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The max-norm minimization in non-synchronous measurements

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Abstract
Acoustic imaging technique based on microphone array is limited by the size and density of the array, and the method of the non-synchronous measurements is a way to beyond these constraints. The core problem of archiving the non-synchronous measurements of microphone array boils down to a matrix completion of a block diagonal spectral matrix. In this paper, the max-norm minimization is investigated for the spectral matrix completion. Second, the rank, nuclear-norm and max-norm have been investigated as complexity measures in the context of spectral matrix completion, and their performances are compared at different frequencies and low signal-to-noise ratio.
Keywords: Max-norm Minimization, Low-rank Model, Microphone Array Signal Processing

1 INTRODUCTION

Acoustic imaging is an array-based measurement method for sound source location and quantification, which has been widely applied in the underwater detection, medical imaging and so on [1, 2]. Two well-known acoustic imaging techniques are Beamforming (BF) and Near-field Acoustical Holography (NAH). Beamforming is an acoustic imaging technique for medium to long measurement distance. The resolution of BF is inversely proportional to the array diameter measured in units of wavelength. The array diameter is required very large at low frequencies, which usually cannot be satisfied. NAH requires short measurement distances, which is less than half a wavelength. The number of measurement points is supposed to be very dense at high frequency[4].

A way for achieving a large array and high microphone density is to scan the object of interest by moving sequentially a prototype array (an arbitrary array), which is referred to as non-synchronous measurements [5]. In comparison to a large array and high microphone density array that can acquire simultaneously all the information of the spectral matrix, in particular all cross-spectra, non-synchronous measurements can only acquire a block diagonal spectral matrix, while the cross-spectra between the non-synchronous measurements remain unknown due to the missing phase relationships between consecutive positions.

The unknown cross-spectra can be estimated by the low-rank matrix completion (MC) techniques [6]. The rank minimization and the minimization of its convex relaxation, such as nuclear-norm minimization, are core processes in the MC algorithms. The rank and nuclear-norm are used in the cross-spectral matrix (CSM) completion problem in Ref. [6, 7] with corresponding cyclic projection (CP) algorithm and FISTA algorithm. It is verified that the nuclear norm outperforms the rank in the condition of different signal-to-noise ratio (SNR). Although it is considered that the max-norm performs better than the rank or the nuclear-norm in the theoretical conditions, there is no analysis of how the max-norm performs in the CSM completion problem. It is noted that the MC techniques of max-norm in aforementioned investigations cannot be directly used for solving the CSM completion problem. The max norm minimization algorithm for CSM completion problem is supposed to be introduced.

2 ACOUSTICAL SOURCE RECONSTRUCTION FROM NON-SYNCHRONOUS MEASUREMENTS

2.1 Problem statement

The sound pressures are measured by the microphone array, and then the CSM in each measurement can be estimated. These CSMs can be methodically rearranged in the block diagonal positions of an incomplete CSM. The scheme of the non-synchronous measurement is shown in Fig. 1. When the CSM is completed, the acoustic image can be easily reconstructed. With the purpose of implementing the non-synchronous measurements, the problem is formulated as to seek a full CSM from the CSM of data missing.

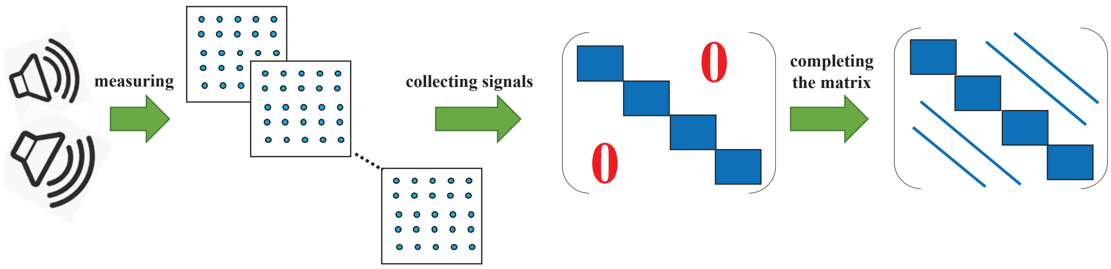


Figure 1. Scheme of non-synchronous measurements: the sound pressure is measured with a moving small microphone array. The collected signals are formulated as a CSM with data missing. The missing elements in the CSM are estimated.

2.2 The matrix complexity measures

Matrix complexity was introduced in Ref. [8] to analyze the low-rank matrix completion problem. It is indicated in Ref. [8] that the complexity can be measured by the rank, nuclear-norm and max-norm. The choice of the complexity measure in the matrix completion problem is highly relevant to the performance of the solving methods. Thus, the properties of each matrix complexity measure (rank, nuclear-norm and max-norm), and the relationships of the complexity measures are supposed to be investigated. For a matrix $\mathbf{S} \in \mathbb{R}^{m\times n}$, the complexity measures are defined as below.

1. Rank$(\mathbf{S})$ is the the minimum k such that $\mathbf{S} = \mathbf{U}\mathbf{V}^*, \mathbf{U} \in \mathbb{R}^{m\times k}, \mathbf{V} \in \mathbb{R}^{n\times k}$.

2. Nuclear-norm $\|\mathbf{S}\|_*$: $\|\mathbf{S}\|_* := \sum_{k=1}^{\min(m,n)} \sigma_k^2(\mathbf{S})$, which is the sum of all eigenvalues values: $\sigma_1^2(\mathbf{S}) \geq \sigma_2^2(\mathbf{S}) \geq \cdots \sigma_{\min(m,n)}^2(\mathbf{S})$. And nuclear-norm can also be equivalently written as:

$$\|\mathbf{S}\|_* = \inf\left\{\sum_j |\sigma_j^2(\mathbf{S})| : \mathbf{S} = \sum_j \sigma_j^2(\mathbf{S}) u_j v_j^T, u_j \in \mathbb{R}^m, v_j \in \mathbb{R}^n, \|u_j\|_2 = \|v_j\|_2 = 1\right\}, \tag{1}$$

   where the u_j and v_j denote eigenvectors of the matrix $\mathbf{S}$ corresponding to the jth eigenvalue σ_j^2.

3. Max-norm $\|\mathbf{S}\|_{\max}$ is defined as matrix factorization: $\|\mathbf{S}\|_{\max} = \min\{\|\mathbf{L}\|_{2,\infty}\|\mathbf{R}\|_{2,\infty} : \mathbf{S} = \mathbf{L}\mathbf{R}^*\}, \mathbf{L} \in \mathbb{R}^{m\times k}, \mathbf{R} \in \mathbb{R}^{n\times k}$, where $\|\cdot\|_{2,\infty}$ denotes the maximum row norm of a matrix: $\|\mathbf{A}\|_{2,\infty} := \max_j (\sum_j \mathbf{A}_{jk}^2)^{1/2}$ and $\mathbf{A}_{jk}$ denotes the element of $\mathbf{A}_{jk}$ at jth row kth column.

2.3 Max-norm minimization

The max-norm is an alternative convex relaxation for the rank in the low-complexity matrix completion problem, which is considered to be better than nuclear-norm in theoretical cases [9]. It is defined that the M is the number of microphones in the prototype array and P the number of non-synchronous measurements. When the sound pressures $p \in \mathbb{C}^M$ are measured by the microphone array, the CSM $\hat{\mathbf{S}}_{pp}^{(i)} \in \mathbb{C}^{M\times M}$ of ith measurement can be estimated. The CSMs $\hat{\mathbf{S}}_{pp}^{(i)},\ i = 1, ..., P$ (spectral matrices for short hereafter) in non-synchronous measurements are methodically rearranged in the block diagonal positions of an incomplete CSM $\hat{\mathbf{S}}_{pp}^{m} \in \mathbb{C}^{MP\times MP}$. The data missing CSM completion based on max-norm can be formulated as the following constrained optimization problem:

$$\begin{aligned} \underset{\mathbf{S}}{\text{minimize}} \quad & \|\mathbf{S}\|_{\max} \\ \text{subject to} \quad & \|\mathscr{A}(\mathbf{S}) - \hat{\mathbf{S}}_{pp}^{m}\|_F \le \varepsilon \\ & \|\mathbf{\Psi}\mathbf{S}\mathbf{\Psi}^* - \mathbf{S}\|_F \le \varepsilon \\ & \mathbf{S}^* = \mathbf{S} \succeq 0, \end{aligned} \tag{2}$$

1. Object function: $\|\mathbf{S}\|_{\max}$ is the max-norm of $\mathbf{S}$.
2. System equation constraint: measurement data fitting is imposed as a constraint $\|\mathscr{A}(\mathbf{S}) - \hat{\mathbf{S}}_{pp}^{m}\|_F \le \varepsilon$, where $\|\cdot\|_F$ denotes the Frobenius norm: $\|\mathbf{X}\|_F = \sqrt{\sum_{ij}\mathbf{X}_{ij}^2}$. The difference between $\mathscr{A}(\mathbf{S})$ and $\hat{\mathbf{S}}_{pp}^{m}$ in Frobenius norm is less than a given tolerance ε that is composed of estimation and measurement errors. The sampling operator $\mathscr{A}$ is defined formally as follows.

$$\mathbf{S}_{pp}^{m} = \mathscr{A}(\mathbf{S}) \quad \text{with} \quad [\mathbf{S}_{pp}^{m}]_{ij} = \begin{cases} [\mathbf{S}]_{ij} & ij \in \mathbf{\Upsilon} \\ [\mathbf{S}]_{ij} & ij \notin \mathbf{\Upsilon}, \end{cases}$$

   where $[]_{ij}$ denotes the (i, j) element of a matrix and $\mathbf{\Upsilon}$ denotes the positions of fixed block diagonal entries.
3. Spatial continuity constraint: in order to ensure spatial continuity of the acoustic field, the information on microphone positions must be encoded in the CSM: $\|\mathbf{\Psi}\mathbf{S}\mathbf{\Psi}^* - \mathbf{S}\|_F \le \varepsilon$, where $\mathbf{\Psi} = \mathbf{\Phi}\mathbf{\Phi}^\dagger$ is a projection basis (the purpose of this representation is to construct an inner structure of the CSM as a constraint to solve a matrix completion problem). The $\mathbf{\Psi}^*$ denotes the transpose-conjugate of $\mathbf{\Psi}$, $\mathbf{\Phi} \in \mathbb{C}^{MP\times K_p}$ and $\mathbf{\Psi} \in \mathbb{C}^{MP\times MP}$, where K_p is the dimension of basis.
4. Hermitian property constraint: $\mathbf{S}^* = \mathbf{S} \succeq 0$, where $\succeq$ denotes positive semi-definite. The Hermitian property is an inherent characteristic of the CSMs, as well as the fact that its eigenvalues are non-negative.

The object function, the max-norm of the CSM, can be further rewritten as

$$\|\mathbf{S}\|_{\max} = \underset{(\mathbf{L},\mathbf{R}):\mathbf{L}\mathbf{R}^*=\mathbf{S}}{\text{minimize}} \quad \max\{\|\mathbf{L}\|_{2,\infty}^2, \|\mathbf{R}\|_{2,\infty}^2\}. \tag{3}$$

The formulation (3) of the max-norm is non-convex due to the constraint on the product $\mathbf{L}\mathbf{R}^*$. The aforementioned problem (2) can be rewritten with the max-norm as a regularization term:

$$\begin{aligned} \underset{\mathbf{S}}{\text{minimize}} \quad & \|\mathscr{A}(\mathbf{S}) - \hat{\mathbf{S}}_{pp}^{m}\|_F + \beta\|\mathbf{S}\|_{max} \\ \text{subject to} \quad & \|\mathbf{\Psi}\mathbf{S}\mathbf{\Psi}^* - \mathbf{S}\|_F \le \varepsilon \\ & \mathbf{S}^* = \mathbf{S} \succeq 0, \end{aligned} \tag{4}$$

where β is a positive penalty parameter. Let the object matrix $\mathbf{S}$ is factorized as $\mathbf{S} = \mathbf{L}\mathbf{R}^*$. A matrix $\mathbf{A}$ is defined to the matrix of factors stacked on top of one another $\mathbf{A} = \begin{bmatrix}\mathbf{L}\\ \mathbf{R}\end{bmatrix}$. Thus, it can be obtained that

$\|\mathbf{A}\|_{2,\infty}^2 = \max\{\|\mathbf{L}\|_{2,\infty}^2, \|\mathbf{R}\|_{2,\infty}^2\}$. Also let $\tilde{f}(\mathbf{A})$ denote $\|\mathscr{A}(\mathbf{S}) - \hat{\mathbf{S}}_{pp}^m\|_F^2$. The original max-norm minimization problem can be further written as

$$\begin{aligned} \underset{\mathbf{S}}{\text{minimize}} \quad & \tilde{f}(\mathbf{A}) + \lambda\|\mathbf{A}\|_{2,\infty} \\ \text{subject to} \quad & \text{rank}(\mathbf{S}) \le d \\ & \|\mathbf{\Psi}\mathbf{S}\mathbf{\Psi}^* - \mathbf{S}\|_F \le \varepsilon \\ & \mathbf{S}^* = \mathbf{S} \succeq 0. \end{aligned} \tag{5}$$

A constraint of rank is added to accelerate convergence in the above max-norm minimization problem (5). A proximal-point algorithm can be obtained to solve the max-norm minimization problem (3) based on Ref. [3].

2.4 Simulation

In general, the spatial sampling frequency of the microphone array is governed by the Nyquist Shannon sampling theorem, which declares that the working frequency range of the array is determined by its size and the microphone density [4]. It is considered according to Nyquist Shannon sampling theorem that the working frequency must be lower than 1700 Hz in the simulation, or the measurements are invalidated. The matrix completion error (MCE) and acoustic reconstruction error (ARE) of the three matrix complexity measures versus different working frequency are shown in Fig. 2. The CSM completion error (MCE) is calculated as

$$\text{MCE} = \frac{\|\mathbf{S}_{pp}^{\{sim\}} - \mathbf{S}_{pp}^{\{com\}}\|_F}{\|\mathbf{S}_{pp}^{\{sim\}}\|_F}, \tag{6}$$

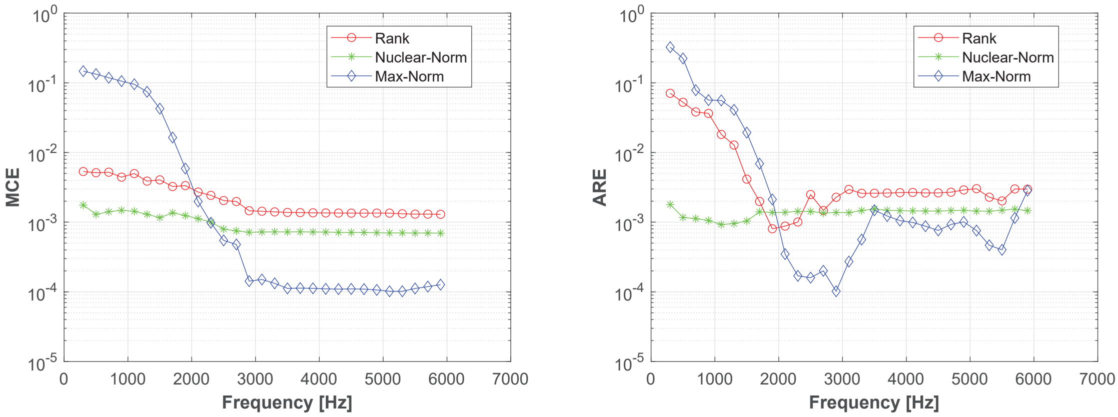
where $\mathbf{S}_{pp}^{\{sim\}}$ is the true (simulated) CSM and $\mathbf{S}_{pp}^{\{com\}}$ is the completed one. The acoustical reconstruction error (ARE) is calculated as

$$\text{ARE} = \frac{\|\mathbf{S}^{\{sim\}} - \mathbf{S}^{\{rec\}}\|_F}{\|\mathbf{S}^{\{sim\}}\|_F}, \tag{7}$$

where $\mathbf{S}^{\{rec\}}$ is the reconstructed CSM from the completed CSM. Figure. 2 shows that the MCE or ARE of the three complexity measures are far lower than 1.0 under any working frequency, which indicate that the non-synchronous measurements make a breakthrough of Nyquist Shannon sampling theorem. The ARE and MCE of the max-norm are relatively high at low frequency (< 2100Hz), and those of nuclear-norm are fairly low at any frequency. However, at high frequency (> 2100Hz), the MCE and ARE of max-norm are significantly lower than nuclear-norm. In other words, max-norm owns superiority at high frequency in the acoustic reconstruction problem.

High noise is added here to compare the performance of the three matrix complexity measures. Fig. 3 shows the MCE and ARE of the three complexity measurements versus different frequency. The MCE of the max-norm are the highest at low frequency (< 2100Hz), while the MCE of the max-norm equals to the MCE of the rank or the nuclear-norm at high frequency (> 2100Hz). The ARE of max-norm is almost the lowest, especially at high frequency (> 2100Hz). The robustness of max-norm is better than rank, and nuclear-norm in the case of noise.

The images of acoustic source reconstruction based on the full CSM and data missing CSMs (completed by rank minimization, nuclear-norm minimization and max-norm minimization) are shown in Fig. 4. Figure. 4 shows that the results of acoustic image reconstruction based on max-norm minimization is significantly better than the reconstruction results of the rank minimization and the nuclear-norm minimization at SNR $= 20$ dB, which indicates that the max-norm minimization is far more robust to noise than the rank minimization and the nuclear-norm minimization.



(a) (b)

Figure 2. Reconstruction error versus working frequency without noise: (a) MCE, (b) ARE. Results of rank are shown by red circles; nuclear-norm by green asterisks. max-norm by blue diamonds.

3 Conclusion

In summary, the max-norm minimization of the acoustic CSM completion problem is investigated in the non-synchronous measurements. The rank, nuclear-norm and max-norm are compared in the simulation. The results of the simulation show that the reconstruction errors of each complexity measures are determined by working frequency. The performance of the max-norm is superior to the rank or the nuclear-norm at high frequency (> 2100Hz).

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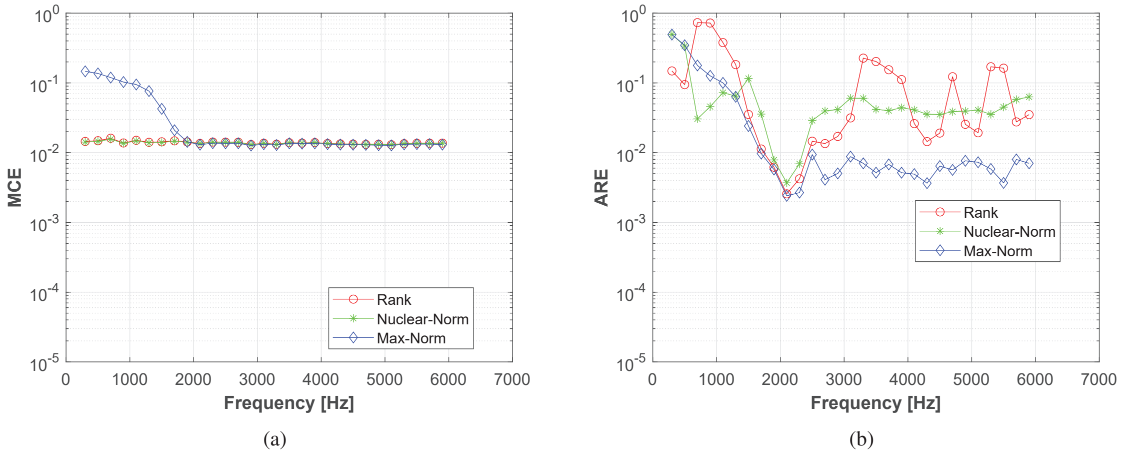


Figure 3. Reconstruction error versus working frequency : (a) MCE versus frequency with SNR= 20 dB, (b) ARE versus frequency with SNR= 20 dB. The results of rank are shown by red circles; nuclear-norm by green asterisks; max-norm by blue diamonds.

REFERENCES

[1] Kompis M, Pasterkamp H, Wodicka G R. Acoustic imaging of the human chest[J]. Chest, 2001, 120(4): 1309-1321.

[2] Sutton J L. Underwater acoustic imaging[J]. Proceedings of the IEEE, 1979, 67(4): 554-566.

[3] Rockafellar R T. Monotone operators and the proximal point algorithm[J]. SIAM journal on control and optimization, 1976, 14(5): 877-898.

[4] Hald J. Combined NAH and beamforming using the same array[J]. Technical, 2005: 11.

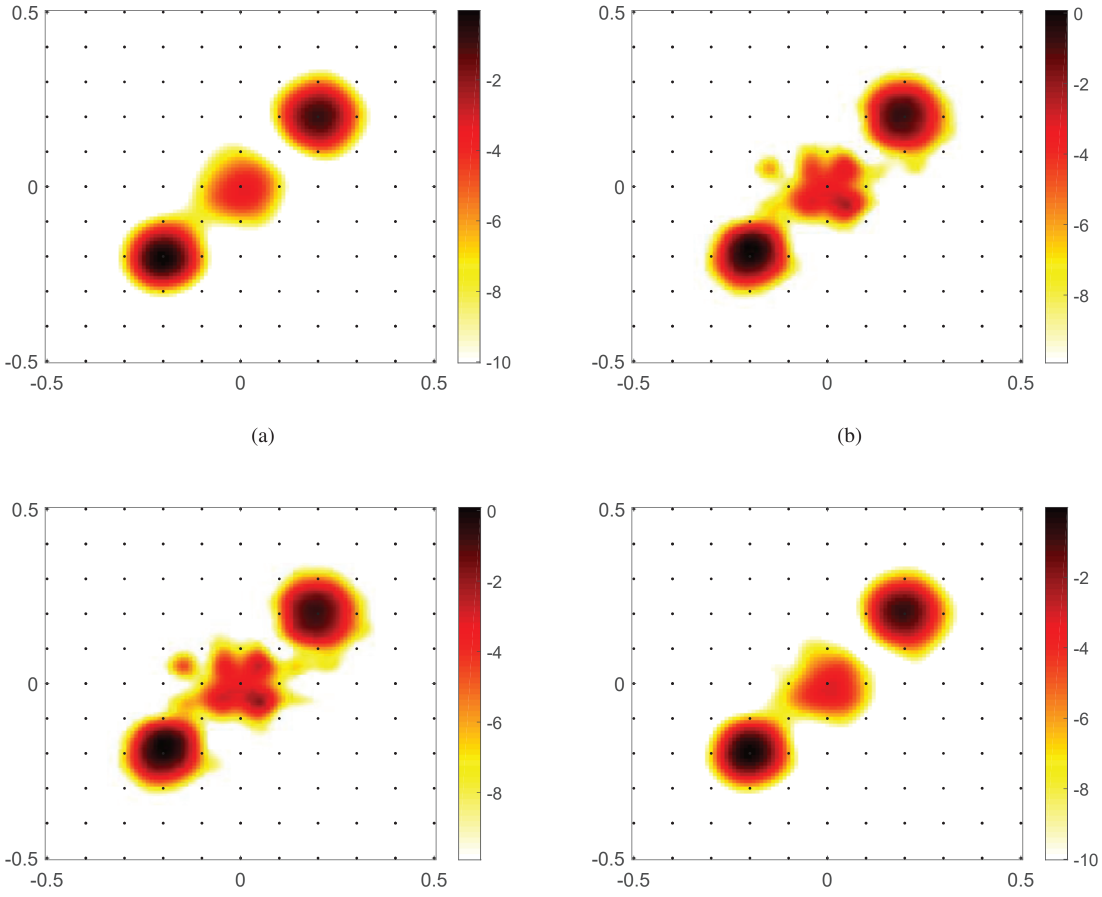
[5] Yu L. Acoustical source reconstruction from non-synchronous sequential measurements[D]. INSA de Lyon, 2015.

[6] Yu L, Antoni J, Leclere Q. Spectral matrix completion by Cyclic Projection and application to sound source reconstruction from non-synchronous measurements[J]. Journal of Sound and Vibration, 2016, 372: 31-49.

[7] Yu L, Antoni J, Leclere Q, et al. Acoustical source reconstruction from non-synchronous sequential measurements by Fast Iterative Shrinkage Thresholding Algorithm[J]. Journal of Sound and Vibration, 2017, 408: 351-367.

[8] Srebro N, Shraibman A. Rank, trace-norm and max-norm[C]//International Conference on Computational Learning Theory. Springer, Berlin, Heidelberg, 2005: 545-560.

[9] Cai T T, Zhou W X. Matrix completion via max-norm constrained optimization[J]. Electronic Journal of Statistics, 2016, 10(1): 1493-1525.



(a) (b) (c) (d)

Figure 4. acoustic source reconstruction based on (a) full spectral matrix, (b) data missing spectral matrix completed by rank minimization, (c)data missing spectral matrix completed by nuclear-norm minimization and (d) data missing spectral matrix completed by max-norm minimization. The $f = 3000$ Hz and SNR$= 20$ dB.