Linear stability analysis for bends and overbends on the blues harmonica

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Introduction

A blues harmonica (blues harp) is a diatonic harmonica. In each of its channels there is a blow reed and a draw reed. The reeds are riveted to reed plates and can oscillate freely through openings in the plates (free reeds). Blowing or drawing with relaxed embouchure results in normal blow or draw notes sounding just below the natural frequency of the blow or draw reed. In the following, channel #4 of a C-harp will be considered as a representative example. In this channel, the normal draw note sounds a whole tone above the normal blow note. It is possible to bend the pitch of the draw note continuously down to just above the pitch of the normal blow note by applying suitable vocal tract geometries (draw bends). Similarly one can play blow notes which sound about a semitone higher than the normal draw note and can be continuously bent up (overblows). The normal blow note can only be bent down rudimentarily. It is impossible to play blow bends loud and with good sound.

In 1979, N. H. Fletcher [1] takes up a long tradition and explains self-excited oscillations of wind instruments by an interaction between a generator (the oscillating reed in the mouthpiece acting as control valve) and a resonator (the air in the bore). A general formula for the generator admittance is derived and discussed. In 1987, R. B. Johnston [2] builds on this work by postulating a formula for the admittance of the overall system of the two reeds in the channel of a blues harp and plotting it for a set of parameters, including comparatively and ad hoc given „large“ values for mean pressures. The resonator admittance does not appear in detail. It is only supposed to have a positive real part because it mirrors energy losses in the resonator. Comparing with the real part of the admittance of the reeds system Johnston postulates intervals for possible playing frequencies in accordance to playing practice.

Over the last several decades, the above-mentioned model for self-excited oscillations has been extended in various directions and also made more precise. In this paper a stringent linear stability analysis will be performed, where a mean playing pressure together with the corresponding playing frequency appear as a pair of solutions of a complex equation, relating the admittances of generator and resonator [3][4]. The resonator admittance is obtained from a toy model built from impedance measurements during saxophone playing.

The goal of this paper is to present a nontrivial test case. Starting with an overblow (for example on channel #4 of a C harp) and then changing from blowing to drawing as quickly as possible (without changing the geometry of the vocal tract), a comparatively deep draw bend will sound. Will the model be able to predict self-excited oscillations with frequencies a whole tone apart based on a common resonator frequency?

The air flow through a blues harmonica

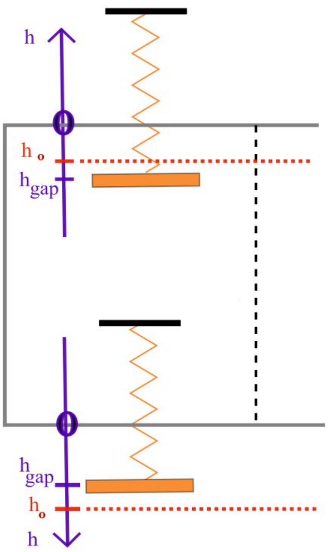

Figure 1: 1-point oscillators modeling the reeds: blow note. Top: blow reed. Bottom: draw reed. The displacement h corresponds to the displacement of the reed tip relative to the reedplate, h_{gap} is the displacement (gap) at rest, h_0 is the mean displacement.

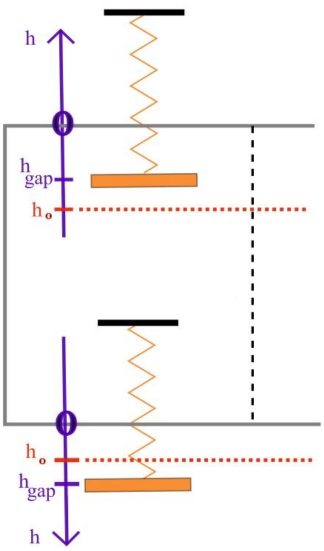

Figure 2: 1-point oscillators modeling the reeds: draw note. Reeds and definition of h , h_{gap} and h_0 as in Fig. 1.

The airflow from a fictitious boundary between instrument and vocal tract through the instrument into the environment (for blow notes) or vice versa (for draw notes) is modeled by a 1-dimensional volume flow. The reeds can be represented as 1-point oscillators, using effective sizes for the parameters

[5]. A stationary Bernoulli equation is supposed to hold on the trajectory through the gaps between the reeds and the reedplates. Using a non-stationary Bernoulli equation as in [1][2] complicates matters unnecessarily from today's point of view. On the other hand, it is now common to consider in addition the volume flow generated by the moving reeds. Pressure inside the reed channel is supposed to be homogeneous. Pressure fluctuations in the surroundings (the emitted sound) are neglected. The pressure at the boundary to the vocal tract thus acts as pressure difference on the reed surfaces.

The volume flow Q_j at reed j (with $j=1$ for the blow reed and $j=2$ for the draw reed) is approximately equal to:

$$Q_j\left(h_j, v_j, \dot{h}_j\right) = (-1)^j W h_j v_j + S\dot{h}_j \tag{1}$$

The displacement h_j of the 1-point oscillator (see Figs. 1 and 2, the indices are omitted in the figures) models the displacement of the reed tip. Wh_j stands for the cross-sectional area, and both this area and the "width" W are effective quantities (which explains the factor 2 in Table 1). The factor $(-1)^j$ compensates the negative sign of h_j for the blow reed (see Figs. 1 and 2). With effective reed surface S, $S\dot{h}_j$ quantifies the volume flow caused by reed motion. The axis for the volume flow points into the instrument for blow as well as for draw notes, due to the fact that an input admittance will be calculated. The total volume flow Q is the sum of the volume flows Q_j passing the two reeds.

In the following, it is assumed that at the boundary between vocal tract and instrument, "small" sinusoidal pressure fluctuations with (circular) frequency ω around a mean value are given. These fluctuations cause oscillations of both reeds with common frequency ω around a mean displacement. A linear approximation of the Bernoulli equation as well as a subsequent linear approximation of (1) provide a corresponding approximation for the volume flows Q_j and thus also for their sum Q. The quotient $\Delta Q / \Delta p$ of volume flow fluctuation ΔQ and pressure fluctuation Δp is the differential admittance $Y_h(p_0, \omega)$ of the system made up by both reeds in the channel of the harmonica. In complex notation one obtains:

$$\operatorname{Re} Y_h = \sum_{j=1,2} A(-1)^j \left(\pm \frac{S}{m_j} \cdot \frac{\omega_{j0}^2 - \omega^2}{D} + \frac{h_{j,gap} + S p_0 / \omega_{j0}^2 m_j}{2|p_0|} \right) + \\ + \frac{S^2}{m_j} \omega^2 \cdot \frac{\omega_{j0} q_j}{D} \tag{2}$$

$$\operatorname{Im} Y_h = \sum_{j=1,2} \pm A(-1)^{j+1} \frac{S}{m_j} \cdot \left(\frac{\omega_{j0} \omega q_j}{D} + S\omega \frac{\omega_{j0}^2 - \omega^2}{D} \right) \tag{3}$$

The abbreviation $A = W\sqrt{2|p_0|/\rho}$ was used as well as:

$$D = \left(\omega_{j0}^2 - \omega^2\right)^2 + \left(\omega_{j0} \omega q_j\right)^2$$

(4)

The index j distinguishes between blow and draw reeds, the plus sign in $\pm$ stands for blow notes, the minus sign for draw notes. For the reed j, $h_{j,gap}$ denotes the reed gap, m_j the effective mass, ω_{j0} the natural (circular) frequency, and q_j the damping constant. My own measurements [9] for channel #4 of a C-harp result in values listed in table 1:

Table 1: Parameters for the reeds system

	blow reed	draw reed
S	$1{,}1 \cdot 10^{-5} m^2$	$1{,}1 \cdot 10^{-5} m^2$
ω_{j0}	$3300 s^{-1}$	$3700 s^{-1}$
q_j	0,004	0,004
W	$2 \cdot 0{,}002 m$	$2 \cdot 0{,}002 m$
$h_{j,gap}$	$-0{,}2 \cdot 10^{-3} m$	$0{,}2 \cdot 10^{-3} m$
m	$3{,}9 \cdot 10^{-6} kg$	$3{,}1 \cdot 10^{-6} kg$

Admittance of the resonator

A major role for the generation of bends and overbends is played by a constriction between the back of the tongue and the palate, as is the case when whistling or speaking vowels [i], [y], or [u]. Therefore, it seems obvious to attribute the role of the resonator to the vocal tract. Although there are by now MRI images of the vocal tract during bending [6], measurements or calculations of the admittance of the vocal tract do not exist to my knowledge.

In contrast, measurements of vocal tract admittance do exist for bending and altissimo playing on clarinet and tenor saxophone [7]. According to my subjective experience, vocal tract geometries for bending on the harmonica can be used for bending on the saxophone (and vice versa).

This gave rise to the idea of fitting the impedance curves measured in [7] for a bend note and for a note in the altissimo range on the saxophone by a 1-mode resonator in order to get a "toy model" of the vocal tract. The curves were each shifted to a resonant frequency of $\omega_0 = 3500 s^{-1}$. For the values given in Table 2, the maximum impedance $Z_{\max}$ was taken from [6], the quality factor q was determined by fitting another impedance value, and finally the modal factor $F = q\omega_0 Z_{\max}$ was calculated.

Table 2: Parameters for the resonator

	Bending	Altissimo
$Z_{\max}$	$1{,}7 \cdot 10^7 Pa \cdot s / m^3$	$3{,}4 \cdot 10^7 Pa \cdot s / m^3$
q	0,32	0,10
F	$1{,}9 \cdot 10^{10} Pa / m^3$	$1{,}2 \cdot 10^{10} Pa / m^3$

Using complex notation, the admittance $Y_{res}(\omega)$ of a 1-mode resonator as a function of playing frequency ω is:

$$Y_{res} = \frac{q\omega_0}{F} + i\frac{\omega^2 - \omega_0^2}{\omega F} \tag{5}$$

Graphical stability analysis

Linear stability analysis in the sense of [3][4] is based on the idea that both volume flow and pressure are continuous at the boundary between instrument and vocal tract. Consequently, the differential input admittance of the reed system and the input admittance of the vocal tract are equal save for a minus sign:

$$-Y_h = Y_{res} \tag{6}$$

Solutions of equation (6) are of the form (p_0, ω), where p_0 is an average blow or draw pressure for which self-excited oscillations with playing frequency ω are possible.

Solution strategy in Figs. 3 and 4 was to use a family of functions $-Y_h$ depending on a parameter p_0 and then solve equation (7):

$$(-Y_h(p_0))(\omega) = Y_{res}(\omega) \tag{7}$$

For this purpose, the corresponding real and imaginary parts from (2), (3) and (4) are plotted using the values from Tables 1 and 2. Equations (7) and (6) are solved with solution (p_0, ω) if and only if the curves of the two real and the two imaginary parts (with parameter p_0 for the reeds admittance) intersect at the same location (frequency) ω.

Fig. 3 shows the evaluation for a draw note on channel #4 of a C harp with values from Table 2 for a bend on the saxophone. The imaginary part of $-Y_h$ is nearly invariant under changes of p_0 (plotted are exemplary curves for the parameter values $-30Pa$, $-67Pa$, and $-100Pa$). Thus the solution (p_0, ω) can easily be found by varying p_0 . Self-excited oscillations are possible for a draw pressure of $p_0 = -67Pa$ and a playing frequency of $\omega = 3485s^{-1}$ which lies slightly below the assumed resonant frequency $\omega = 3500s^{-1}$ of the vocal tract.

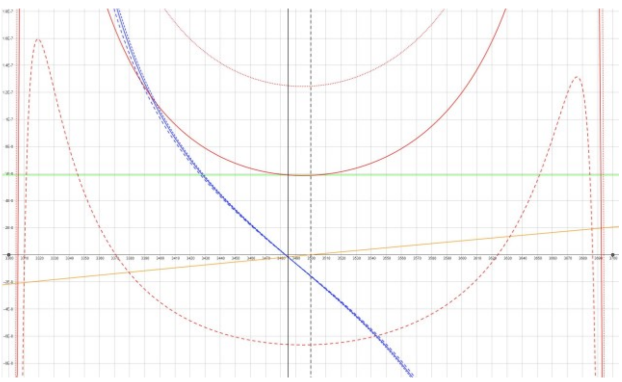

Figure 3: Draw bend on channel #4 of a C-harp. Horizontal: (angular) frequencies in s^{-1}. Marked are the frequencies $3300s^{-1}$ for the blow reed and $3700s^{-1}$ for the draw reed. The solid vertical line indicates the playing frequency $\omega = 3485s^{-1}$, the dashed line the resonator frequency $\omega_0 = 3500s^{-1}$. Vertical: admittances in $m^3 / Pa \cdot s$. Red resp. green: real part of $-Y_h$ resp. of Y_{res} . Blue resp. yellow: imaginary part of $-Y_h$ resp. of Y_{res} . Solid curves: Self-excited oscillations are possible for $p_0 = -67Pa$. No solutions of the equation system “red = green” and “blue = yellow” exist for $p_0 = -100Pa$ (dotted curves) or for $p_0 = -30Pa$ (dashed). Parameters for the reeds system as in Table 1, parameters for the resonator as for bending on the saxophone in Table 2.

The requirement that phase angles of $-Y_h$ and of Y_{res} should be equal [2] proves to be an unsuitable criterion in the context of a linear stability analysis. For $p_0 = -100Pa$ and $\omega = 3487s^{-1}$ these phase angles would be equal, but not the real parts and consequently not the complex admittances (cf. Fig. 3).

As mentioned in the introduction, one can switch between draw bend and overblow by quickly changing the direction of breathing while the vocal tract is kept fixed. In doing so, the overblow sounds a marginal semitone above the normal draw bend. Using the values from Table 2 for a bend on the saxophone, the solution of for a blow note would be the pair $(1200Pa, 4180s^{-1})$. The note would sound more than two semitones above the normal draw note, and the blowing pressure would also be unrealistically high. Playing overblows imposes high demands on the geometry of the vocal tract, perhaps comparable to playing the saxophone in the altissimo range. Using corresponding values from Table 2 (second column) hardly changes the results for a draw bend found in Fig. 3. However, the overblow corresponding to the solution $(390Pa, 4046s^{-1})$ in Fig. 4 now sounds only one and a half semitones above the draw note, and the blowing pressure is also much more realistic.

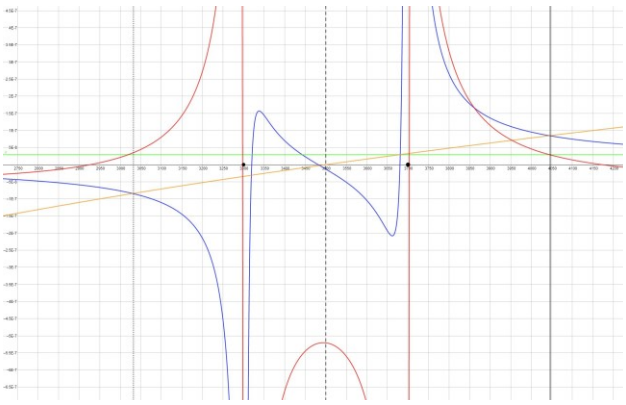

Figure 4: Overblow and blow bend on channel #4 of a C-harp. Axes and colors as well as parameters for the reed system as in Fig. 3, parameters for the resonator as for the altissimo playing on the saxophone in Table 2. For $p_0 = 390Pa$ an overblow sounds with $\omega = 4046s^{-1}$. With $p_0 = 355Pa$ a blow bend with $\omega = 3032s^{-1}$ would be just as conceivable.

As indicated by the dotted vertical line in Fig. 4, not only an overblow, but also a blow bend could sound at approximately the same blowing pressure. In practice, low volume blow bends are indeed possible, often sounding together with an overblow. With higher volume one can only play overblows in a decent way. This might be an example that linear stability analysis can predict instabilities, but not the possibly resulting nonlinear stable region.

Review and outlook

The real part of the resonator admittance (5) is positive, which correlates with the energy losses in the resonator. Therefore, self-excited oscillations are only possible if the real part of the reed admittance is negative. Thus, plots like Figs. 3 and 4 confirm the restriction of possible playing frequencies already described by Johnston [2], which is apparently robust with respect to particular model assumptions or to the parameters used.

In contrast to [2], this paper was primarily concerned with quantitative statements. For a "typical" channel of a blues harp, possible playing frequencies were given together with the blowing or drawing pressures required for them within the framework and validity range of a linear stability analysis.

The model was able to represent alternating playing of draw bends and overblows with equal vocal tract geometry. It turns out that the draw bend sounds practically at the resonant frequency of the vocal tract, while the overblow is about a whole tone above it. This might explain why playing an overblow is subjectively more difficult. If the brain is used to associating a certain vocal tract geometry with a certain pitch of a draw bend, imagined and actual pitch no longer match when playing the corresponding overblow.

Considering all this, of course, one must not forget that measurements were applied for playing on the saxophone. And, of course, it would be desirable to know the vocal tract admittance when playing on the blues harmonica.

Finally, one should ask whether linear stability analysis in frequency range can explain the occurrence of self-excited oscillations on the blues harmonica at all. The model tacitly assumes sinusoidal motions of both reeds with common frequencies. Measurements [9] show instead rather irregular and independent movements of the two reeds at the very beginning, followed by a rapid exponential growth of sinusoidal oscillations into the saturation range. Thereby the reeds „agree“ very quickly on common pseudo frequencies resp. frequencies, but common frequencies are not given a priori. These observations motivate investigations in time domain.

L. Millot [5][10] deals with modeling and numerical simulations of free reeds instruments in time domain. An alternative approach with emphasis on the emergence of collective behavior of the system as a whole might be some ansatz similar to the Impulse Pattern Formulation IPF [11].

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