Modeling the Body of a Classical Guitar Using the Finite Element Method and Experimental Modal Analysis

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Introduction
The classical guitar is not only fascinating musicians and audiences worldwide, but also scientists are interested in the interplay of the coupled oscillatory systems that make up the instrument. A numerical finite element model of a classical guitar body enhanced by experimental data is presented in this contribution. The very detailed model includes all necessary physical influences, and the guitar’s geometry is reverse-engineered from computed tomography scans. Five different wooden materials used in the guitar are modeled with their corresponding anisotropic material properties. Furthermore, the fluid-structure interaction between the guitar body and the enclosed air volume is considered in the coupled finite element model. Moreover, a setup for an experimental modal analysis employing a scanning laser Doppler vibrometer is proposed, and modal parameters are identified utilizing the complex mode indicator function and enhanced frequency response functions. Finally, the experimentally identified modal parameters are used to improve the numerical model via a model updating procedure in which the properties of the wooden materials are identified. The model updating procedure includes a highly parallelized sampling procedure to cover the ample parameter space. The results substantiate the numerical model and, in addition, enhance the knowledge about the instrument by delivering the beforehand only coarsely known material properties.

Experimental Modal Analysis
In this contribution, a mid-priced guitar of type Yamaha GC-12 with a cedar top and made of all solid woods is used for the investigations. Firstly, an experimental modal analysis is conducted on the instrument to identify the modal parameters, i.e., eigenfrequencies, eigenmodes, and modal damping ratios of the instrument. These modal parameters will be used as a reference solution for the numerical model developed in the further sections. Figure 1 depicts the experimental setup used for the experimental modal analysis. The guitar is suspended by soft springs to create approximately free boundary conditions, with the eigenfrequencies of the approximate rigid body movement being well below the eigenfrequencies of the guitar. An automatic impulse hammer excites the guitar reproducibly up to 1000 Hz with maximum forces of 2.5 N measured by an integrated circuit piezoelectric sensor. A Polytec PSV-500 scanning laser Doppler vibrometer is used to measure the velocity on the guitar’s surface. The experiment is designed such that the guitar can be turned around, ensuring an excitation and a measurement of the front and the back of the guitar. This is necessary since the modes of the guitar will be only unambiguous if both the oscillation on the front and the back of the guitar are measured.

During the experiment, the mobility

\[ Y_{jk}(\omega) = \frac{V_j(\omega)}{F_k(\omega)} \]

is calculated for 452 input-output combinations with the Fourier transformed velocity \( V_j(\omega) \) at position \( j \) and the Fourier transformed input force \( F_k(\omega) \) at position \( k \). Two input positions and 226 output positions are used. For the identification of the modal parameters, a method based on the complex mode indicator function (CMIF) and enhanced frequency response functions (EFRF) is used. In short, the CMIF extracts the singular values of the matrix of mobilities and, hence, shows peaks at all eigenfrequencies. The left eigenvectors at these eigenfrequencies can then be used to project the matrix of mobilities into the modal domain to form an EFRF for each eigenfrequency. A standard least-squares method can then be used to identify the modal parameters from the EFRFs. The method is advantageous, firstly, because it uses the large amount of data gathered in the experiment and, secondly, because even modal parameters of peaks very close to each other can be reliably identified.

In Figure 2 the magnitude and the phase of a mobility excited on the bottom right corner of the sound-
The excellent agreement between reconstruction and measurement proves the success of the parameter identification. A thorough description of the experimental setup and the application of the modal parameter identification method can be found in [1]. The modal parameter identification method is discussed in detail in [2].

The modal assurance criterion (MAC)

\[ \text{MAC}_{ij} = \frac{\sum_{k=1}^{N} \psi_{ik}^{H} \phi_{kj} \phi_{kj}^{H} \psi_{ik}}{\sum_{k=1}^{N} \sum_{l=1}^{N} \psi_{ik}^{H} \psi_{il} \phi_{lj}^{H} \phi_{lj} \psi_{il}} \]

is shown. The excellent agreement between reconstructed and measured mobility with magnitude and phase is depicted. The measurement is taken between the soundhole and the bridge with an excitation on the bottom right corner of the soundboard.

**Finite Element Model**

In the following, the finite element model of the guitar investigated in the experiment is presented briefly. A sophisticated guitar model contains three main ingredients. First and foremost, the geometry of the guitar needs to be modeled in a high level of detail, including the bracing pattern and the shape of the braces. Secondly, the fluid-structure interaction between the guitar body and the enclosed air needs to be taken into account, and, finally, the orthotropic material properties of the different parts of the guitar have to be approximated.

A detailed geometry model is created from computed tomography (CT) scans, which could be made with the help of the Department of Diagnostic and Interventional Radiology of the Klinikum Stuttgart. The CT scans were created with a medical CT scanner of type Siemens SOMATOM, see Figure 3. On the foundation of the CT scan images detailed models of all parts were created, meshed, and then bound together with the commercial FE software Abaqus. In Figure 4 the resulting FE model is illustrated with different parts in different colors, and the different materials of the parts are indicated. Furthermore, linear volume elements of type C3D8 and linear shell elements of type S4 are used as specified in Figure 4.

The whole volume inside the guitar body is discretized with linear acoustic finite elements of type AC3D8 to include the influence of the air inside the guitar box on the structural dynamics. On the surface of the soundhole, radiating boundary conditions are applied by utilizing acoustic infinite elements of type ACIN3D4. The system of equations for the undamped system dynamics without external forces results in

\[
\begin{bmatrix}
M_F & \rho_F C_{fsi}^T \\
0 & M_S
\end{bmatrix}
\begin{bmatrix}
\ddot{p} \\
\dot{q}
\end{bmatrix}
+ \begin{bmatrix}
K_F & 0 \\
-C_{fsi} & K_S
\end{bmatrix}
\begin{bmatrix}
p \\
q
\end{bmatrix} = 0
\]

with the mass matrix \( M \) consisting of the mass matrix of the fluid \( M_F \), the mass matrix of the solid \( M_S \), and the coupling matrix \( C_{fsi} \) multiplied with the density of the fluid \( \rho_F \). The stiffness matrix \( K \) further contains the stiffness matrix of the fluid \( K_F \) and the stiffness matrix of the solid \( K_S \), and the vector of generalized coordinates \( \mathbf{q} \) consists of the pressure degrees of freedom \( \mathbf{p} \) and the translations and rotations of the structural nodes \( \mathbf{u} \).

In a first iteration, the orthotropic material parameters for the different wooden materials used in the guitar are taken from [3]. Alas, the material properties of wood are known to vary largely between different specimens and, thus, they are assumed not to be accurate. Nevertheless, the eigenmodes of the initial model proposed in this section shall be compared with the experimentally identified ones. The modal assurance criterion (MAC)

\[
\text{MAC}_{ij} = \frac{\left| \psi_{i}^{H} \phi_{j} \right|^2}{\psi_{i}^{H} \psi_{i} \phi_{j}^{H} \phi_{j}}
\]

is taken between the soundhole and the bridge with an excitation parameter identification method can be found in [1]

![Figure 2: Measured and reconstructed mobility with magnitude and phase](image1)

![Figure 3: Photo of the guitar while being scanned with a Siemens SOMATOM medical CT scanner.](image2)

![Figure 4: The detailed geometric model resulting from the CT scan images](image3)

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which uses a normalized scalar product of experimental modes $\psi_i$ and modes of the numerical model $\varphi_j$ to compare them. The values of the MAC can be arranged in a matrix and displayed as a heatmap as done in Figure 5, where numerical modes and experimental modes are compared. Ideally, a unity matrix would indicate a perfect fit. Although this is not the case, the MAC values are pretty good as five of the first six modes can be correlated with MAC values above 0.8. However, especially above the 8th mode, the modes cannot be correlated anymore between numerical model and experiment.

To circumvent these problems, in the first step, the numerical parameters that need to be identified. On top of that, one model evaluation computing the modal parameters of the numerical model takes a recent workation about 5 minutes, and during an identification process, typically thousands of evaluations are necessary. This combination of a large parameter space and high computational costs renders a straightforward identification process impossible.

To circumvent these problems, in the first step, the number of independent parameters is reduced. For each material region $m$ only two parameters

$$p_m = [p_m^{\rho}, p_m^{\text{stiff}}]$$

remain, one for the density ($p_m^{\rho}$) and one for all material parameters influencing the stiffness ($p_m^{\text{stiff}}$). The Young’s moduli in all directions and the shear moduli in all directions for each material region $m$ are modified uniformly by multiplication with the factor $p_m^{\text{stiff}}$. This approach reduces the number of independent parameters to 12.

Figure 6: Workflow for the parameter identification process.

To overcome the problem of high computational costs, the parallelized sampling approach visualized in Figure 6 is applied for the identification. Beginning from the FE model characterized in the previous chapter, 2000 sample input files for the Abaqus solver are created. The parameter samples are calculated from a Sobol sequence. Sobol sequences are quasi-random sequences to form successively finer uniformly distributed points in an s-dimensional hypercube [5]. These sequences can cover a parameter space well already with a small number of parameter samples. A parallelization approach is favorable because the calculation of 2000 samples on one workstation would still need about one week of computational time. Hence, GNU Parallel is used to distribute the evaluation of the sample input files to 19 workstations in the computer network of the institute [4]. The parallelization reduces the computational time to about nine hours, making an overnight computation of all samples possible. Finally, the modal parameters of all parameter samples are gathered on one machine and the objective function

$$\varepsilon = \sum_{r=1}^{R} \left( \frac{\omega_{r,\text{Exp}} - \omega_{r}(p)}{\omega_{r,\text{Exp}}} \right)^2 + \left( \frac{1 - \text{MAC}_{rr}(p)}{\text{MAC}_{rr}(p)} \right)^2$$

is evaluated. The objective function compares the eigenfrequencies $\omega_{r}(p)$ of a sample with there corresponding experimentally identified eigenfrequencies $\omega_{r,\text{Exp}}$ and further penalizes low MAC values on the diagonal with the second term.

The relative parameter changes to the initial parameters for each material region are depicted in Figure 8. A summary of the changes can be given as follows. The neck and fretboard get heavier and less stiff as assumed before, while the soundboard and the bridge get lighter and stiffer. Furthermore, the bracing gets stiffer, and the back and sides get heavier and stiffer. This gives rise to the conclusion that the different parts needed to be tuned relative to each other. When looking at the specific mode pairing displayed in Figure 7, the influence of the parameter changes on the modes is obvious. The most similar mode to the 8th mode in the experiment with the
The paper conveys two preeminent findings. In the first place, the three substantial ingredients for a high-fidelity FE model of a guitar are found to be a detailed geometry model, the fluid-structure interaction between the guitar body and the enclosed air, and identified orthotropic material parameters for the specific analyzed instrument. Secondly, a sophisticated, parallelized model updating procedure based on Sobol sequences to identify the material parameters has been proposed, and it could be shown that the FE model with updated parameters produces very realistic modal parameters up to 500 Hz.

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References