PROCEEDINGS of the
23rd International Congress on Acoustics
9 to 13 September 2019 in Aachen, Germany

Numerical investigation of acoustic radiation damping in sandwich structures

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Abstract

Acoustic radiation damping is the primary energy dissipating mechanism for lightweight structures with large radiating surfaces. Attempts to reduce the vibrational response of those lightweight structures by additional mechanical damping can only be successful if the extent of mechanical damping is comparable or larger than the extent of radiation damping. In this regard, engineers are in need of reliable and flexible methods for the quantification of radiation damping as well as for the modeling of its effect in an early stage of the design process. In this paper, we address the equations of time-harmonic elastodynamics and acoustics by means of finite and boundary element methods respectively. The acoustic radiation damping of a honeycomb sandwich panel is compared to the energy loss due to hysteretic material damping. Moreover, the influence of radiation damping on geometrical as well as material properties is studied.

Keywords: Structural acoustic interaction, Acoustic radiation damping, Honeycomb sandwich structures

1 INTRODUCTION

Acoustic radiation damping refers to the energy dissipation of structures due to sound radiation to the acoustic far-field. In many engineering applications, radiation damping is rather insignificant and negligible. However, it is relevant in cases where other damping mechanisms are only weakly pronounced, such as sandwich structures exhibiting a high stiffness-to-weight ratio [2]. The quantification of radiation damping relies on the physical coupling between the fluid and the dynamic behavior of the structure. Therefore, it is hardly amenable to analytical solutions except for plates with uniform thickness and density [4, 6, 7]. Moreover, experimental damping determination necessitates reference measurements inside vacuum chambers to deduce the effect of radiation damping, which are therefore accompanied by an excessive effort. In this regard, numerical structure acoustic interaction analyses are a suitable way to systematically study the extent of acoustic radiation damping in sandwich structures with different material and geometric configurations.

In this paper, we use the finite element method (FEM) and the collocation boundary element method (BEM) for modeling the structural acoustic interaction. The theory is applied to the example of an air-loaded honeycomb sandwich panel.

2 NUMERICAL MODELING OF STRUCTURAL ACOUSTIC INTERACTION

We consider the fully coupled structural acoustic interaction of sandwich structures using FEM [11] and BEM [8] for discretizing the equations of linear time-harmonic elastics and acoustics, respectively. The resulting linear systems of equations for the structural and for the acoustic subdomain read

$$\left(\mathbf{K}\left(1-\mathrm{i}\eta_{\mathrm{h}}\right)-\omega^2\mathbf{M}\right)\mathbf{u}=\mathbf{f}_{\mathrm{s}}+\mathbf{f}_{\mathrm{f}}, \tag{1}$$

$$\mathbf{H}(\omega)\mathbf{p}=\mathbf{G}(\omega)\mathbf{v}_{\mathrm{s}}. \tag{2}$$

The column vectors $\mathbf{u}$ and $\mathbf{p}$ contain the unknown displacement and sound pressure degrees of freedom at the nodes. The stiffness and mass matrices of the structure are denoted as $\mathbf{K}$ and $\mathbf{M}$. Structural damping is modeled using the hysteretic loss factor η_h. The boundary element matrices $\mathbf{H}(\omega)$ and $\mathbf{G}(\omega)$ are obtained by a collocation discretization of the Kirchhoff-Helmholtz integral equation, relating the structural particle velocity $\mathbf{v}_\mathrm{s}$ to the sound pressure. These matrices are implicitly dependent on the angular frequency $\omega = 2\pi f$, where f is the frequency in Hz. The structure is excited by nodal forces $\mathbf{f}_\mathrm{s}$. Equations (1) and (2) are coupled on the sound radiating boundary. There, the structure is subject to normal tractions due to the acoustic sound pressure, and the particle velocity in (2) equals the time derivative of the normal displacement on the boundary. The coupling conditions can be expressed as

$$\mathbf{f}_\mathrm{f} = \mathbf{C}_\mathrm{sf}\mathbf{p} \quad \text{and} \quad \mathbf{v}_\mathrm{s} = -\mathrm{i}\omega\mathbf{C}_\mathrm{fs}\mathbf{u}, \tag{3}$$

where $\mathbf{C}_\mathrm{sf}$ and $\mathbf{C}_\mathrm{fs}$ are the mesh coupling matrices obtained by a Galerkin projection [10]. The force vector $\mathbf{f}_\mathrm{f}$ can be interpreted as the acoustic loading on the structural nodes and i denotes the imaginary unit. Finally, the global system of equations containing the coupling conditions emerges as

$$\begin{bmatrix} \mathbf{K}(1-\mathrm{i}\eta_\mathrm{h}) - \omega^2\mathbf{M} & -\mathbf{C}_\mathrm{sf} \\ -\mathrm{i}\omega\mathbf{G}(\omega)\mathbf{C}_\mathrm{fs} & \mathbf{H}(\omega) \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_\mathrm{s} \\ \mathbf{0} \end{bmatrix}. \tag{4}$$

When only the structural response is of interest, (4) can be reformulated by forming the Schur complement and thereby omitting the pressure degrees of freedom [9], i.e.

$$\left[\mathbf{K}(1-\mathrm{i}\eta_\mathrm{h}) - \omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C}_\mathrm{sf}\mathbf{H}(\omega)^{-1}\mathbf{G}(\omega)\mathbf{C}_\mathrm{fs}\right]\mathbf{u} = \mathbf{f}_\mathrm{s}, \tag{5}$$

where the expression $\mathrm{i}\omega\mathbf{C}_\mathrm{sf}\mathbf{H}(\omega)^{-1}\mathbf{G}(\omega)\mathbf{C}_\mathrm{fs}$ can be interpreted as the effect of acoustic loading.
Once (4) or (5) is solved, the harmonic radiation loss factor η_r can be evaluated, quantifying the extent of energy dissipation due to sound radiation. It is defined as

$$\eta_\mathrm{r} = \mathrm{Re}(P) / |-\mathrm{i}\omega E_\mathrm{p}|, \tag{6}$$

where P and E_p denote the radiated sound power and the potential energy of the vibrating structure, respectively. Only the real part $\mathrm{Re}(\cdot)$ of the sound power corresponds to the far-field sound radiation and hence to energy dissipation. Recent results [1] show that spurious numerical damping could lead to an overestimation of damping phenomena when studying them with BEM. However, the occurrence of numerical damping does not seem to be an issue in exterior acoustics.
The radiated sound power and the potential energy can be determined from the acoustic and the structural responses, i.e.

$$P = \frac{1}{2}\mathbf{p}^\mathrm{T}\mathbf{\Theta}\mathbf{v}_\mathrm{s}^*, \qquad E_\mathrm{p} = \frac{1}{2}\mathbf{u}^\mathrm{T}\mathbf{K}\mathbf{u}^* - \frac{1}{2}\mathbf{f}_\mathrm{s}^\mathrm{T}\mathbf{u},$$

where $\mathbf{\Theta}$ denotes the boundary mass matrix that is obtained from numerical integration of the boundary element interpolation functions. In the case that structural damping is considered, the total loss factor is the sum of radiation and structural loss factors. The latter is defined as the relation between the power loss due to structural damping and the power corresponding to the total potential energy. It is given as

$$\eta_\mathrm{s} = \frac{1}{2}\omega\eta_h\frac{\mathrm{Re}\left(\mathbf{u}^\mathrm{T}\mathbf{K}\mathbf{u}^*\right)}{|-\mathrm{i}\omega E_\mathrm{p}|}. \tag{7}$$

3 RADIATION DAMPING OF A HONEYCOMB SANDWICH PANEL

The acoustic radiation damping of a honeycomb sandwich panel ($a = 0.696\,\mathrm{m}, b = 0.464\,\mathrm{m}$) in air is studied using coupled structural acoustic analyses. The material properties of the baseline panel are given in Tab. 1. Besides radiation damping, additional mechanical damping resulting from a hysteretic loss factor of $\eta_\mathrm{h} = 0.01$ is considered in all simulations. The panel is free of any structural boundary conditions and modeled without an

Table 1. Baseline properties of the honeycomb sandwich panel

Aluminum face sheets		
Thickness	t	0.2 mm
Density	ρ_a	2780 kg/m^3
Young's modulus	E	73 GPa
Poisson's ratio	ν_a	0.34
Aluminum honeycomb core		
Thickness	h	20 mm
Density	ρ_c	44.8 kg/m^3
Young's modulus	E_x, E_y	18.9 MPa
Young's modulus	E_z	1.89 GPa
Shear modulus	G_{xy}	3 MPa
Shear modulus	G_{yz}	200 MPa
Shear modulus	G_{xz}	200 MPa
Poisson's ratio	ν_c	0.1

acoustic baffle. The structural mesh consists of shell and solid elements to model the face sheets and the core, respectively. The discretized model has a total of 16071 displacement and 3392 pressure degrees of freedom. The radiation loss factor of the panel resulting from a point-force excitation at a corner node is displayed in Figure 1 along with the structural loss factor. At low frequencies, radiation damping is rather small and insignificant but it increases gradually until reaching its maximal value in the coincidence region. At higher frequencies, the radiation loss factor shows an asymptotic behavior. The comparison to the structural loss factor clearly demonstrates the significance of radiation damping for this type of sandwich structures.

Figure 2 shows the radiation loss factors of the panel with different face sheet thicknesses. All other material and geometric parameters are left unchanged as given in Tab. 1. The results indicate that the frequency range in which coincidence occurs is not affected by the face sheet thickness. This is a reasonable result when considering that the critical frequency is proportional to the mass and inversely proportional to the flexural rigidity of the panel (recall that the flexural rigidity in turn is approximately proportional to the thickness of the face sheet). However, it can also be seen that the extent of radiation damping significantly decreases with increasing thickness of the face sheets. Above coincidence, this is mainly due to the effect of added mass of the panel per unit area.

Many authors note that the acoustic properties of sandwich structures are highly sensitive to the shear stiffness of the core [3, 5]. However, the results in Fig. 3 indicate that the radiation damping of the panel is not significantly affected by the shear stiffness of the core - neither below nor above coincidence.

Lastly, the influence of the core thickness is displayed in Fig. 4. Below coincidence, the resonance effects dominate the radiation damping properties, and a clear classification of the different panels is hardly possible. Above coincidence, again the effect of added mass leads to a decrease of the radiation loss factor with increasing core thickness.

4 CONCLUSION

The numerical framework based on FEM and BEM enables flexible and systematic structural acoustic analyses without any limitations on the geometry, material and boundary conditions. Using the numerical framework, the extent of acoustic radiation damping of a honeycomb sandwich panel has been studied. The comparison to structural loss factors indicates the significance of radiation damping for honeycomb sandwich structures. Regarding the influence of material configurations, it has been found that the radiation damping exhibits a high sensitivity to the face sheet and core thicknesses. In fact, the radiation loss factor above coincidence is

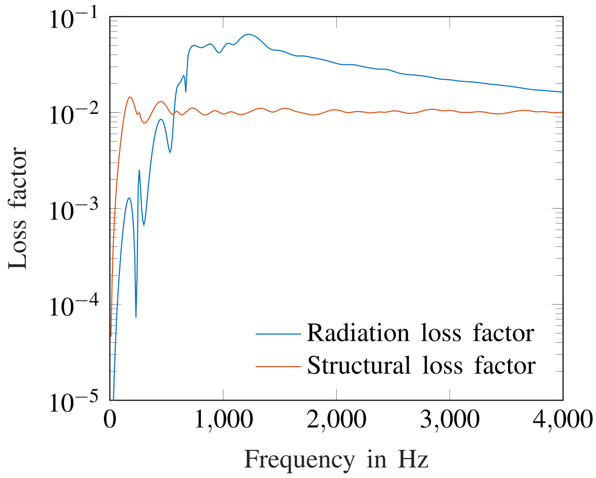

Figure 1. Comparison between the radiation loss factor and the structural loss factor ($\eta_{\mathrm{h}} = 0.01$) of the honeycomb sandwich panel.

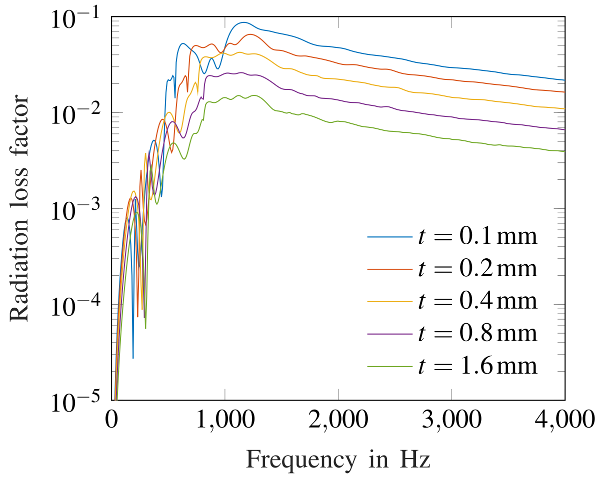

Figure 2. Radiation loss factor of the honeycomb sandwich panels with different face sheet thicknesses.

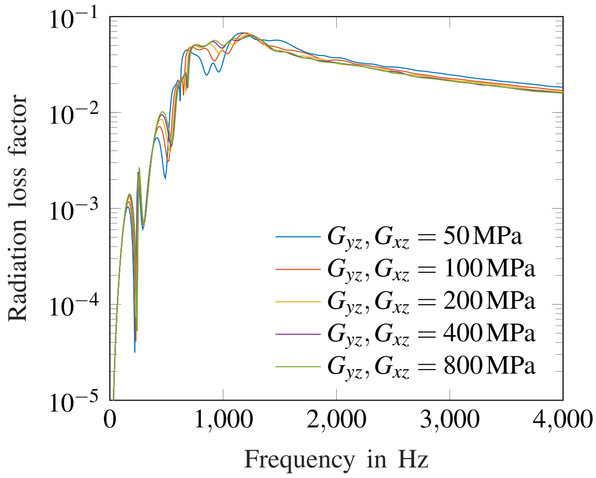

Figure 3. Radiation loss factor of the honeycomb sandwich panels with different shear stiffnesses of the core.

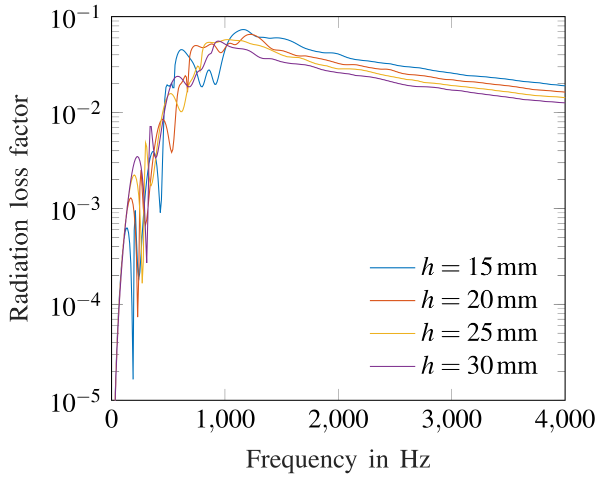

Figure 4. Radiation loss factor of the honeycomb sandwich panels with different shear thicknesses of the core.

approximately inversely proportional to the surface density of the panel. Further investigations are required to fully understand the effect of the stiffness properties of the core and the face sheets on radiation damping.

ACKNOWLEDGEMENTS

This work was conducted with the support of the German Research Foundation (DFG) in the context of the priority program 1897 "Calm, Smooth and Smart - Novel Approaches for Influencing Vibrations by Means of Deliberately Introduced Dissipation."

REFERENCES

[1] S. K. Baydoun and S. Marburg. Quantification of numerical damping in the acoustic boundary element method for two-dimensional duct problems. *Journal of Theoretical and Computational Acoustics*, 26(3):1850022, 2018.

[2] B. L. Clarkson and K. T. Brown. Acoustic radiation damping. *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, 107:357–360, 1985.

[3] H. Denli and J. Q. Sun. Structural-acoustic optimization of composite sandwich structures: A review. *The Shock and Vibration Digest*, 39(3):189–200, 2007.

[4] F. Fahy and P. Gardonio. *Sound and Structural Vibration*. Academic Press, Oxford, United Kingdom, 2007.

[5] Y.-J. Kim and J.-H. Han. Identification of acoustic characteristics of honeycomb sandwich composite panels using hybrid analytical/finite element method. *Journal of Vibration and Acoustics*, 135(1):11006, 2013.

[6] G. A. Kriegsmann. Assessment of a new radiation damping model for structural acoustic interaction. *Journal of the Acoustical Society of America*, 86(2):788–794, 1989.

[7] R. A. Mangiarotty. Acoustic radiation damping of vibrating structures. *Journal of the Acoustical Society of America*, 35(3):369–377, 1963.

[8] S. Marburg and B. Nolte. A unified approach to finite and boundary element discretization in linear time-harmonic acoustics. In S. Marburg and B. Nolte, editors, *Computational Acoustics of Noise Propagation in Fluids. Finite and Boundary Element Methods*, pages 1–34. Springer, Berlin, Germany, 2008.

[9] H. Peters, N. Kessissoglou, and S. Marburg. Modal decomposition of exterior acoustic-structure interaction. *Journal of the Acoustical Society of America*, 133(5):2668–2677, 2013.

[10] H. Peters, S. Marburg, and N. Kessissoglou. Structural-acoustic coupling on non-conforming meshes with quadratic shape functions. *International Journal for Numerical Methods in Engineering*, 91(1):27–38, 2012.

[11] O. C. Zienkiewicz, R. L. Taylor, and D. Fox. *The Finite Element Method for Solid and Structural Mechanics*. Butterworth-Heinemann, Oxford, United Kingdom, 2014.