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Selective identification of structural force distribution

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ABSTRACT

On the one hand, in the field of structural source identification, some methods have been developed during the last decades like the Force Analysis Technique, the Virtual Fields Method, the 2D spatial Fourier transform or the structural holography. The proposed method clearly falls in the same framework and is close to the philosophy behind the Virtual Fields Method.

On the other hand, in the field of acoustic source identification, the inverse Patch Transfer Function has been recently proposed to reconstruct the acoustic fields on a complex shape source in any (stationary) acoustic environment. This method is based on the concept of virtual acoustic volume. The method proposed here tries to adapt this concept to structural identification.

Therefore, the proposed method is based on the use of a virtual testing structure solved by a Finite Element solver. It allows selecting what kind of distribution force one wants to identify. In the example of a plate with welded stiffeners, if one is only interested in localizing and quantifying the applied force, the virtual testing structure has to include the stiffeners. If one is interested in forces due to the welded points, the virtual testing structure has to exclude the stiffeners.

Keywords: inverse method, vibration, force distribution

1. INTRODUCTION

Based on the same concept of virtual domain as the recently proposed inverse Patch Transfer Functions method (1) in acoustics, the present method (2) aims at reconstructing the force distribution applied on a plate-like structure with any shape and any boundary conditions.

As the Force Analysis Technique (3), the Virtual Fields Method (4), the 2D spatial Fourier transform or the structural holography (5), the proposed method is based on the equation of motion of a thin plate. The concept of virtual structure introduced here is close to the virtual fields defined in (4). However, as mode shapes of the virtual structure are used to reconstruct the force distribution in a zone of the structure, it is easy to choose what to include in the model and so what to exclude of the force distribution. The example of a razor-shaped structure with stiffeners excited by a point force is given. If the stiffeners are included in the virtual structure, only the point force appear in the identified force distribution. If the stiffeners are not included then the force distribution exhibits some force spots at rigid connections between the stiffeners and the structure. The proposed method is thus considered as a selective structural source identification method.

2. THEORETICAL BACKGROUND

Let us consider a plate-like structure with any shape and boundary condition as shown in Figure 1. A force distribution $F(M,\omega)$ is applied on the structure generating a displacement field $W(M,\omega)$ on the structure. The displacement field respects the following equation of motion

$$-m\omega^2 W(M,\omega) + D\Delta^2 W(M,\omega) = F(M,\omega) \quad (1)$$

where m and D are respectively the surface mass and the bending stiffness of the plate and ω is the angular frequency of excitation. In the following, the objective is to identify the force distribution $F(M,\omega)$ by measuring the transverse displacement $W(M,\omega)$.

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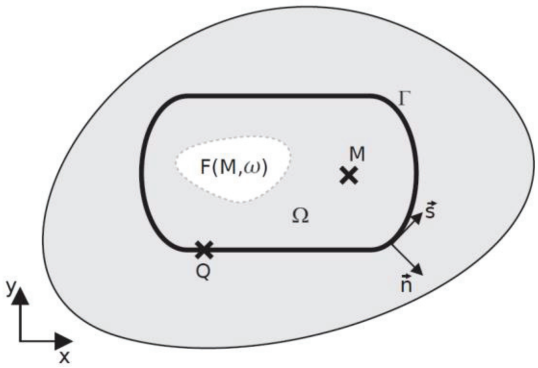


Figure 1: Tested plate of unknown shape and boundary conditions excited by a force distribution $F(M,\omega)$. A contour Γ delimits a surface Ω on this plate.$\vec{n}$ and $\vec{s}$ are the normal and tangential vectors of contour Γ.

A contour Γ of arbitrary shape delimiting a zone Ω on the surface of the structure is defined. As expressed in (6), the Green's identity applied on the equation of motion (1) can be written as

$$\int_{\Omega} F(M,\omega)\eta(M)dM = \int_{\Omega} \big(-m\omega^2\eta(M) + D\Delta^2\eta(M)\big)W(M,\omega)dM$$
$$+\int_{\Gamma} W(Q,\omega)\mathcal{T}\big(\eta(Q)\big) - \eta(Q)\mathcal{T}\big(W(Q,\omega)\big)dQ \quad (2)$$
$$-\int_{\Gamma} \frac{\partial W(Q,\omega)}{\partial n}\mathcal{M}_f\big(\eta(Q)\big) - \frac{\partial \eta(Q)}{\partial n}\mathcal{M}_f\big(W(Q,\omega)\big)dQ$$

where $\eta(M)$ (resp. $\eta(Q)$) is an arbitrary function (continuous and twice differentiable) defined on Ω (resp. Γ). $\mathcal{M}_f(\blacksquare)$ and $\mathcal{T}(\blacksquare)$ are respectively the bending moment and shear force operators (1).

In Equation (1), the terms $W(Q,\omega)$, $\frac{\partial W(Q,\omega)}{\partial n}$, $\mathcal{M}_f\big(W(Q,\omega)\big)$ and $\mathcal{T}\big(W(Q,\omega)\big)$ are considered as measurable quantities. On the contrary, $\eta(Q)$ is testing function that can be chosen arbitrarily. To determine $F(M,\omega)$ in Equation (2), $\eta(Q)$ has been chosen as a mode shape of a virtual plate with clamped boundary conditions on the contour Γ. The mode shapes then respect the following equations

$$-m\omega_p^2\phi_p(M) + D\Delta^2\phi_p(M) = 0 \qquad M \in \Omega \quad (3)$$
$$\phi_p(Q) = 0 \quad Q \in \Gamma \quad (4)$$
$$\frac{\partial \phi_p}{\partial n}(Q) = 0 \quad Q \in \Gamma \quad (5)$$

Considering Equations (3) to (5) in Equation (2), it yields

$$F_p(\omega) = m\big(\omega_p^2 - \omega^2\big)\int_{\Omega} \phi_p(M)W(M,\omega)dM$$
$$+\int_{\Gamma} W(Q,\omega)\mathcal{T}\big(\eta(Q)\big)dQ - \int_{\Gamma} \frac{\partial W(Q,\omega)}{\partial n}\mathcal{M}_f\big(\eta(Q)\big)dQ \quad (6)$$

where $F_p(\omega)$ are the modal masses of virtual mode p also defined as $\mathbf{F_p} = \mathbf{AF}$ where $\mathbf{F_p}$ is the vector of modal masses, **A** is the matrix of mode shapes ϕ_p and **F** is the force distribution on the degrees of freedom in domain Ω. Therefore, the modal masses can be estimated measuring displacement on surface Ω and contour Γ and rotation $\frac{\partial W(Q,\omega)}{\partial n}$ on contour Γ. Finally, as $\phi_p(M)$ constitute an orthonormal basis of functions, the force distribution is simply obtained by

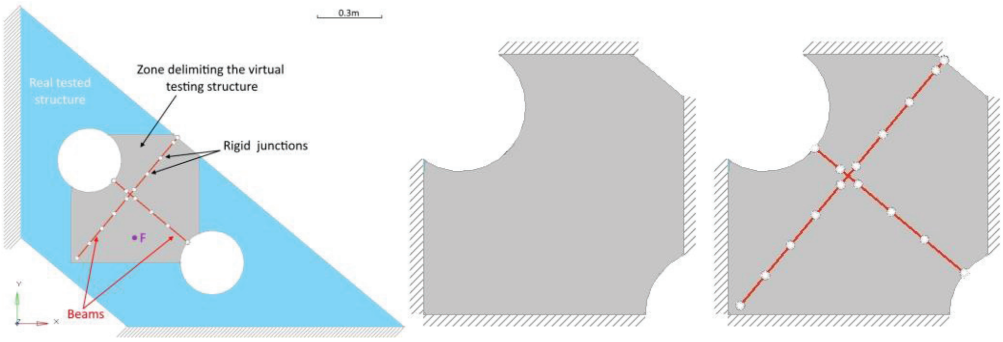
$$\mathbf{F} = \mathbf{A}^{\mathbf{T}}\mathbf{F}_{\mathbf{p}} \tag{7}$$

where $\blacksquare^T$ stands for the transpose of a matrix.

3. NUMERICAL EXAMPLE

3.1 Structure under study

To illustrate the method proposed previously, a razor-shaped structure excited by a point force is investigated as presented in Figure 2. This structure is clamped on two edges and is pierced by two circular holes of same dimensions. Two stiffeners are fixed on the plate by 10 rigid connections for the longest one and by 6 for the shortest. This structure will be referred in the following as the "tested plate". The objective is here to reconstruct the force distribution on the grey zone in Figure 2. To achieve this result, two testing structures will be used. The first one, presented in Figure 2(b) doesn't have any stiffeners and is clamped on most of its edges. The second one, presented in Figure 2(c), exhibits the same stiffeners as those of the tested structure.



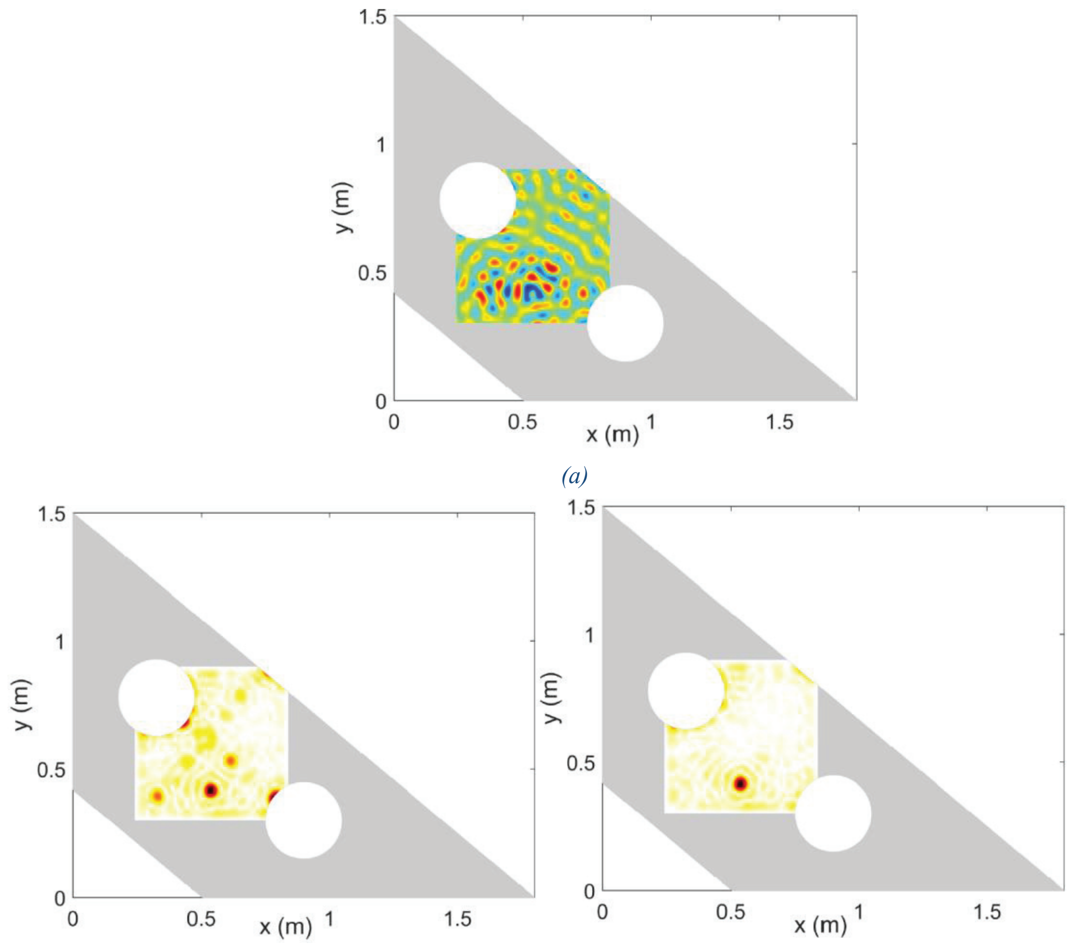
(a) *(b)* *(c)*

Figure 2: (a) Tested structure excited by a point force F; (b) virtual structure without stiffeners, clamped on some edges; (c) virtual structure with stiffeners, clamped on some edges

Mode shapes of the two testing plates are extracted with a commercial FE software and Equations (6) and (7) are applied to reconstruct the force distribution.

3.2 Results

The results are presented in Figure 3. Figure 3(a) presents the displacement field "measured" on the tested structure on the surface Ω. Figure 3(b) and Figure 3(c) present respectively the force distribution obtained from mode shapes of the structure without stiffeners and from mode shapes of the structure with stiffeners.



(a)

(b) (c)

Figure 3: (a) displacement field (m) measured (numerical experiment) on the real structure in the zone of the virtual structure at 2100Hz; (b) force distribution (N/m²) identified using the virtual structure without stiffeners (Figure 2 (b)); (c) force distribution identified using the virtual structure with stiffeners (Figure 2 (c))

One can see in Figure 3 that some force spots appear in the first force distribution. They are due to the point force itself but also to the forces implied by the rigid connections of the stiffeners. It is easy to evaluate here that 4 rigid connections among the 16 generate non negligible force levels. In the second case, only one spot due to the point force appears. The influence of rigid connections is canceled because they are taken into account in the mode shapes of the virtual structure. Figure 3 demonstrates that the proposed method can act as a selective structural source identification method.

4. CONCLUSIONS

The proposed method aims at reconstructing the force distribution applied on a plate-like structure. This method is based on the concept of virtual structure. The mode shapes of this virtual structure with clamped boundary conditions on the contour of the tested zone are used to reconstruct the force distribution in this zone.

Thanks to the use of this orthonormal basis of functions, no inversion is needed making the proposed method hardly sensitive to background noise.

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