On the Impact of Loudspeaker Nonlinearities on Room Acoustic Measurements

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Introduction

When measuring reverberation time and other room acoustic parameters, different signal processing techniques can be used. Popular choices include the interrupted noise method which allows for a direct estimate of the energy decay curve, but requires multiple decays due to the stochastic nature of the noise signal used. Another method is the exponential sweep method, which allows measurement of the room impulse response and from that reverberation time or other room acoustic parameters, can be estimated. The equipment used, especially the loudspeakers and amplifier have seen dramatic improvements in linearity over the years and for the amplifiers also efficiency has been greatly improved. One measurement challenge still remains and that is to generate sufficient sound pressure level at low frequencies, especially if the loudspeaker should be easy to carry around, as a large volume velocity output is needed at low frequencies. In the quest for a good signal to noise ratio one risks activating the nonlinearities in the loudspeaker and this may potentially influence the measurement result. This paper investigates to which extent this can be an issue and suggest ways to detect such artifacts.

Interrupted noise

The interrupted noise method measures the decay curve directly and is therefore in practice immune to any nonlinear behaviour. The nonlinear distortion components may alter the spectrum of the emitted noise signal slightly, but will not affect the decay process itself and thus any effects of nonlinearity is expected to be negligible.

Exponential sweep

The exponential sweep [1] has many advantages: It uses a deterministic signal, reducing the need for repetitions and has a lower crest factor than noise leading to improved signal to noise ratio. The sweep is

$$x(t) = \sin(2\pi f_0 L(e^{t/L} - 1)) \tag{1}$$

where f_0 is the starting frequency and L determines the sweep rate, i.e. a doubling of frequency will happen for every $log(2)L$ seconds. The spectrum of the sweep is found

$$X(f) = \int_0^T \sin(2\pi f_0 L(e^{t/L} - 1))e^{-j2\pi ft}dt \tag{2}$$

where T is the sweep duration. The spectrum of the inverse filter may in principle be found as

$$X_{inv}(f) = \frac{1}{X(f)} \tag{3}$$

Note that this naive definition of the inverse may lead to unwanted amplification of noise and regularisation techniques should be applied in practice. The time domain impulse response may be found as

$$x_{inv}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_{inv}(f)e^{j2\pi ft}df \tag{4}$$

The measurement signal, which consist of the sweep played through the loudspeaker and room is then convolved with the inverse filter impulse response

$$y(t) = (x(t) * h_{ls}(t) * h_{room}(t)) * x_{inv}(t) \tag{5}$$

Where $*$ represents convolution. Under the assumption of linearity this simplifies to

$$y(t) = h_{ls}(t) * h_{room}(t) \simeq h_{room}(t) \tag{6}$$

as the loudspeaker impulse response normally decays much faster than the room

Maximum length sequences

A third alternative is maximum length sequences (MLS) which are flat spectrum deterministic signals that have the property of being their own inverse filter, ie.

$$x_{MLS}(t) * x_{MLS}(t) = \delta(t) \tag{7}$$

And thus they also rely on linearity to extract the room impulse response from the measurement

Exponential sweeps, inverse filter and nonlinear distortion

When using an exponential sweep the inverse filter basically have to compensate for the fact that low frequency energy appears before high frequencies. As a consequence any harmonics of the sweep frequency the loudspeaker emits, see figure 1, will be moved to negative time by the inverse filter as shown in figure 3. Assuming the sweep

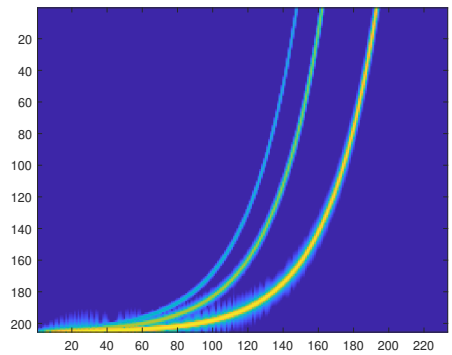

Figure 1: Exponential sweep output from simple nonlinear system

rate is low enough all the pre echoes generated by the

harmonics can be removed by time windowing without affecting the response at the fundamental around $t = 0$. It is tempting to conclude that this eliminates all nonlinear distortion components in the remaining impulse response, but this not true, as nonlinear artifacts also corrupt the impulse response [2]. This happens because the nonlinear distortion affects the response of the loudspeaker at the input frequency, normally leading to an overall reduction in the output.

A nonlinear system is one where the input signal (or a filtered version of it) gets multiplied onto itself. If we look at a system where the input signal is squared

$$(A\cos(\omega t))^2 = \frac{A^2}{2} + \frac{A^2}{2}\cos(2\omega t) \tag{8}$$

We see that the double frequency is generated in addition to a DC value. Thus no component is generated at the input frequency, and it fact this is the case for all nonlinear components of even powers. However for at third order nonlinearity

$$(A\cos(\omega t))^3 = \frac{3A^3}{4}\cos(\omega t) + \frac{A^3}{4}\cos(3\omega t) \tag{9}$$

we see that in addition to a 3rd harmonic a component at the input frequency is generated and this component is 3 times a strong as the 3rd harmonic. This pattern repeats for all odd powers of the input signal, ie for order 5

$$(A\cos(\omega t))^5 = \frac{10A^3}{16}\cos(\omega t) + \frac{A^3 5}{16}\cos(3\omega t) + \frac{A^3}{16}\cos(5\omega t) \tag{10}$$

So one will underestimate the nonlinear components at the input frequency by a lot from looking at the level of the harmonics only.

In order to better understand the influence of the nonlinearities on the fundamental let us first look at a simple nonlinear system, one without memory or dynamics, i.e. a system that is not frequency dependent.

$$y(t) = 0.21(x - 0.5x^3 + 0.12x^5) \tag{11}$$

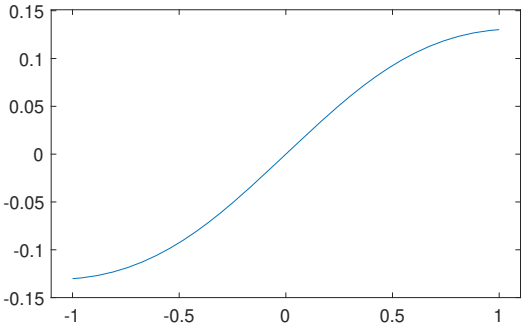

Figure 2: Instantaneous nonlinear input-output function

the input output relation is also shown in figure 2 and a soft compression effect can be seen. Figure 1 shows the spectrogram of output, which shows fairly strong 3rd and 5th harmonic.

After convolving the output with the inverse filter the impulse response in figure 3 is obtained and we see that the 3rd and 5th harmonic appear as two clear pre echo delta functions. After timewindowing, octave band filtering and backwards integration the decay curves in figure 4 are obtained, and as no room was added in this simulation very short reverberation times are obtained, essentially only showing the reverberation in the octave band filters. Since in this case the influence of the nonlinearity is independent of frequency the effect in the time domain can be represented as a delta function, i.e. it does not create any additional reverberation.

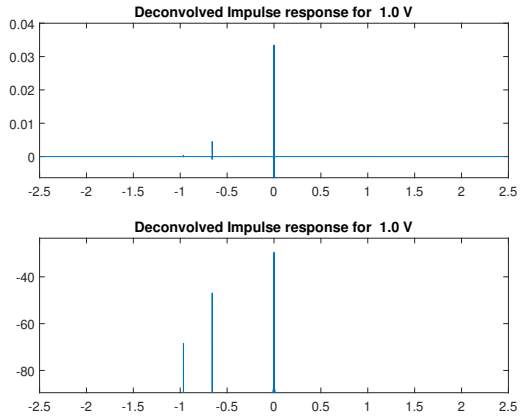

Figure 3: Impulse response of instantaneous nonlinearity after inverse filter is applied

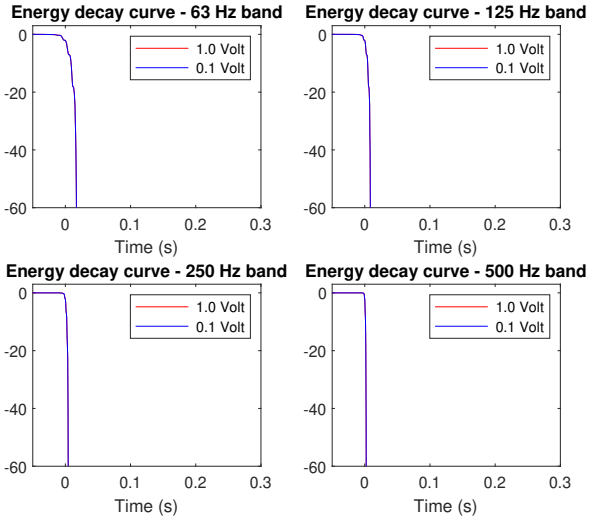

Figure 4: Octave band energy decay curves for instantaneous nonlinearity

Loudspeaker nonlinearities - simulation

An electrodynamic loudspeaker is a fairly complicated device with several physics involved (electric, magnetic, mechanic and acoustic) which all can contribute to nonlinear behavior. It is well established [3] that the force factor, the inductance of the voice coil, and the stiffness of the suspension exhibit strong non-linear behavior and

are responsible for the main part of the non-linear behavior. As the nonlinear behaviour is mainly driven by the position of the voice coil, which depends on frequency (a second order lowpass function) we must expect that the nonlinearities may introduce additional reverberation in the response.

Figure 5 shows the nonlinear force factor, inductance and suspension stiffness as a function of displacement used as parameters in a time domain state space model the nonlinear dynamics of the loudspeaker.

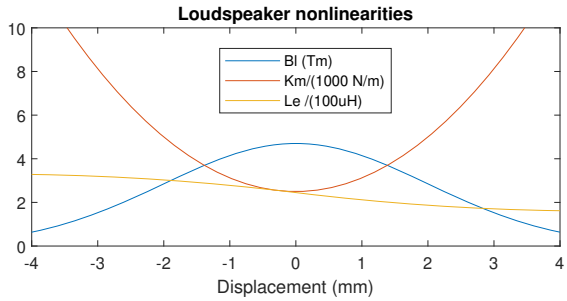

Figure 5: Loudspeaker nonlinearities as a function of voice coil displacement as used in the simulation

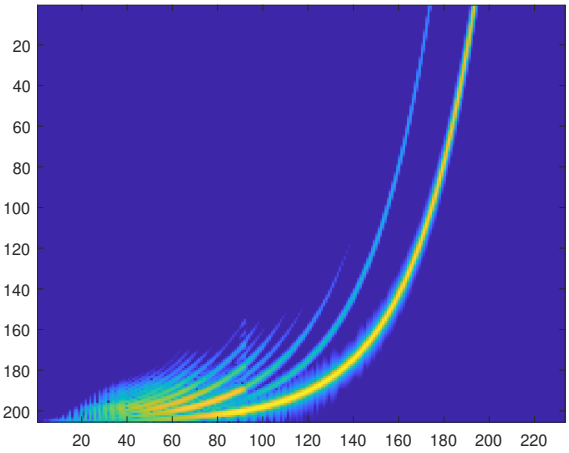

Figure 6: Spectrogram of output from simulated nonlinear loudspeaker in free field

Figure 6 shows spectrogram of the near field pressure. Strong harmonic distortion is seen at the lowest frequencies where the displacement is high.

Figure 7 shows the octave band decay curves. Here there is a clear additional reverberation for the high voltage excitation, as a result of the nonlinear behavior. This is most clearly seen for the 125 Hz octave band, less so for 63 Hz and 250 Hz and no effect is seen for 500 Hz. This confirms that the nonlinear behaviour is mainly happening at the lowest frequencies where the displacement is highest. It is worth noting that the ringing of the loudspeaker in the linear range decays very fast and does not influence the measurement, so what we see is not just the consequence of the frequency response of the loudspeaker.

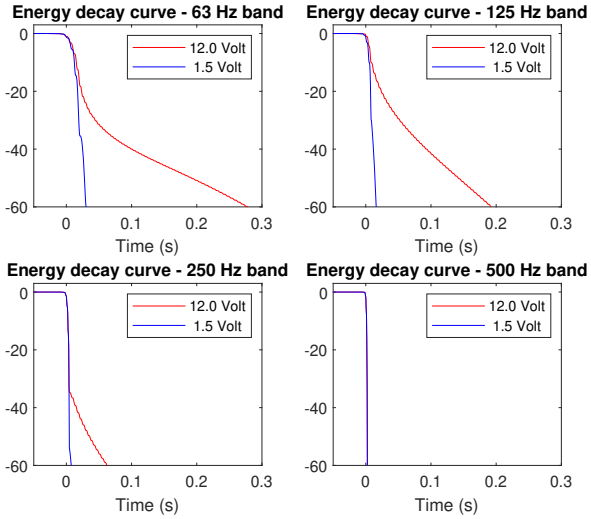

Figure 7: Octave band energy decay curves for simulated loudspeaker in free field

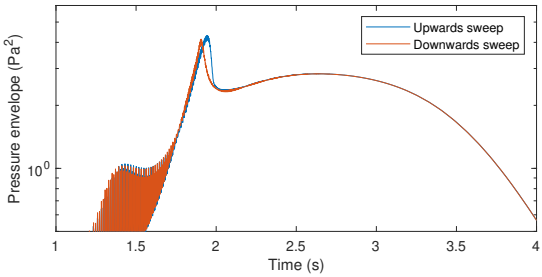

Figure 8: Pressure envelope of upwards and downward (plotted timereversed) sweep responses from nonlinear loudspeaker

Figure 8 shows envelope of the pressure signal for both a normal upwards sweep and a downwards sweep (plotted timereversed). It is striking that while the two envelopes do coincide in most parts of the plot they do differ significantly around $t = 2s$, demonstrating that the response of the loudspeaker depends on the direction of the sweep(!). This is a well known phenomena, called bifurcation, in nonlinear oscillators with a stiffening spring. The effect of the stiffening spring is an increase of the effective resonance frequency with increasing displacement. This means that for a certain range of frequencies and input levels two different responses are possible and which one you get depends on the past of the input signal. For the upwards sweep we can see a dramatic drop in response where the system changes from the "high displacement-high resonance frequency" to the "low displacement-low resonance frequency" response. It is this rapid change in response level in the frequency domain that causes the ringing in the time domain.

Measurements

In order to test if the effect observed in the simulations also occur in the real world a test of several drivers was made, here the decay curves measured in the near field are shown for a high quality 6" driver in figure 9. Here we can see that no additional ringing is introduced at

the higher measurement level of 10.7 volt. The only difference we observe is a slight improvement in signal to noise ratio. Figure 10 shows the near field decay curves for a medium quality 5" driver and here we do see differences between the two measurement levels, but nowhere as dramatic as we saw in the simulation.

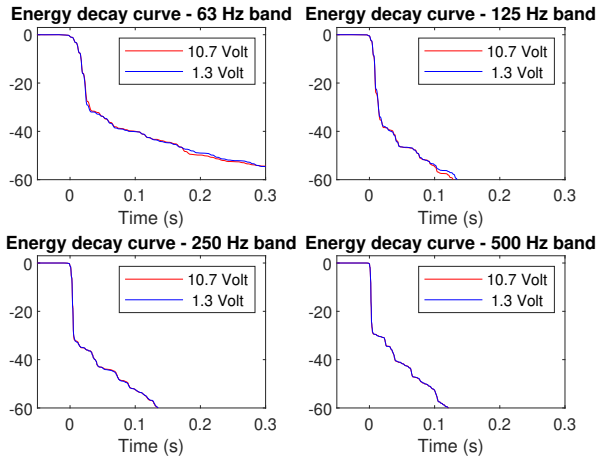

Figure 9: Measured near field octave band energy decay curves for 6" loudspeaker

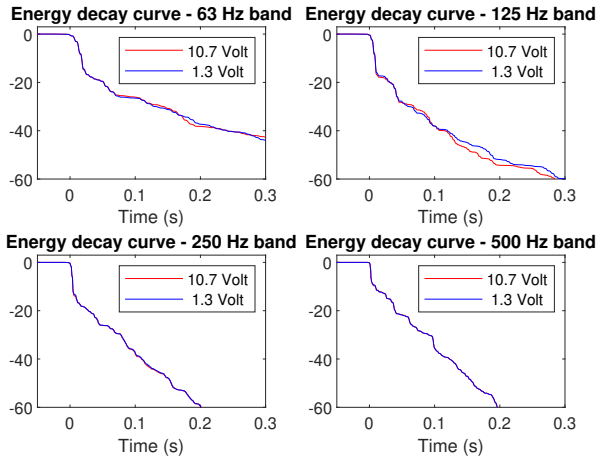

Figure 10: Measured near field octave band energy decay curves for 5" loudspeaker

Discussion

The fact that the measurements show much reduced influence of the nonlinearities compared to the simulations is a welcome result, but begs the question of what is causing this. It seems that something is preventing the effect of the stiffening spring to happen. A plausible explanation is that the standard nonlinear model does not contain all relevant physical phenomena. While it does take the geometrical stiffening of the suspension into account, the effect in the suspension material is not explicitly included. Especially rubber seems to exhibit a nonlinear effect of gradual dry friction which leads to a softening effect [5], as can be observed in electrical measurements of the resonance frequency [4][5]. It seems that in the measurement the softening effect of the material of the suspension is strong enough to counteract the geometrical stiffening such that the bifurcation effect does not occur. Finally it should be mentioned the methods for seperating the linear response from the nonlinear components by combining measurements at different levels have been suggested [6], but this is beyond the scope of this paper.

Conclusions

Unlike interrupted noise measurements, exponential sweep are potentially affected by nonlinearities in the loudspeaker. Time windowing of the pre echoes in the impulse response only removes the harmonics from the measurement signal and the influence on the input frequency remains. If the nonlinearities are such in nature that they cause rapid changes in the response level, such as can happen with a stiffening suspension, ringing in the time domain will occur. If this happens one can observe a different response when compared upwards and downwards sweeps and fast changes in the envelope of the near field sound pressure. Measurements on a small number of drivers revealed that this effect seem much less prone to happen in real world speakers compared to the simulation result. A likely explanation for this is the softening effect in materials, such as rubbers typically used for the outer suspension, seems to counteract the geometrical stiffening of the suspension. Some caution is therefore recommended when selecting the measurement level that gives the best signal result and to check the near field response for any signs of sudden jumps in level.

References

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