Tone Hole Adjustment for a Wooden Saxophone

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Abstract

A wooden saxophone body requires a wall thickness of about 6 mm. The corresponding tone hole chimney height increase is relevant for the tuning. The task is therefore to adapt the tone hole layout while keeping the main bore unchanged so that the resonance properties of a wooden saxophone correspond to those of a normal saxophone.
Firstly, geometry was determined from computer tomography images of a tenor saxophone. Using a 1D waveguide model, input impedance curves were calculated for this geometry and compared with measurements. Next, the sensitivity of the first resonance frequencies to variation in the three tone hole parameters distance from mouthpiece, diameter, and chimney height was examined for each fingering. In good agreement with Nederveens 'Acoustical Aspects of Woodwind Instruments', we found that the pitch shift following a chimney height increase of 6 mm can be compensated by a 20 to 50 % diameter increase, which would affect the timbre. As a step towards redesign, tabulated results of necessary parameter changes for each single tone hole are presented. Further steps are to include the effect of closed tone holes, and construct a prototype. Tests under controlled playing conditions could help formulating impedance targets for algorithmic optimization of reed woodwind tuning.

Introduction

Problem Statement

A metal saxophone typically has a wall thickness of about 0,7 mm. To manufacture a sax bore in wood, the wall thickness must be increased to provide sufficient durability. We consider a minimum wall thickness of at least 6 mm for a wooden saxophone. This means a considerable increase in tonehole chimney height. If the main bore remains unchanged, the resonance frequencies would be lowered and the instrument would sound out of tune. The objective is to find out, which adjustment of the tone hole layout is needed, that the pitch of a wooden saxophone corresponds to that normal metal saxophone. This can be solved by enlarging the tone hole radius [1, 2].
Preserving the original positions was aspired for easier adaption of the key mechanics to a wooden saxophone prototype.
The questions are: How much radius enlargement is needed to preserve the tuning? And is it at all possible to retain the original tone hole positions?

Material and Method

Impedance Model

The link between the tone hole geometry and the acoustical behaviour is given by the acoustic input impedance. As a first step to tone hole redesign, we need to measure the original saxophone geometry and compute its acoustic response. Therefore the geometry of a Jupiter (JTS-789) tenor saxophone was obtained from computer tomography scans [2]. To calculate the acoustic response, we use the 1D transmission line (plane wave) model outlined in [3]. In this the instrument is represented as a cascade of duct and tone hole elements, each of which is described by a local transmisson matrix T_{duct} or T_{hole} (Eq. 1), which are multiplied to give the global transmission matrix T_{1n} (Eq. 2). The input impedance Z results from Eq. (3). A tone hole element consists of chimney height (t), tone hole radius (b) and bore radius (r). The link between t, b, r, and the transmission matrix is given by Eqs. (4) to (6).

$$T_{duct|hole} = \begin{bmatrix} A_{duct|hole} & B_{duct|hole} \\ C_{duct|hole} & D_{duct|hole} \end{bmatrix} \quad (1)$$

$$T_{1n} = \prod_{i=1}^{n} T_{i,i+1} \quad (2)$$

$$Z = \frac{A_{1n}\, Z_n + B_{1n}}{C_{1n}\, Z_n + D_{1n}}. \quad (3)$$

$$\begin{aligned} \delta &= \tfrac{b}{r} \\ t_a &= -b\,0.282\delta^2 \\ t_i &= b(0.75\delta^{2.7} - 1.4\delta^2 + 0.82) \\ t_m &= b(0.026\delta^3 + 0.125\delta) \end{aligned} \quad (4)$$

$$\begin{aligned} Z_c &= \rho\, c/(\pi r^2); \qquad Z_{ch} = \rho\, c/(\pi b^2) \\ Z_a &= j\, Z_c\,(k\, t_a) \\ Z_h &= j\, Z_{ch}\,(k\, t_i + \tan(k(t_m + t + t_r))) \end{aligned} \quad (5)$$

$$\begin{aligned} A_{hole} &= 1 + \tfrac{Z_a}{2Z_h} \\ B_{hole} &= Z_a + \tfrac{Z_a^2}{4Z_h} \\ C_{hole} &= \tfrac{1}{Z_h} \\ D_{hole} &= 1 + \tfrac{Z_a}{2Z_h}. \end{aligned} \quad (6)$$

Impedance Measurement

To validate the model, we compare calculated input impedance results to measurements. The input impedance measurements are done with the FMTC (four microphone three calibration) method and for that we use a four microphone probe, built in our lab [5] after a design suggested by Dickens et al. [4].
The comparison in Figure 1 shows that model and measurement agree very well for the fundamental impedance peak, but less good for higher frequencies. A possible reason for this mismatch is an inaccuracy in the modeling of the radiation impedance. However, this does not affect our tone hole calculations where we are focusing on the first air column resonances.

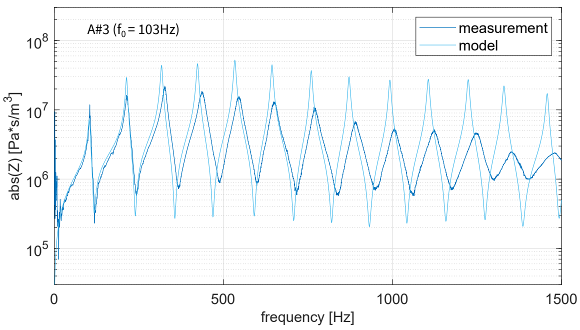

Figure 1: Comparison of measured and modeled input impedance for the lowest note A#3 on a tenor saxophone

Reed Correction

The input impedance peak frequency measured inside the mouthpiece is always slightly higher than the playing frequency [1]. This is due to the fact that the reed motion induces additional flow. We account for the effect of the reed flow by adding a volume of 6 ml to the acoustical transmission line. The peak shift due to this correction rises with pitch. That is, the effect of the reed correction is larger for the smaller tone holes which are closer to the reed.

Tonehole-Redesign

We use the model to calculate the input impedance of a tenor saxophone. While successively enlarging the tone hole radius, the fundamental impedance peak of the original saxophone is compared to the current design layout with increased chimney. We find the optimal radius based on the criterion of deviation of impedance magnitude peak frequencies. The match appears by minimal peak deviation, determined via zero-cross search.
For impedance peak detection a polynomial curve is fitted at the peaks into the numerical curve. The input impedance of 'model + reed correction' is taken as impedance peak matching target. In Figure 2 the black dashed line indicates zero deviation from the nominal pitch corresponding to the fingerings on the horizontal axis. The redesign is an iterative note-by-note process starting at the note of the lowest tone hole (B3) and successively proceeding to the holes closer to the mouthpiece ending at the hole for C#5.

Besides the changed tuning of the measurement due to the addition of the reed correction, Figure 2 also shows the influence of the chimney increase at the tone holes of the first register and its compensation for the 1st input impedance peak frequency by enlarging the tone hole radius.

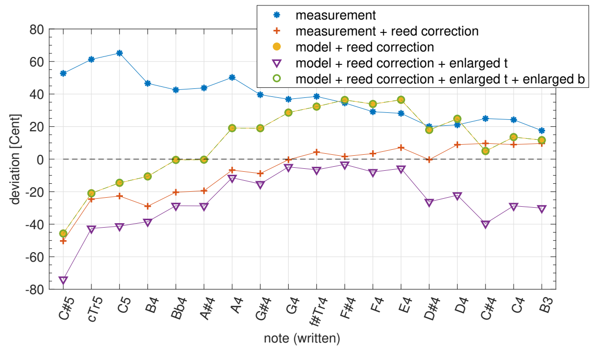

Figure 2: Discrepancy between the first impedance peak frequency and the expected sounding frequency for the first register notes of the tenor saxophone

Results of Optimization

In Figure 3 the normal tone holes of the metal sax and the radius-increased tone holes of the wood sax are compared. We show the result after 2 iterations of tone hole from B3 to C#5 with the aim to match the 1st impedance peak.

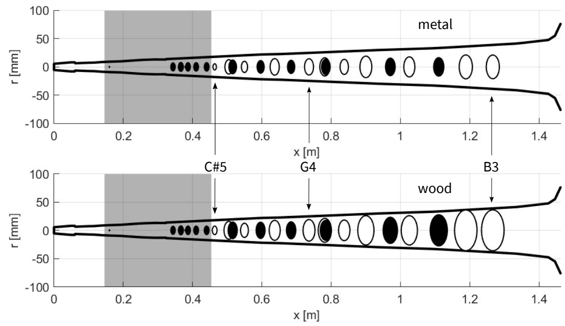

Figure 3: Bore layout for metal (top) and wooden saxophone (bottom). Register holes (gray region) were disregarded. Note that the tone hole scaling on the x-axis is not true to scale.

The tone hole ratio $\delta = b/r$ for a wooden saxophone is 0.75 (±0.13) as compared to 0.54 (±0.08) for a normal saxophone (in Figure 3).

For a greater acoustical similarity between wooden to normal saxophone, more than the 1st impedance peak could be taken into account. One example is depicted in Figure 4 where we use a weigthed average of peak deviations of the first and second impedance peak. We chose to weight $|Z1|$ by 0.7 and $|Z2|$ by 0.3 to account for the assumed greater importance of the fundamental. However, this leads to even larger tone hole radii than the original redesign (see Figure 3) basing on the fundamental only. More research is necessary to investigate the importance of higher air column modes for the settling pitch of a stable regime. The decision of the weighting depends also on the agreement of measurement and model.

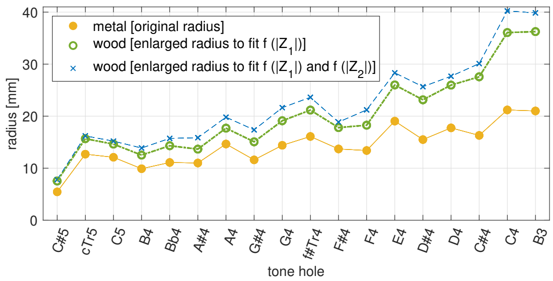

Figure 4: Enlarged tone hole radii (b) for a wooden saxophone to compensate the prolonged chimney height of 6 mm

The tone hole position was kept constant in this exploratory study to see if an enlargement of the tone hole radii would be sufficient. Our results show, that a reconsideration of the tone hole position is advisable to build a wooden saxophone. Care must also be taken that the spacing between tone holes is sufficient.

Conclusion

Prolonged tone hole chimneys in a wooden saxophone can be compensated by strong tone hole enlargement. However, we can expect that such a wooden saxophone will have a different timbre.
The theoretical enlarged tone hole radius would at some holes be greater than the main bore of the saxophone to preserve the tuning. Therefore a compensation of the longer chimney height is not possible if the original tone hole positions are to be preserved. The tone hole position needs to be shifted towards the reed, to minimise the radius enlargement for building purpose and to approximate the original tone hole radius ratio δ.

Eventually, the original main bore must also be slightly shortened, as the prolonged tone hole chimneys lower the pitch of the lowest note, where all holes are closed. Therefore may an enlargement of the bell radius is necessary (or a strong correction of the player at the reed).

For this exploratory study, we have only adressed the first register. Further steps should include the implementation of octave keys and higher registers.
Also, appropriate target function for the optimization can be studied. So far, we have limited the work to the aspect of intonation, looking only at the fundamental resonances. Higher resonances could be taken into account, but it is difficult to decide about their weighting, because their contribution of the higher resonances to the evolution of a pitch is still unclear. In the optimization, one could also seek for a global match of a wide band impedance spectrum [6], i.e. an "acoustical copy", but this would require a better fit between model and measurement.
Until then, playability and timbre are to be explored systematically with building and testing prototypes.

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