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Feedforward Control of Fan Noise in Ducts using Multichannel Order-reduced Inverse Filters

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ABSTRACT

Single-channel feedforward control is a commonly used approach for active noise control (ANC). In this paper, a time-domain underdetermined multichannel inverse filtering technique is proposed for the feedforward active control (ANC) of duct noise. In the commonly used filtered-x least-mean-square (FXLMS) algorithm, the feedforward control problem is formulated as an overdetermined inverse filtering problem which generally results in non-zero residual noise. By introducing multiple secondary sources, the problem can be reformulated into an underdetermined system, which admits infinite number of exact solutions with zero residual noise. Linearly constrained minimum variance (LCMV) method is employed in the controller design. However, as a major shortcoming of the time-domain approach, the length of the filters tends to be too long to admit digital signal processor (DSP) implementation. To address the problem, the least absolute shrinkage and selection operator (LASSO) algorithm, is exploited to effectively reduce the controller orders. A two-channel system is implemented to suppress the fan noise of an air-cleaner. Simulation and experiment results have demonstrated that the proposed approach has achieved significantly higher noise reduction than the conventional FXLMS algorithm.

Keywords: Active noise control, Multiple channels, Inverse filtering

1. INTRODUCTION

Active noise control (ANC) has been a field of active research for several decades. ANC is a method for attenuating undesired noise by secondary sources, which is particularly effective in low frequency range (1, 2). There are three control structures of ANC system: the feedforward structure (3, 4), the feedback structure (5, 6), and the hybrid structure (7, 8, 9) which combines the former two structures. This paper is focused on the widely used feedforward structure.

The feedforward active noise control problem is an overdetermined inverse filtering problem which typically leads to non-zeros residual noise. A method of multiple-input/output inverse theorem (MINT) was suggested (10) to convert the overdetermined problem to a square one by adding additional channels of secondary sources. In this paper, the MINT is extended to a time-domain underdetermined inverse filtering (TUMIF) approach (11), where a regulated underdetermined problem is posed in a model-matching framework.

Traditionally, feedforward control for ducts is implemented with adaptive filters such as the filtered-x least mean square (FXLMS) algorithm (12, 13). In contrast, the TUMIF controllers are implemented as fixed filters obtained using optimizations methods, e.g., the least squares (LS) with an L2-norm regularizer, also known as the Tikhonov regularization (TIKR) (14, 15). However, the resulting FIR filters are prohibitively long. To overcome this difficulty, a sparse coding (SC) technique (16) named the least absolutely shrinkage and selection operator (LASSO) algorithm (17) is employed to reduce the controller's complexity.

To further enhance the TUMIF controllers, the linearly constrained minimum variance (LCMV) (18) approach is exploited by considering the noise covariance matrix and multiple linear constraints posed in the TUMIF. The LCMV algorithm seeks to minimize the array output power subject to linear constraints. The major contribution is to implement the LCMV-TUMIF approach on an active fan noise control problem of duct. In addition, the orders of controllers are reduced by using sparse coding techniques to enable real-time implementation. Experimental results are given and discussed.

2. THE TIME-DOMAIN UNDERDETERMINED MULTICHANNEL INVERSE

FILTERS (TUMIF)

2.1 Two-channel TUMIF system

The idea of TUMIF is to incorporate N-channels ($N > 1$) of secondary sources into the ANC system. This section focuses on a two-channel system. For a general N-channel TUMIF-based feedforward system, we refer the interested readers to a previous paper (11).

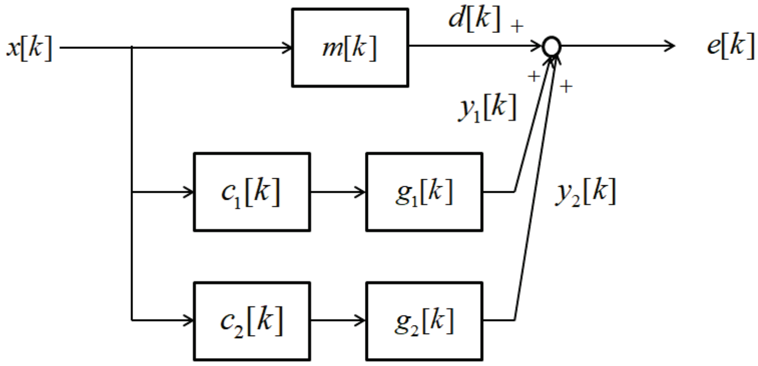


Figure 1 – The block diagram of a two-channel feedforward ANC system.

A two-channel feedforward TUMIF system is shown in Figure 1. This can be viewed as a model-matching problem, as described in the following equation:

$$c_1[k] * g_1[k] + c_2[k] * g_2[k] = -m[k], \tag{1}$$

where k denotes the discrete time index and "$*$" denotes the convolution. The impulse response $m[k]$ represents the primary path, whereas $g_1[k]$ and $g_2[k]$ denote the secondary paths. The aim is to minimize the residual noise $e[k]$ by using the feedforward controllers $c_1[k]$ and $c_2[k]$. Assume that the primary path, two secondary paths, and two feedforward controllers are modeled by L_m, L_g, and L_c-tapped finite impulse response (FIR) filters. In matrix notations, the model-matching problems can be written as

$$\mathbf{G}_T \mathbf{c}_T + \mathbf{m} = \mathbf{0}, \tag{2}$$

where

$$\mathbf{G}_T = \left[\mathbf{G}_1\, \mathbf{G}_2\right] \in \mathbb{R}^{L_m \times 2L_c}, \tag{3}$$

$$\mathbf{G}_m = \begin{bmatrix} g_m[0] & \cdots & 0 \\ g_m[1] & g_m[0] & \vdots \\ \vdots & g_m[1] & \\ g_m[L_g-1] & \vdots \quad \ddots & \\ & g_m[L_g-1] & g_m[0] \\ & \ddots & g_m[1] \\ \vdots & & \vdots \\ 0 & \cdots & g_m[L_g-1] \end{bmatrix} \in \mathbb{R}^{L_m \times L_c}, \quad m = 1,2; \tag{4}$$

$$\mathbf{c}_n = \left[c_n[0] \quad c_n[1] \quad \cdots \quad c_n[L_c-1]\right]^T, \quad n = 1,2; \tag{5}$$

$$\mathbf{c}_T = \left[\mathbf{c}_1^T\, \mathbf{c}_2^T\right]^T \in \mathbb{R}^{2L_c}, \tag{6}$$

$$\mathbf{m} = \left[m[0] \quad m[1] \quad \cdots \quad \cdots \quad m[L_m-1]\right]^T, \tag{7}$$

$$L_m = L_c + L_g - 1 < 2L_c, \tag{8}$$

$$L_c > L_g - 1, \tag{9}$$

where "T" denotes matrix transpose.

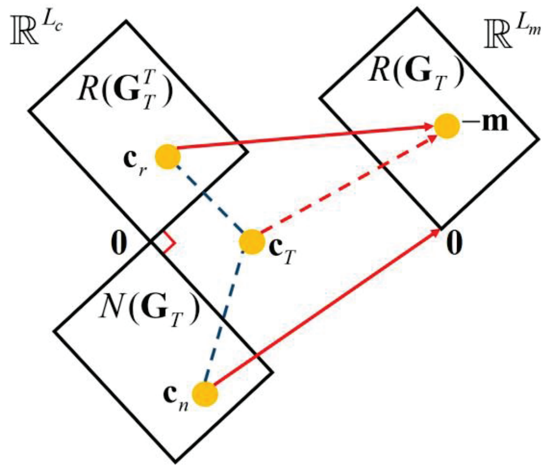


Figure 2 - Vector subspace for the full-rank underdetermined problem. There exist infinite number of exact solutions. $\mathbf{c}_r$ is the projection of $\mathbf{c}_T$ in $N(\mathbf{G}_T^T)$ and $\mathbf{c}_n$ is the projection of $\mathbf{c}_T$ in $N(\mathbf{G}_T)$.

The TUMIF system described by Equation 2 is an underdetermined system if the inequalities of Equations 8 and 9 are fulfilled by choosing appropriate filter lengths, L_g, L_c and L_m. Assuming that $\mathbf{G}_T$ has full-rank, the vector subspaces (19) associated with $\mathbf{G}_T$ is illustrated in Figure 2. $N(\mathbf{G}_T^T)$ shrinks to $\mathbf{0}$, so $\mathbf{m} \in R(\mathbf{G}_T^T)$, suggesting solutions always exist. Furthermore, $N(\mathbf{G}_T)$ is not $\{\mathbf{0}\}$, so infinite number of exact solutions exist. The minimum-norm solution is

$$\mathbf{c}_{LS} = -\mathbf{G}_T^T(\mathbf{G}_T\mathbf{G}_T^T)^{-1}\mathbf{m}. \tag{10}$$

The controller gain can be regularized by the TIKR method

$$\mathbf{c}_{TIKR} = -(\mathbf{G}_T^T\mathbf{G}_T + \beta^2\mathbf{I})^{-1}\mathbf{G}_T^T\mathbf{m}, \tag{11}$$

where β is the regularization parameter to trade-off the controller gain and noise reduction performance.

To enhance the feedforward ANC system, the LCMV design is employed to minimize the control effort.

$$E\left[\left|\mathbf{c}_1^T\mathbf{x}\right|^2\right] + E\left[\left|\mathbf{c}_2^T\mathbf{x}\right|^2\right] = \mathbf{c}_T^T\mathbf{R}_{xx}\mathbf{c}_T, \tag{12}$$

where $E[\cdot]$ denotes the expected value, $\mathbf{R}_{xx}$ consist of $\mathbf{R}_{ss}$, and $\mathbf{x}$ is the input noise vector.

$$\mathbf{x} = \begin{bmatrix} x[k] & x[k-1] & \cdots & x[k-L_c+1] \end{bmatrix}^T \tag{13}$$

$$\text{and } \mathbf{R}_{xx} = \begin{bmatrix} \mathbf{R}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{ss} \end{bmatrix}, \tag{14}$$

where $\mathbf{R}_{ss}$ is the noise covariance matrix. That is, the LCMV optimization problem can be written as

$$\min_{\mathbf{c}} \mathbf{c}_T^T\mathbf{R}_{xx}\mathbf{c}_T \quad st. \quad \mathbf{G}_T\mathbf{c}_T = -\mathbf{m}. \tag{15}$$

The optimal controllers of the preceding constrained optimization problem are

$$\mathbf{c}_{LCMV} = -(\mathbf{R}_{xx} + \varepsilon\mathbf{I})^{-1}\mathbf{G}_T^T(\mathbf{G}_T((\mathbf{R}_{xx} + \varepsilon\mathbf{I})^{-1})\mathbf{G}_T^T + \mu\mathbf{I})^{-1}\mathbf{m}, \tag{16}$$

where ε and μ denote the regularization terms in TIKR approach.

2.2 Order Reduction of Feedforward Controllers

The preceding controllers tend to be too long to be implemented in real-time. To overcome the difficulty, a sparse coding technique will be exploited to reduce the order of controller. The

TUMIF-based ANC problem can be posed as a constrained optimization problem:

$$\min_{\mathbf{c}} \|\mathbf{c}\|_0 \quad st.\ \mathbf{G}_T\mathbf{c} + \mathbf{m} = \mathbf{0}, \tag{17}$$

where $\|\mathbf{c}\|_0$ denotes the zero-norm, or the cardinality of the solution vector, $\mathbf{c}$. To accommodate noise robustness, inequality constraint is used instead.

$$\min_{\mathbf{c}} \|\mathbf{c}\|_0 \quad st.\ \|\mathbf{G}_T\mathbf{c} + \mathbf{m}\|_2 < \eta, \tag{18}$$

where η is a threshold and $\|\cdot\|_2$ donates the L2-norm. This problem can be solved by using relaxation methods or greedy methods. This paper adopts one of relaxation methods, the LASSO algorithm. Instead of the zero-norm, the LASSO approach uses the L1-norm in the cost function to obtain a convex optimization problem (20, 21, 22) as shown

$$\min_{\mathbf{c}} \|\mathbf{c}\|_1 \quad st.\ \|\mathbf{G}_T\mathbf{c} + \mathbf{m}\|_2 < \eta, \tag{19}$$

where $\|\cdot\|_1$ denotes the L1-norm. Besides, Equation 19 can also be converted into an equivalent unconstrained convex optimization problem.

$$\min_{\mathbf{c}} \ \frac{1}{2}\|\mathbf{G}_T\mathbf{c} + \mathbf{m}\|_2^2 + \lambda\|\mathbf{c}\|_1, \tag{20}$$

where λ denotes the regularization parameter that serves as a weighting for sparsity.

To extend this to the LCMV controllers, we rewrite Equation 15 into a modified LASSO form:

$$\min_{\mathbf{c}} \ \frac{1}{2}\|\mathbf{G}_T\mathbf{c}_T + \mathbf{m}\|_2^2 + \lambda\|\mathbf{c}_T \odot \mathbf{r}_{xx}\|_1, \tag{21}$$

where $\odot$ denotes the Hadamard product and vector $\mathbf{r}_{xx}$ is derived from the noise covariance matrix

$$\mathbf{r}_{xx} = \frac{\mathrm{dvec}(\mathbf{R}_{xx})}{\|\mathrm{dvec}(\mathbf{R}_{xx})\|_\infty}, \tag{22}$$

where $\mathrm{dvec}(\cdot)$ denotes the operation to convert diagonal entries of a matrix into a column vector, and $\|\cdot\|_\infty$ denotes the infinity norm. The convex optimization problems can be solved by the package CVX (23) or the alternating direction method of multipliers (ADMM) (24) method.

3. EXPERIMENTAL RESULTS

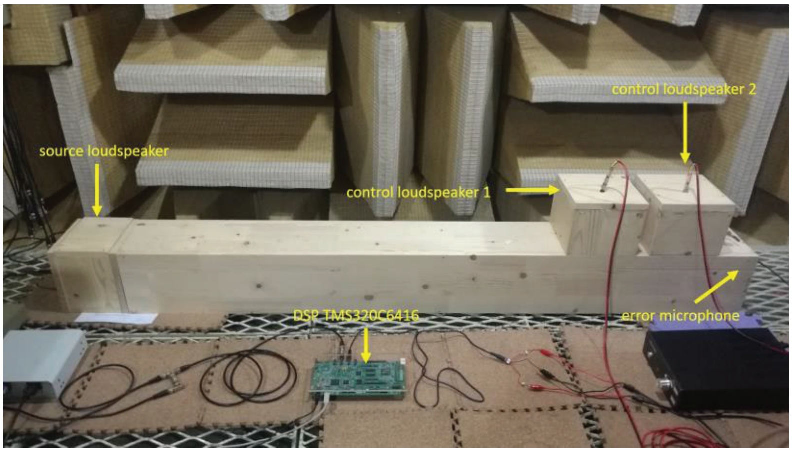


Figure 3 – The experimental arrangement of duct ANC system in the anechoic room.

In this section, several experiments were performed to validate the two-channel feedforward ANC system in anechoic room, as shown in Figure 3. The noise attenuation performance of controllers is compared with or without order reduction method. The duct is 1.75 m long, with rectangular cross-section 0.16 m×0.16 m and wall thickness 0.018 m as shown in Figure 4. The corresponding cutoff frequency of the duct is 1078 Hz that is also regarded as the control bandwidth in which the sound field in the duct can be treated as plane waves.

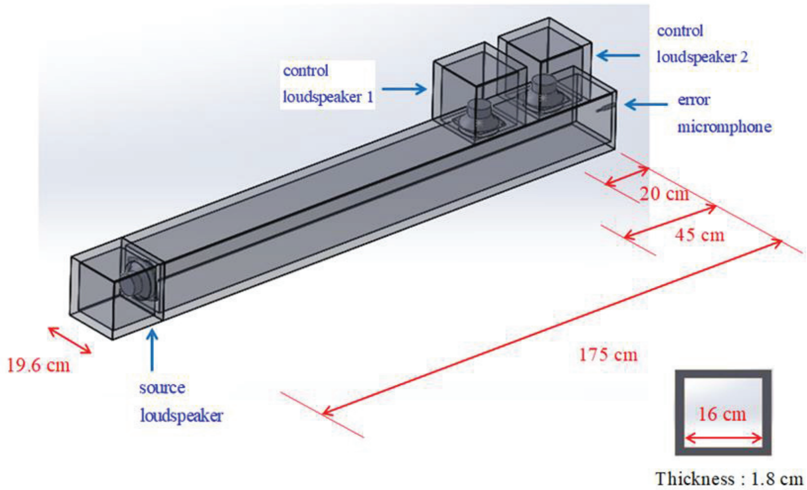
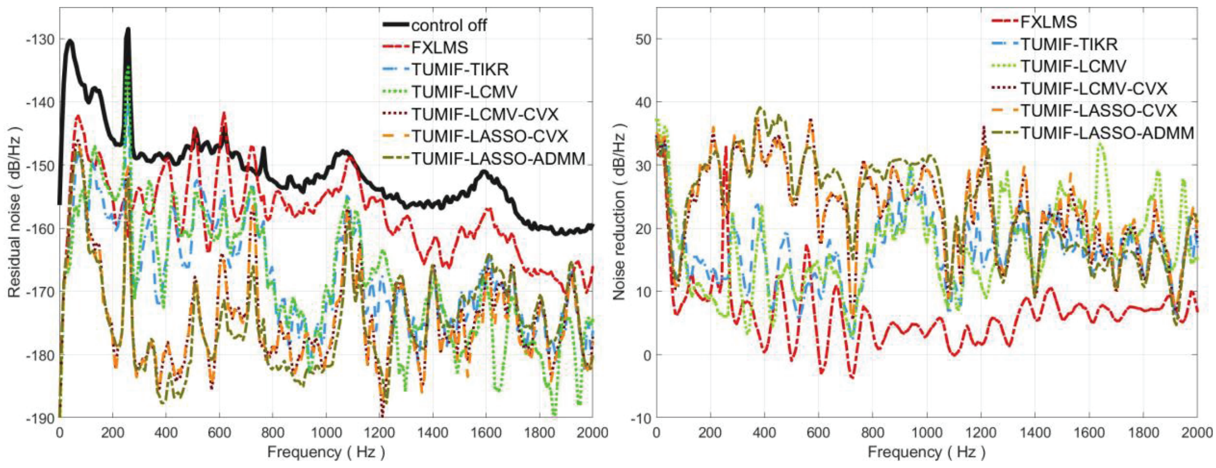


Figure 4 – Dimensions of the wooden duct.

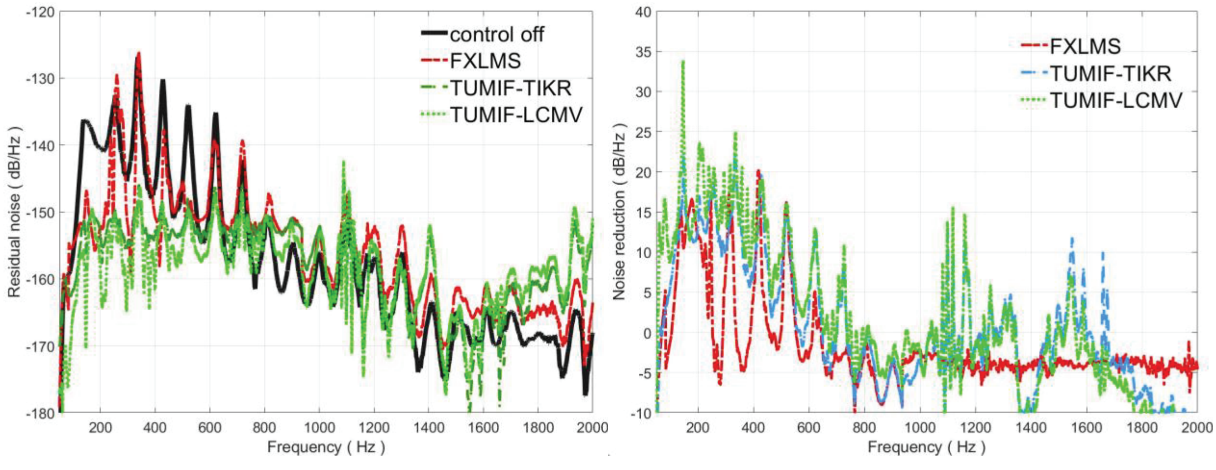
In this paper, fan noise band-limited to 4 kHz is used as the noise source. In experiment, all of the controllers which are offline designed under 8 kHz sampling rate are implemented on the TMS320C6416 of Texas Instrument® for two control loudspeakers. The conventional FXLMS algorithm for control-loudspeaker 2 is adopted as a benchmarking method. However, with the limitation of TMS320C6416's computation power, the allowable maximum number of taps of controller is 400. Therefore, the controllers in this work are selected to be 400 taps in experiment and simulation. In addition, the step size of the FXLMS is 0.01, the regularization parameter of the TUMIF-LCMV is 0.01, the TUMIF-TIKR is 0.1, and the other sparse coding approaches is 0.001.



(a) (b)

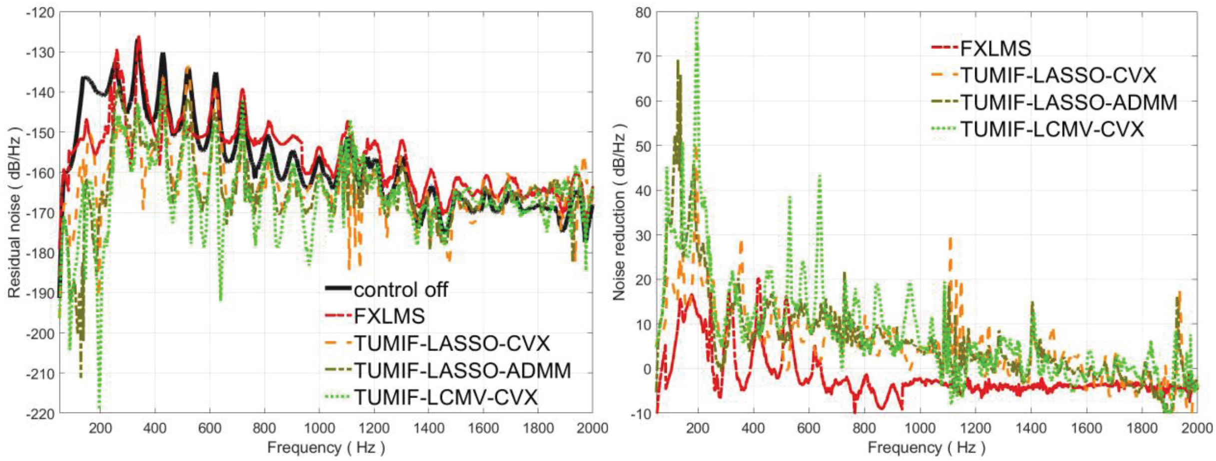
Figure 5 – Simulation results obtained using the FXLMS, the TUMIF-TIKR, the TUMIF-LCMV, the TUMIF-LCMV-CVX, the TUMIF-LASSO-CVX and the TUMIF-LASSO-ADMM. (a) residual noise, and (b) noise reduction performance (control off – control on).

Figure 5 of simulation results show that the order-reduced controllers perform well in noise reduction. Because of the limitation of DSP board's computation power that mentioned previously, the performance of the unreduced controllers will decrease severely. However, with the noise covariance matrix, the TUMIF-LCMV approach has a better and more robust performance. In experiment, the unreduced TUMIF-LCMV controllers attains 0-15dB noise suppression performance more than the benchmarking controller in frequency range 100-770 Hz. Moreover, the order-reduced TUMIF-LCMV achieves 0-60dB noise attenuation performance more than the benchmarking controller in frequency range 100-1100 Hz. Therefore, Figures 6 and 7 of experimental results indicate that the order-reduced TUMIF controllers attain larger noise attenuation than the unreduced controllers. In particular, the order-reduced TUMIF-LCMV approach attains 5-80 dB noise attenuation in frequency range 100-1100 Hz, which is the best among all methods.



(a) (b)

Figure 6 – Experimental results obtained using the FXLMS, the TUMIF-TIKR and the TUMIF-LCMV. (a) residual noise, and (b) noise reduction performance (control off – control on).



(a) (b)

Figure 7 – Experiment results obtained using the FXLMS, the TUMIF-LASSO-CVX, the TUMIF-LASSO-ADMM and the TUMIF-LCMV-CVX.

(a) residual noise, and (b) noise reduction performance (control off – control on).

4. CONCLUSIONS

In this paper, we apply the linearly constrained minimum variance (LCMV) method to reformulate the time-domain underdetermined inverse filters for feedforward ANC. In order to simplify the FIR controllers, we use the sparse coding technique to reduce the order of the controllers such that in real-time implementation is possible.

Through experiments, the proposed LCMV-TUMIF controller has demonstrated superior noise attenuation performance over other methods. However, the TUMIF-LCMV-CVX controllers will behave like the TUMIF-LASSO controllers if regularization parameter defined in Equation 21 is slightly under-regularized.

Given a DSP board with limited computational power, the sparsely coded TUMIF controllers significantly outperforms the FXLMS algorithm in noise attenuation. In the future, more efficient implementation of the LCMV-TUMIF controllers will be investigated.

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