Abstract
A Finite-Difference Time Domain (FDTD) model of an experimental guitar with a fan bracing consisting of hollow fans is presented. The model couples the differential equations of a plate to the air column in the fan and back to the plate. The fan is placed at different positions on the plate. For comparison, a solid fan is also used. The hollow fan has a considerably more lively spectrum compared to the plain plate without fan. This is caused by a disturbance of the modes due to an asymmetry of the plate and therefore to less cancellation of mode radiation to a virtual microphone position. Integrating over 500 Hz bands, a much more smoothed spectrum appears. Although with solid fans such an asymmetric mode disturbance is also expected, and indeed present, using the solid fan does not lead to a smoothed spectrum. Therefore, the hollow fan behaves considerably different compared to a solid fan. Also, as a hollow fan has less mass, the flexibility of the top plate is increased leading to a louder radiated sound.

Introduction
Acoustic metamaterials[6] have been used in musical instruments with drums[3] or guitars[1]. Still the complex geometry of musical instruments might lead to reconsider them as such [4][5]. Although the fan bracing of guitars or the bracing of piano soundboards is built mainly for the purpose of stability, such regular substructures might lead to a behaviour meeting conditions of the concept of metamaterials. Thereby, the radiation of complex geometries might be very complex, and regular modes might take much longer than the initial transient time of the sound[2].

Metamaterials in musical instruments can be used to change the instrument sound considerably. Changing existing instrument geometries can lead to added band gaps in their spectrum, and using several of such band gaps will lead to a designed sound. With percussion instruments musical articulation is realized by striking or knocking at different positions on e.g. drums or cymbals. By adding metamaterial structures to them the variability of such sounds can be increased considerably.

The paper proposes a new method for building a metamaterial guitar. It suggests altering the sound of a classical guitar by replacing the traditional fan bracing by hollow fans. Regular fans are solid, leading to increased stability of the top plate. They add mass to the top plate and therefore alter the mode shapes and vibrational distribution on the plate. Hollow fans, on the other side, couple an air column to the top plate, leading to unexpected behavior as discussed below.

Methods
The experimental guitar is built to be able to attach and deattach a top plate by screwing the top plate to a solid body. The advantage of this setup compared to gluing the top plate to a body is that the top plate can be modified without partially destroying the top plate during the act of deattaching it from the body again. Secondly, to ensure flexible experimental setup with regular and parameterized geometry alternations, the experimental guitar used has a rectangular body shape.

The experiment is beyond the scope of this paper. Therefore, only the Finite-Difference Time Domain (FDTD) model is discussed below.

Finite-Difference Time-Domain (FDTD) model
The plate is modelled with displacement $u(x,y)$ like

$$\frac{Eh^3}{12(1-\nu^2)} \left( \frac{\partial^4 u}{\partial x^4} + 2 \frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} \right) = \frac{\partial u^2}{\partial t^2}, \quad (1)$$

with Young’s modulus $E$, plate thickness $h = 0.003$ m, density $\varrho = 890$ kg/m$^3$, and Poisson’s ration $\nu = 0.3$.

For the air column with air pressure $p$ it holds

$$c^2 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) = \frac{\partial^2 p}{\partial t^2}, \quad (2)$$

with $c = 343$ m/s the speed of sound in air.

The coupling between the plate and the air column is implemented coupling the plate velocity $v$ and the air column pressure like

$$\frac{\partial p}{\partial x} = \frac{\partial v}{\partial t}, \quad (3)$$

over the air column length at the positions on the plate the air column is placed.

The discrete model uses Young’s modulus in two directions, where $E_x = 13 \times 10^9$ Pa and $E_y = 1.3 \times 10^9$. The discrete version of the plate then at first calculate the mass-weighted accelerations like
with bending moments $M$ as second-order differentials with respect to space like for differentiations

$$M^x_{x+n,y} = \frac{-2u_{x+n,y} + u_{x+n+1,y} + u_{x+n-1,y}}{\Delta x^2}$$ (4)

$$M^y_{x,y+m} = \frac{-2u_{x,y+m} + u_{x,y+m+1} + u_{x,y+m-1}}{\Delta y^2}$$ (5)

$$M^{xy}_{x,y+m} = \frac{-2u_{x,y+m} + u_{x+1,y+m} + u_{x-1,y+m}}{\Delta x^2}$$ (6)

and

$$M^{zx}_{x+m,y} = \frac{-2u_{x+m,y} + u_{x+m+1,y} + u_{x+m-1,y}}{\Delta y^2}$$ (7)

The air column is implemented accordingly like, calculating a mass-weighted acceleration like

$$a^{\text{Air}} = c^2 \frac{-2p_i + p_{i+1} + p_{i-1}}{\Delta x^2},$$ (8)

with discrete pressure values at each node point $i$. The coupling between the air column and the plate then couples plate accelerations $a$ to the first-order derivative of air column pressure. For each time step of iteration and each coupling point the plate acts into the air column by adding the plate acceleration to the first-order derivative of the air column pressure like

$$\frac{p_{i+1} - p_i}{\Delta x} = \varrho a_{x,y},$$ (9)

and the plate acceleration is coupled into the air column like

$$a_{x,y}^+ = \frac{1}{\varrho} \frac{p_{i+1} - p_i}{\Delta x}.$$ (10)

Therefore, the nodal points $i$ of the air column are in between those of the plate, as coupling happens through the first-order differential. Still both share the same spatial discretization $\Delta x$ (or $\Delta y$ respectively).

From the acceleration $a_{x,y}^t$ at each node point $x,y$ and time point $t$ the velocities $v_{x,y}^t$ are calculated and added to the velocities of the previous time point $v_{x,y}^{t-1}$ like

$$v_{x,y}^{t+1} = v^t + a_{x,y}^t \delta t,$$ (11)

with time constant $\delta t$. These velocities are damped using a logarithmic increment $\delta$ like

$$v_{x,y}^{t+1} = v_{x,y}^{t+1} \delta t.$$ (12)

Finally, for each time step, the new displacements $u_{x,y}^{t+1}$ are calculated again integrating over time like

$$u_{x,y}^{t+1} = u_{x,y}^{t} + v_{x,y}^{t+1} \delta t.$$ (13)

The plate has dimensions of 39.2 cm $\times$ 39.2 cm according to the experiment. A spatial discretization of 2.45 cm is used to represent frequencies at least up to 4 kHz.

The boundary conditions of the plate are taken as clamped as in the experiment the plate is skewed to the guitar body at its four corners and its boundaries rest on the guitar body. In the FDTD model these boundary conditions are modelled by assuming two nodal points at all boundaries to have zero displacement over time, resulting in a momenta $M^x = M^y = 0$ at the boundaries automatically. As only the first nodal point on the boundaries apply for $u(x,y,t) = 0$, an additional nodal point is used outside the geometry on all boundaries to ensure boundary conditions. This leads to a discretization of 16 $\times$ 16 nodal points and in the calculation to 18 $\times$ 18 nodal points.

The air column is taken to be 29.4 cm in length in the calculation while in the experiment the rod has a length of 28.4 cm. This is due to the discretization of $\Delta x = 2.45$ cm used. Still the slightly enlarged rod length is compensating the air columns end-correction to a certain extent.

In this paper, due to page restrictions, only simulations are shown.

**Simulation setups**

The simulation was performed for

- only plate
- only air column
- plate coupling to air column but not back
- plate coupling to air column and back
- plate with bars but no air column

The simulations of only plate and only air column are references. The coupling of plate to air column is performed with and without back coupling to estimate the impact of this back coupling. The simulation with air column and with bar are performed to estimate the differences between a traditional fan bracing adding mass.
Abbildung 1: Spectra of FDTD simulations of the plate, integrated to a virtual microphone position 1 m above the plate, comparing the uncoupled plate (blue) to the plate coupled to an air-filled fan (yellow), the uncoupled air column (green), the air column coupled to the plate (purple), and the air column fully coupled to the plate back and forth (red). The coupling leads to a much more lively spectrum due to disturbed modes.

to the top plate and the a coupled air column. When attaching a hollow fan to a top plate, of course, both are added, mass and air column. This is also simulated but due to page restrictions not shown here.

Results

Fig. 1 shows the eigenmode spectra of the plate and the air column. To compare them, in the figure the amplitudes of the curves have been shifted. All spectra are taken by integrating the plate vibration at a virtual microphone position 1 m above the plate opposite to the plate’s center.

The blue curve shows the spectrum of the plane plate while the yellow curve shows that of the plate with air column coupled both directions but without a bar, so without added mass. Clearly, the coupled case is much more lively and complex. Also several frequency ranges are enhanced or attenuated.

The green curve shows the plain air column to estimate the positions of the resonances. The case of incoupled plate to the air column but without back coupling is shown in purple. There at positions of the the first, second, and fifth air column eigenfrequencies the spectrum has larger amplitudes, still not for the third and fourth one. When also back coupling the air column to the plate, shown in red, the amplitude of higher modes is reduced, as expected as energy leaves the air column again, still also the enhanced spectral regions of the purple case of no back coupling has mainly vanished.

At the air column eigenfrequency positions, the yellow curve of the radiated plate sound does not show enhanced amplitudes. Therefore, we find that the effect of air-filled fans does not lead to an enhancement of frequencies of the air column’s eigenfrequencies.

To estimate the reason for the coupled plate spectrum being more lively can be found when examining the plate modes with and without coupled air column shown in Fig. ???. The eigenmodes are considerably disturbed in the presence of an air column. This leads the eigenmodes to become unsymmetric and therefore cancellation of radiated modes is reduced. Then, eigenmodes which only have low amplitudes will to cancellation at a microphone position are now more prominent there.

Still this is also expected when attaching bars to a plate, so with regular fan bracing. Therefore, the air column simulation is compared to a simulation without air columns but with bars. This time we are interested in the guitar sound playing strings. Therefore, all six open strings were
played in the model resulting in six sounds.

Fig. 3 shows the summed spectra of these six sounds integrated over 500 Hz bands. Although it is also interesting to go into more spectral details comparison with the bar case is most clear with this wide integration bandwidth.

In the figure, the black curve is the case without air column. The other curves all have air columns attached to the plate and are placed at different positions pos 1 - 10, where pos 1 and pos 10 are at the plates side and pos 5 and pos 6 are most close to its middle. Consequently, the effect becomes stronger when the air column is placed in the middle rather than at the plate’s sides.

The main effect shown is a considerable smoothing of the radiated spectrum. The uncoupled spectrum (black curve) has an amplitude drop at about 1500 Hz and 3000 Hz which are practically gone when attaching the air column at the plate’s middle.

This smoothing effect of the air column does not appear with the attached bar. Although the spectrum changes, the spectral bands in all cases are much closer together.

A possible reason for the difference between air column and bar can be seen in the mode changes shown in Fig. 2. The air column often enhances the amplitudes at the air column positions. This becomes more obvious when integrating all modes (not shown here). Contrary, adding a bar or fan adds mass to the respective plate position making this region more stiff, so less flexible and therefore less radiating. The bar or fan therefore also changes the modes shapes, leading to unsymmetric modes, still the reduced flexibility does not lead to a smoothing of the spectrum as does the air column.

Conclusions
Replacing the traditional fan of guitar top plates by a hollow fan adding an air column has a considerably different effect to the guitar sound compared to the traditional solid fan. The air column leads to a smoothed spectrum with less spectral variations compared to the plain plate case and to the case of a plate with fan.

The added air column does not lead to enhanced amplitudes at the positions of the air column eigenfrequencies. This effect is also not known from traditional solid fans. Still, due to the increased stiffness of a bar compared to air in an air column, the lowest eigenfrequency of a fan is much higher than that of an air column of same length. Therefore, the impact of fan eigenfrequencies is not expected to be considerable anyway. Still, with the air column, such an effect was originally considered but did not turn out in the model.

Therefore, replacing the top plate solid fan with a hollow one seems to be a promising alternative. As the fan is mainly there for stability, and a hollow fan is expected to have about the same stability properties, the replacement seems feasible. Additionally, a hollow fan has less mass leading to increased plate flexibility and therefore to a louder guitar sound. As loudness has always been a big problem in guitar building, only this is a considerable improvement.

Literatur