

# Symmetry and asymmetry in guitar bracings and related sound tuning parameters

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## Abstract

The low-frequency range strongly determines sound quality in classical Spanish guitars. In the range above the fundamental air mode  $A0$  at roughly 100 Hz, and below 400 Hz, most guitars reveal one or two main plate/body resonances, only few guitars have up to four resonances. While guitar models come along with similar geometrical dimensions, the existence of multiple resonances traces back to construction: the symmetry/asymmetry of the bracing, the coupling of the top and the back plate, and the geometrical relation of bridge mass to the braces' attack points. The sparse and very asymmetrical bracing in a guitar from Faustino Conde encourages the development of analytical models. A simple 'top-plate-only' model yields analytical solutions for the mentioned low frequency range. Another more complete 'top-and-back-plate' model yields numerical solutions. Both models agree with findings on the guitar and with an experimental setup. The coupling between resonances grows with asymmetry, yielding modulated, lively sounds in asymmetrically braced guitars. All guitars could have multiple resonances independent from aspects of symmetry, but the parametrical space of related factors is wide and sound tuning is more delicate in asymmetrically braced guitars than it is in symmetrically braced guitars.

## I. Introduction

The guitar has already been extensively researched in terms of its resonance box feature (Caldersmith, 1978), its modal properties and coupling (Richardson, 1984), the relation of modes and musical tones played (Richardson, 1994), as well as issues of radiation (Lai and Burgess, 1990; Hill et al., 2004). Lumped models of three to four masses explain the lowest air mode when coupled with the lowest one or two body modes (Christensen and Vistisen, 1980; Popp, 2012; Richardson et al., 2012). A guitar built by Faustino Conde in 1964 represents a progressive construction to combine classical with flamenco style construction, introducing the *flamenco negra*, as the rosewood used is darker than cypress. The guitar has four strong resonances between the air resonances  $A0$  and  $A1$ , where other guitars have just one or two, and it has a unique construction: the fan bracing is inverted in direction and overly asymmetric.

## II. Empirical data

### A. The 1964 Faustino Conde guitar and its sound

The top plate is made of spruce with regions as thin as 1.6 mm. The plantilla, the total size of the top plate including sound hole, is 0.1396 m<sup>2</sup>, and the volume is 12.9 liters. The calculated rigid-body Helmholtz resonance is 137 Hz. Instead of an odd number of braces, Conde used four braces,  $b2$  through  $b5$  in Fig. 1b, without a central brace, and two short supplementary braces outside the bridge region,  $b1$  and  $b6$ . In the center though, instead of a brace, there is only a thin (1 mm) backing for the joint,  $cb$ . The second feature is

the asymmetry. The bar underneath the sound hole, bar  $B1$ , is strongly skewed (21°), and the bracing is inverted. Therefore, the two pairs of main braces  $b2/b3$  and  $b4/b5$  have a considerable length difference.

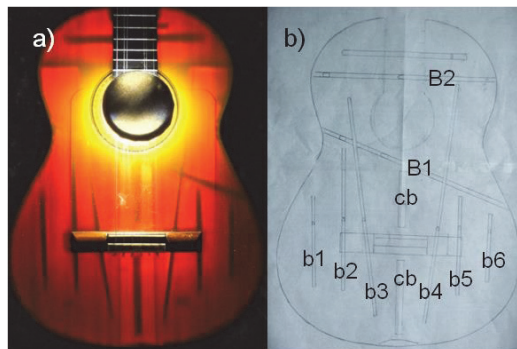


FIG. 1. Photo of the top plate of the guitar made by Faustino Conde in 1964 (a) and its sketched bracing (b). Bars  $B1$  and  $B2$  are 11 mm high and stiffer than the braces  $b1$  through  $b6$ , which are only 5 mm high. The central backing  $cb$  is only 1 mm thick.

### B. Bridge mobility measurement

Mobility measurements were taken according to a matrix of three tapping positions (between pairs of strings at the bridge) and two sensing positions (in proximity to E2 and E4 string attachment). Tapping was done with a nine gram impulse-hammer (DYTRAN 5800) while sensing with 1 gram acceleration probes (Kistler 8778A500).

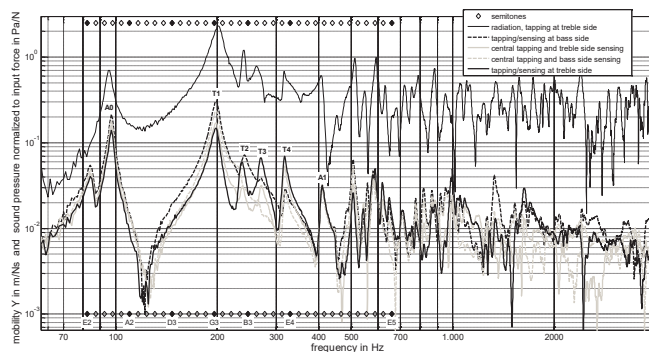


FIG. 2. Sound pressure of the Faustino Conde guitar measured at 1 m distance normalized to input force (upper trace), and bridge mobility (lower traces) measured at two bridge positions, while tapping at three positions at the bridge, revealing four prominent resonances  $T1$  to  $T4$  between the air modes  $A0$  and  $A1$ .

The bridge mobility clearly reveals four strong vibrational modes between the air modes  $A0$  and  $A1$ , Fig 4. Other guitars from a wider field study reveal only one or two prominent modes in the same frequency range. Other publications also report only one or two modes between  $A0$  and  $A1$  (Firth 1977, Fig. 2, Elejabarrieta et al., 2002; Rossing and Eban, 1999), or refer to a 'low-frequency triplet', including  $A0$  (Jansson 1971; Stetson 1981, Richardson 1984). Thus, the Conde guitar reveals a quintet of modes including  $A0$ . The notation will follow earlier work, (Firth 1977, Caldersmith 1978, Chen et al., 2012).

Name	Freq.(Hz)	Description
$A0$	98	fundamental air resonance in the flexible resonance box
$T1$	200	top plate (1, 1) and back plate (1, 1) couple in common phase motion
$T2$	240	top plate (1, 1) and back plate (1, 1) couple in anti-phase motion, a breathing mode
$T3$	277	cross mode (2, 1) of the top
$T4$	317	(1, 3) mode of the top plate, bridge rotating around its longitudinal axis
$A1$	408	(1, 2) air resonance

TABLE 1. Signature modes of the Faustino Conde guitar

$T1$  is usually strong in every guitar, and also  $T2$  appears, most likely as the back plate resonance without quantified coupling to the top plate (Richardson, 1984; Richardson et al. 2012), or  $T3$  develops together with  $T1$  but not  $T2$  (Richardson, 2010). Here,  $T2$  is strong at the top plate and strongly radiates from the top, even though it is the main resonance of the back plate. This paper models  $T1$  though  $T3$  only, as  $T4$  is a rotational mode around the longitudinal axis of the bridge, with its own tuning conditions.

The extensive discourse on the so-called signature modes ( $CBR$ ,  $B1+$ ,  $A1$  and  $B1-$ ) in violin acoustics (Bissinger 2003; Gough 2010) should likewise happen for guitar acoustics. The multiple adjacent resonances deserve such investigation, they are spaced well, wide enough to cover the range from the open  $G3$  to the open  $E4$  string, and close enough to support mutual coupling and thus modulation of sound.

### III. Simple analytical model

In the first place, the analysis begins with the top plate alone, ignoring the ‘inferior’ contribution of the back plate (Richardson 1984; Romanillos, 1997, p. 238). The striking length difference of braces  $b3$  and  $b4$  leads to the question: would such asymmetry allow the bridge to ‘dance’ on its ‘two feet’ at distinguishable frequencies?

The model to begin with is very simple as it only considers the bridge and the braces, ignoring the top plate. The bridge is an object of length  $2R$ , mass  $m$ , and has a general moment of inertia  $J$ . Two springs are attached to the two ends of the object, having the spring constants  $k_1$  and  $k_2$ . See (Mores, 2020) for illustration of a general model, the differential equations and their solution. Solving for  $k_1=k_2=k$ , for two degrees of freedom, up-/down movement and rotation around the center of mass, one obtains only two frequencies, basically related to the two degrees of freedom. This only supports explanations for  $T1$  and  $T3$  but not for  $T2$ . The same model can be used to explore cases of  $k_1 \neq k_2$ , or, in a more general form,  $k_1=k$ ,  $k_2=k\delta$ , allowing  $\delta$  to express a tuning parameter, the spring coefficient difference as a feature of asymmetry. Again, there result only two eigenfrequencies.

$$\omega_{1/2} = \sqrt{\frac{2k}{m} (1 + \delta \pm \sqrt{1 - \delta + \delta^2})} \quad (1)$$

With a closer look to the bridge’s geometry, one has to introduce another tuning parameter. The bridge wings appear to overhang beyond the effective attack point of the braces. Let  $h$  and  $2L$  be the dimensions of the effective rectangle cross-section that represents the same mass and the same moment of inertia as the real bridge in the guitar, Fig. 3a.

The bridge length of 180 mm (height 9 mm at the block and 4 mm at the wings) translates to an effective length of  $2L = 151.9 \dots 160$  mm (at a height  $h$  of 7.4 ... 7 mm). The actual attack points  $R_{b2} = R_{b5} = 85$  mm and  $R_{b3} = R_{b4} = 51$  mm translate to an effective attack point for the pairs  $b2/b3$  and  $b4/b5$ , estimated at  $R = 68$  mm. Therefore, the relation  $\mu = R/L$  is estimated with  $\mu_{min} = 2 \times 68 / 160 = 0.85$ ,  $\mu_{max} = 2 \times 68 / 151.9 = 0.895$ . Measurement results for the Conde guitar will be represented at  $\mu = 0.87$  in illustrations Fig. 3b shows the experimental setup.

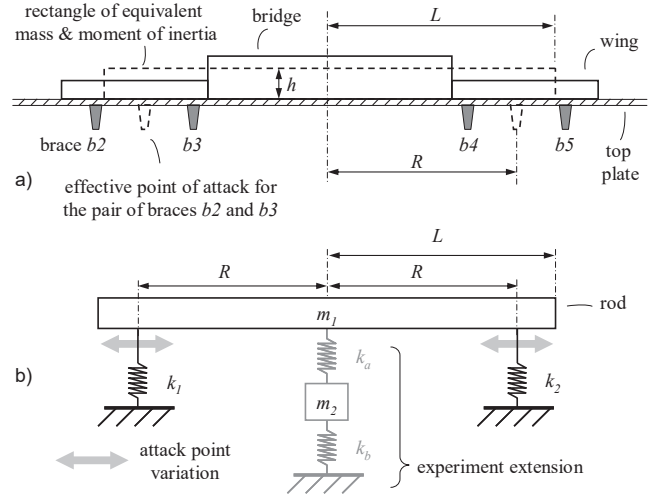


FIG. 3. Geometry of the bridge overhang at (a) the guitar, and (b) in the experiment. Empirical data for the top plate alone (without extension) see Fig. 4, and for top and back plate coupled together (with extension) see Figs. 5 and 6.

The factor  $\mu = R/L$  now represents the location of the spring attachment relative to the body length, or,  $1/\mu$  represents the bridge overhang. Let  $\delta$  now be defined in the sense of  $k_1=k(1+\delta)$  and  $k_2=k(1-\delta)$ , the eigenfrequencies are,

$$\omega_{u/l} = \sqrt{\frac{k}{m} (1 + 3\mu^2 \pm \sqrt{(1 + 3\mu^2)^2 - 12\mu^2(1 - \delta^2)})} \quad (2)$$

An upper  $\omega_u$  and a lower  $\omega_l$  are denoted here because the modes couple and there is no strict correspondence to up/down motion, or rotation. Fig. 4 illustrates this analytical result and compares it with empirical results. The different traces indicate various relative spring constants, expressed by  $\delta$ . Measured data result from experiment with a metal rod of negligible diameter with  $2L=30$  cm,  $m=0.12$  kg, attached to two springs at the position  $\mu$  relative to its length. The spring constants are  $k_1=2.8$  N/cm, and  $k_2=1.4$  N/cm.

An important finding is that frequencies can be tuned by factors  $\delta$  and  $\mu$ . Another very important result for the Conde guitar is that the two modes couple only if the springs’ constants are different,  $\delta \neq 0$ . At  $\delta=0$  the two modes, up/down and rotational, superimpose without coupling, and without modulation. At  $(1+\delta)/(1-\delta)=2$  the two modes couple. Even more important, the two modes of vibration are an up/down motion locked with rotation at  $\omega_u$  at the side of the harder spring, and, superimposed, an up/down motion locked with rotation at  $\omega_l$  at the side of the weaker spring. The center of rotation for these two modes is not the center of the bar, or rod, but the respective opposite end. This is an important advantage in terms of radiation as the vibration has the characteristics of a monopole rather than that of a dipole.

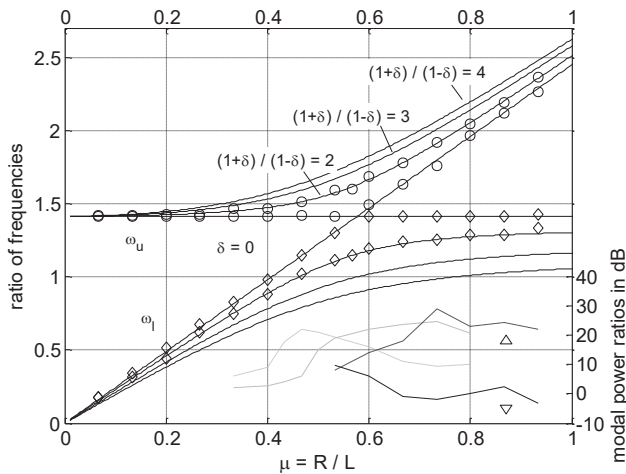


FIG. 4. Main set of traces: ratio of frequencies for a bar of length  $2L$  and unit mass  $m$  when attached to a pair of springs connected at distance  $R$  from its center, according to Eq. (2). Spring constants deviate from unity by  $\delta$  to represent spring constant ratios of 2, 3, and 4. Empirical data results from using a 30 cm long rod,  $m=0.12$  kg, and two types of springs,  $k_1=280$  N/m,  $k_2=140$  N/m, according to Fig. 3. Secondary set of traces: modal power ratio (gray) of  $\omega_u$  over  $\omega_l$  when tapping at the rod's end with the spring  $k_1=1.33k$  (solid) and of  $\omega_l$  over  $\omega_u$  when tapping at the rod's end with the spring  $k_2=0.67k$  (dashed); power ratio (black) of  $\omega_u$  over  $\omega_l$  when tapping at a position corresponding to the treble tapping position at the bridge (solid) and of  $\omega_l$  over  $\omega_u$  when tapping at a position corresponding to the bass tapping position at the bridge (dashed); power ratios for the Conde guitar as extracted from bridge mobility measurement for  $T1$  ( $\Delta$ ) and for  $T3$  ( $\nabla$ ).

#### IV. Advanced analytical model

The complete system for the signature modes  $T1$  through  $T3$  can also be modeled together. The additional tuning parameter  $\xi$  is introduced to because a spring  $k_a$  represents the coupling via the ribs, while  $\xi$  will relate  $k_a$  to unit spring coefficient,  $k_a=\xi k$ . See (Mores, 2020) for the matrix. The resulting complete matrix

$$\det \begin{bmatrix} m_1\lambda^2 + k(2 + \xi) & -\xi k & 2\delta k R \\ -\xi k & m_2\lambda^2 + \xi k + k_b & 0 \\ 2\delta k R & 0 & J\lambda^2 + 2kR^2 \end{bmatrix} = 0 \quad (3)$$

delivers numerical results for the eigenfrequencies. Two of the tuning parameters are directly represented by  $\delta$  and  $\xi$ , with  $\xi k$  representing the ‘spring’ between the masses  $m_1$ , top plate, and  $m_2$ , back plate. The third tuning parameter,  $\mu=R/L$ , is only indirectly contained in this matrix. The general moment of inertia  $J$  incorporates the length  $2L$  of the rotating bridge,  $J=m_1 L^2/3$ , while  $R$  reflects the effective attack point of the two springs  $k_1+k_2=2k$  at distance  $R$  from the center of the rotating body. The braces at the back plate are represented by the spring constant  $k_b$ .

The numerical results for the modal frequencies in this system are illustrated in Fig. 5. To reduce the dimensional space and focus on the tuning parameters, let  $m_1 = m_2 = m =$  unit mass, and  $k_1 + k_2 = 2k = k_b = 2$  unit springs. One important result from the analysis is that in symmetric guitars the rotational and the anti-phase mode do not couple with each other. This has an effect on sound: there is no mutual modulation for those who prefer a lively tone. In the asymmetric guitar, on the other hand, the modes couple.

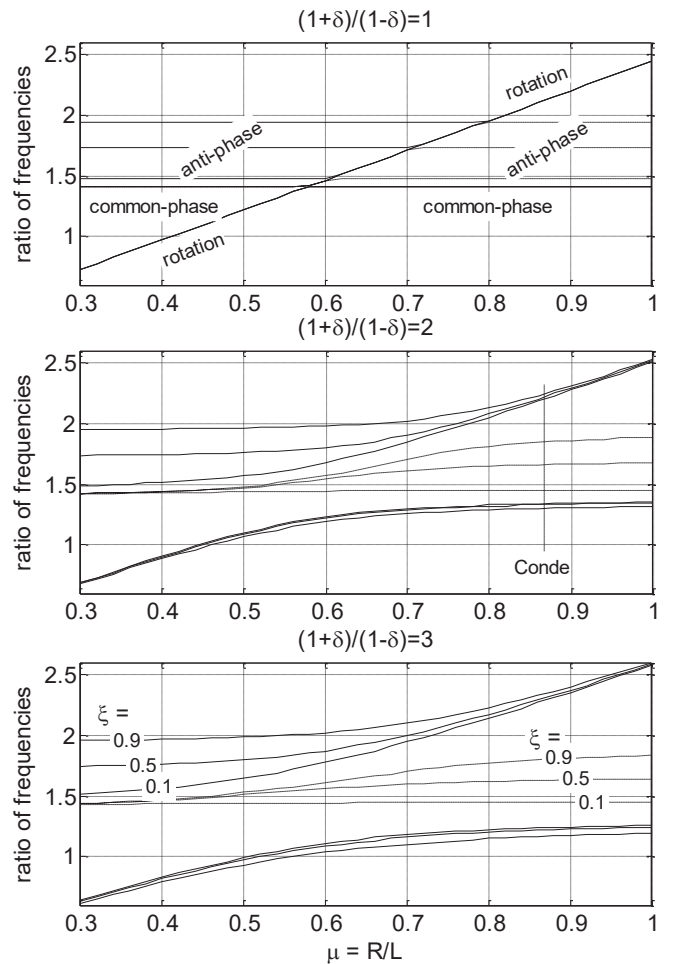


FIG.5. Numerical results for the ratio of the three eigenfrequencies versus bridge overhang  $\mu = R/L$  for  $(1+\delta)/(1-\delta) = 1, 2$ , and  $3$ , according to Eq. (3). The varying coupling between top plate and back plate,  $\xi = 0.1, 0.5$ , and  $0.9$ , as indicated in the bottom graph applies likewise to all graphs. The estimates for the Conde guitar are  $\mu = 0.87$ , and  $(1+\delta)/(1-\delta) = 2$ , indicated in the center graph (- -). The top graph represents symmetric guitars, modes do not couple, but might approach each other. In asymmetric guitars, modes couple strongly and their frequencies depart from each other the larger the asymmetry, compare center and bottom graph.

Another interesting point is Conde's choice of  $\delta$ . He avoided  $(1+\delta)/(1-\delta) = 1$ . But he also avoided too much difference in the bracing. E.g.  $(1+\delta)/(1-\delta) = 3$  will cause a wide spacing of the resonance frequencies, e.g. a factor of 1.8 at  $\mu = 0.87$ , bottom graph. Translated to the guitar,  $T3$  would be found at 360 Hz rather than at 277 Hz when  $T1$  is at 200 Hz. At  $(1+\delta)/(1-\delta) = 2$ , resonances are both, dense and coupled. The bridge overhang will define how far  $T3$  will reach down to  $T1$ , and  $\xi$  will tune  $T2$  perfectly in between.

#### V. Power of the vibrational modes

Frequency tuning is one important factor, but tuning the strength of the vibrational modes is likewise important. In most guitars, the back plate does couple with the top plate, but the vibration is too weak. Modal strength in relation to tuning is investigated in the experimental setup. As a result, the empirical data and the data from the model match well for both, frequencies and modal power, for the complete range of  $\delta$ ,  $\xi$ , and  $\mu$ . In terms of the power ratio of  $T2$  to  $T1$ : the modal power of the anti-phase mode strongly depends on the bridge overhang  $\mu = R/L$ . A too strong bridge overhang seems to diminish anti-phase vibration. How do the

vibrational power of the anti-phase and the rotational modes relate to the three developed tuning parameters in general? While presenting numerical results for a wider parametrical space, a fourth tuning parameter is introduced: guitar makers usually tune the top plate and the back plate somewhat apart, by a semi-tone or maybe even one tone.

Fig. 6 represents the results from computationally executing matrix Eq. (3). The related code includes the geometry for tapping and sensing positions (Mores, 2020). While using properties of real guitars, approximately, the focus is on two cases for  $\delta$ , a symmetrical bracing,  $\delta=1$ ,  $k_1=k_2=k$ , and an asymmetrical bracing,  $\delta=0.33$ ,  $k_1/k_2=(1+\delta)/(1-\delta)=2$ . The vibrational power of the rotational mode and of the anti-phase mode in relation to the power of the common mode is plotted versus the bridge overhang  $\mu=R/L$  for various cases of back plate/top plate coupling, and back plate/top plate frequency tuning. Referencing to data taken from the Conde guitar, data from the model and from the experiment agree well.

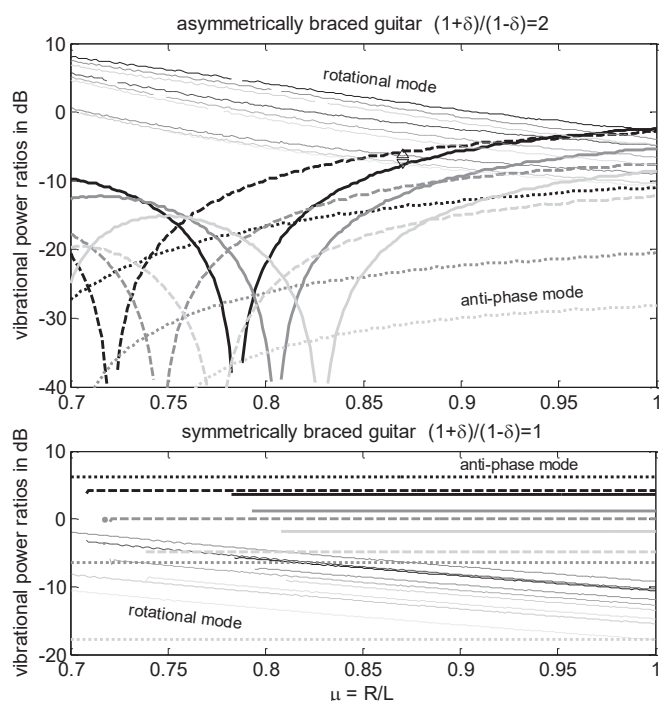


FIG. 6. Numerical results for tapping and sensing a guitar's bridge at the treble side, yielding the power of the rotational mode (thin) and of the anti-phase mode (thick) in relation to the power of the common mode. Mass of the top plate and bridge,  $m_t = 0.13$  kg, mass of the back plate  $m_b = 0.2$  kg, spring property of the top plate,  $k_1 + k_2 = 2k = 30000$  N/m, and the back plate  $k_b = 45000$  N/m. Power versus bridge overhang  $\mu = R/L$ , for various degrees of coupling between the plates,  $k_a = \zeta k$ ,  $\zeta = 0.2$  (••),  $0.6$  (---), and  $1.0$  (—). Non-black lines indicate the case of the back plate being tuned a semitone (dark gray), or a whole tone (light gray) higher than the top plate. Reference data of the Conde guitar taken from the mobility measurement, power of anti-phase ( $\nabla$ ) and rotational mode ( $\Delta$ ) in relation to common mode.

Several general observations now form a wider context.

- 1) The rotational mode appears with growing asymmetry.
- 2) The power of the rotational mode also grows with  $1/\mu$ .
- 3) The power of the anti-phase mode depends on both, the coupling factor  $\zeta$  and the bridge overhang  $\mu$ . In Conde's guitar,  $\zeta$  is promoted by stiffness between top plate and ribs. He used wide wooden triangle chunks to increase stiffness.

- 4) Asymmetrical bracing implies delicate tuning. Cases of  $\mu$ -specific strong suppression of the anti-phase mode are confirmed by experiment. This asks knowledgeable tuning.

## VI. Conclusions

Asymmetric bracing is one of the answers to how multiple resonances can appear in a guitar. A simple model which consists only of the bridge, a few main braces, and the back plate is good enough to understand related tuning issues. The analytical model is confirmed by an experimental setup.

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