves are plotted in Fig. 3. Note that only those points marked by solid dots are actually measured values. Generally, all these curves descend as frequency increases, and almost all of the optimized diffusers displayed here produce better diffusion than stepped QRDs below approximately 1000 Hz. Beyond that, the wideband-optimized diffusers do not necessarily show superiority over the stepped QRD at every individual frequency bands.

Fig. 4 demonstrates the polar responses of wideband-optimized diffusers with 7 and 19 wells, again with comparison to stepped QRDs and plane surfaces. Contrasting them to the data in Fig. 3, we find that a more 'uniform' polar response, such as that of 7 well - 500 Hz and

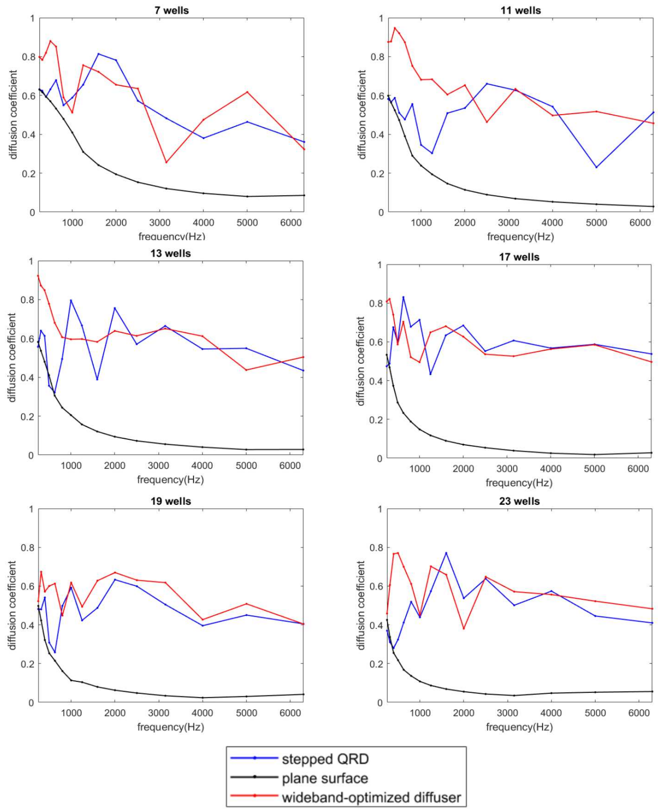


Figure 3: Diffusion coefficient curves

19 well - 2000 Hz, corresponds to a higher diffusion coefficient, even if the shape of the polar plot is bumpy. This accords with the evaluation criteria in previous discussion. If obvious notches are present in the polar response, the diffusion coefficient would be reduced, such as the case for 19 well - 500Hz and 7 well - 2000 Hz. For 7 well - 4000 Hz and 19 well - 4000 Hz, as the SPLs at the sides (-90°and 90°) of the diffusive surface are fairly small, the diffusion coefficients are also suppressed. Another feature to note is that with the same excitation source, more energy is reflected back by the longer surfaces (19 wells) than the shorter ones (7 wells).

1/3-octave-optimized diffusers

Table 2 displays the maximum diffusion coefficient values at individual 1/3-octave bands found by the optimization engine. For short diffusers and low frequencies, fairly high diffusion coefficients (over 0.9) could be achieved. At frequencies over 2500 Hz, the best performances of these diffusers are not so outstanding, with the highest diffusion coefficient of around 0.7.

Fig. 5 demonstrates the diffusion coefficient curves of 1/3-octave-optimized diffusers across the frequency range from 500 to 4000Hz. It is obvious that the target frequency for optimization is not necessarily where highest diffusion in the whole frequency range happens, but the optimized diffusers do produces better diffusion than the others at their target frequencies. Overall, although these diffusers are optimized for specific frequencies, they behave satisfyingly across a wide frequency range despite one or two low values.

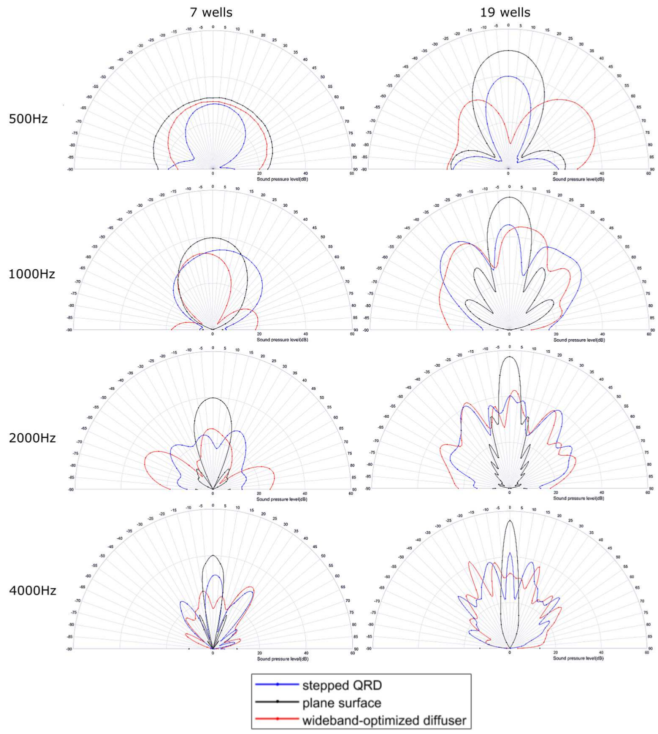


Figure 4: Polar plots of wideband-optimized diffusers

Table 2: Diffusion coefficients produced by 1/3-octave-optimized diffusers

Number of wells	1/3-octave center frequency (Hz)				
	500	630	800	1000	1250
7	0.88	0.89	0.91	0.91	0.76
11	0.96	0.87	0.80	0.78	0.87
13	0.87	0.74	0.86	0.81	0.77
17	0.87	0.83	0.74	0.82	0.71
	1600	2000	2500	3150	4000
7	0.81	0.78	0.70	0.59	0.68
11	0.75	0.68	0.76	0.67	0.59
13	0.71	0.82	0.68	0.70	0.62
17	0.77	0.75	0.70	0.64	0.61

Conclusion

This paper presents a numerical technique for optimizing the geometry of acoustic diffusers, utilising a 2-D wave-based FDTD simulation engine coded in MATLAB. The results reveal that shorter QRD-like diffusers tend to behave better than longer ones, but when choosing the diffuser size, the preferred amount of reflected energy should also be taken into consideration. Scattering for low frequencies benefits the most from geometry optimization, and it is more difficult to produce good diffusion at high frequencies. The optimization engine is parameterized, therefore, it is ready for application in optimization of stepped diffusers under different conditions according to the users' demand.

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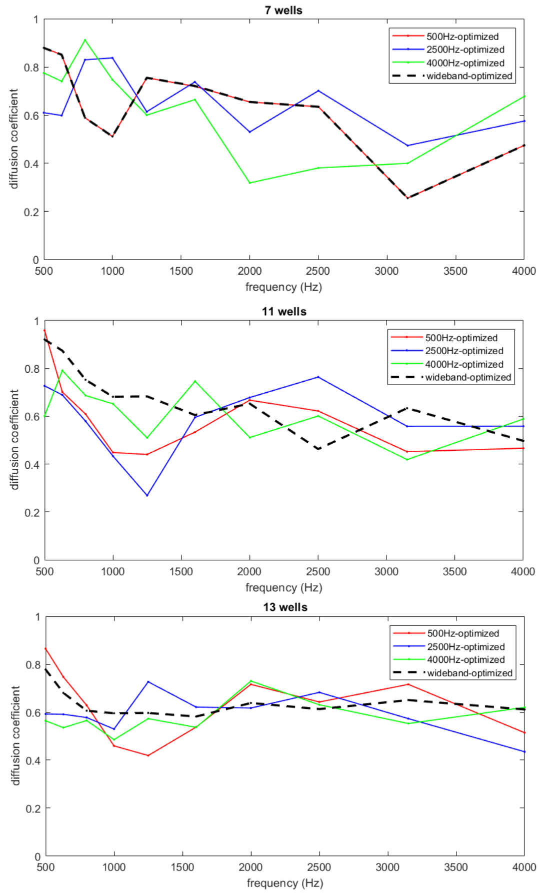

Figure 5: Diffusion coefficients of 1/3-octave-optimized diffusers at different frequencies.