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A target direction search algorithm based on microphone array

Xubin LIANG[1]; Difeng SUN[1]; Tianqing ZHAO[1]; Liangyong ZHANG[1]; Houlin FANG[1]; Fang ZHANG[1]

[1] Northwest Institute of Nuclear Technology, China

ABSTRACT

The use of a microphone array to obtain the direction of a far-field target is an important means of impacting-point detection. In this paper, a new search algorithm based on time-delay orientation is proposed, which USES the time-delay and position vector of the microphone pair in the array to estimate the unit vector of the target direction by using the least square method, and then calculates the azimuth and pitch Angle of the target through this vector. On this basis, calculate the Euclidean distance between the projection of the position vector in the target direction and the difference of the sound path between the microphones. By setting the distance threshold, use a certain search rule to eliminate the time delay when the participation calculation results in a relatively large distance. Extend, thereby finalizing the azimuth and elevation of the target. The simulation analysis of the proposed algorithm is carried out. Compared with the orientation results of the unused search algorithm, the proposed algorithm not only has high orientation accuracy, but also has strong stability and high reliability. It can avoid the problem of angle calculation error caused by the damage of the microphone or individual time delay calculation errors. It has important reference significance for the practical engineering application of using time-delay orientation such as the target impacting-point measurement.

Keywords: Microphone array, Time-delay orientation, Search algorithm

1. INTRODUCTION

Orienting the target with the microphone array is the basis for the target detection and positioning. Accurately obtaining the direction information of the target is the key and basis for obtaining the target position information. At present, there are two methods for obtaining the target direction by using the microphone array. One is to obtain the time delay between the microphone array and the reference microphone (1-3), and use the geometric relationship of each microphone in the array to solve the target direction (4-6); The other is to use the multiple signal subspace method (MUSIC) (7) to solve the direction of multiple targets, but the signal must be continuous and stable. For the first orientation method, the time delay of the microphone pair must be obtained. If the desired time delay is wrong due to poor performance or damage of a certain microphone, or individual delay calculation errors caused by other circumstances, the orientation result will be inaccurate or even wrong. But this kind of situation are often found in engineering applications. If you can find a directional method that can avoid such situations, and does not affect the directional performance, it will be of great significance to engineering applications.

In this paper, a target direction search algorithm based on microphone array is proposed, and the orientation model of arbitrary array is introduced. On this basis, certain search rules are used to eliminate some wrong time delay, and the target direction is finally determined. Finally, the algorithm is simulated and compared.

2. ARBITRARY ARRAY ORIENTATION MODEL

Figure 1 shows the arbitrarily distributed N-element microphone array and sound source geometry in three-dimensional space, where N microphones are isotropic, the target sound source is located at the far field P point, and the azimuth and elevation angles are represented by φ and θ, respectively. The position vector of the m-th element is represented by $\mathbf{r}_m = [x_m, y_m, z_m]^T$, and the direction vector of the j-th microphone to the i-th is represented as $\mathbf{r}_{ij}=\mathbf{r}_j - \mathbf{r}_i$, where $\{i\in[1, N\text{-}1], j\in[1, N], j > i\}$, a total of Q direction vectors can be formed, where Q is

[1] liangxubin@nint.ac.cn

$$Q = C_N^2 = N \times (N-1)/2 \tag{1}$$

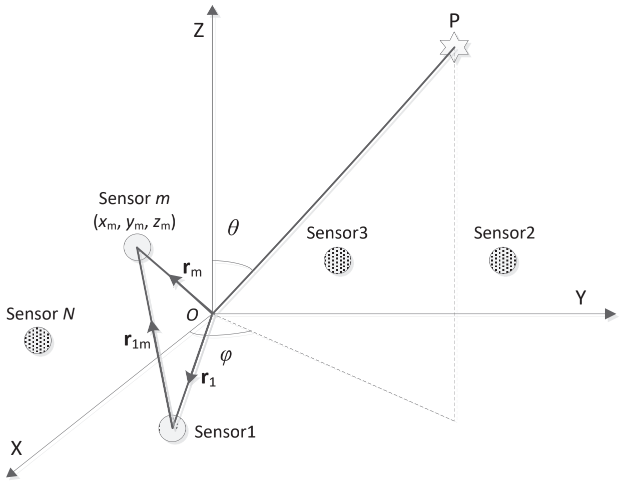


Figure 1 – Spatial N-element microphone array and sound source geometry

Each pair of N microphones obtains a time delay τ_{ij}, which represents the time delay of the microphone of the j-th with respect to the microphone of the i-th, and can obtain Q delays, which are expressed as vectors:

$$\boldsymbol{\tau} = [\tau_{12}, \tau_{13}, \cdots, \tau_{(N-1)N}]^{\mathrm{T}} \tag{2}$$

Then the sound path difference vector $\mathbf{d} = c \cdot \boldsymbol{\tau}$, where c is the speed of sound. The unit vector from the sound source to the origin O is defined as $\mathbf{k}$

$$\mathbf{k} = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} = \begin{bmatrix} \sin(\theta)\cos(\varphi) \\ \sin(\theta)\sin(\varphi) \\ \cos(\theta) \end{bmatrix} \tag{3}$$

It is assumed that the target sound source P is located in the far field and the sound wave propagates as a spherical wave. When the array radius is small, it is considered that the sound wave propagates as a plane wave when passing through the array. The direction vector of Q microphone pairs is expressed as a matrix as follows:

$$\mathbf{R} = \begin{bmatrix} \mathbf{r}_{12} & \mathbf{r}_{13} & \cdots & \mathbf{r}_{(N-1)N} \end{bmatrix}^{\mathrm{T}} \tag{4}$$

Then there is

$$\mathbf{Rk} = c \cdot \boldsymbol{\tau} \tag{5}$$

As shown in equation (3), c and $\mathbf{R}$ are known quantities, $\boldsymbol{\tau}$ can be obtained by time delay estimation, and the problem of solving the target azimuths φ and θ is converted into the solution direction vector $\mathbf{k}$. Here, N must be greater than or equal to 3, that is, at least a 3-element array. When N=3, the matrix $\mathbf{R}$ is a non-singular square matrix. The inverse matrix $\mathbf{R}^{-1}$ of $\mathbf{R}$ can be directly obtained to obtain $\mathbf{k}=c\cdot\mathbf{R}^{-1}\boldsymbol{\tau}=c\cdot(\mathbf{R}^{\mathrm{T}}\mathbf{R})^{-1}\mathbf{R}^{\mathrm{T}}\boldsymbol{\tau}$; when $N > 3$, equation (5) is an overdetermined equation, there is no solution that strictly satisfies the equation, only the least square solution with the lowest norm (8).

$$\hat{\mathbf{k}} = \min_{\mathbf{k}} \arg(\|\mathbf{Rk} - c \cdot \boldsymbol{\tau}\|) = c \cdot (\mathbf{R}^{\mathrm{T}}\mathbf{R})^{-1}\mathbf{R}^{\mathrm{T}}\boldsymbol{\tau} \tag{6}$$

Therefore, when N⩾3, $\hat{\mathbf{k}} = c \cdot (\mathbf{R}^{\mathrm{T}}\mathbf{R})^{-1}\mathbf{R}^{\mathrm{T}}\boldsymbol{\tau}$. Thus, the sound source azimuth angle φ and the pitch angle θ can be calculated by the direction vector $\mathbf{k}$.

$$\hat{\varphi} = \arctan(\hat{k}_y / \hat{k}_x) \qquad -\pi < \hat{\varphi} \le \pi \tag{7}$$

$$\hat{\theta} = \arctan\left(\sqrt{\hat{k}_x^2 + \hat{k}_y^2} \Big/ \hat{k}_z\right) \qquad 0 \le \hat{\theta} \le \pi/2 \tag{8}$$

3. ANGLE SEARCH ALGORITHM

3.1 Search Rule

In the case where all microphones have good performance and all the Q time delays are calculated correctly, the direction of the target can be correctly calculated using equations (6)-(7). However, in the case of one or more time delay errors, using equations (6)-(7) to calculate the orientation will result in large errors or even errors, and certain rules are needed to eliminate the erroneous delay and thus estimate the correct **k**. Let $\Delta\mathbf{d}$ be

$$\Delta\mathbf{d} = \mathbf{R}\hat{\hat{\mathbf{k}}} - c\cdot\boldsymbol{\tau}, \qquad \hat{\hat{\mathbf{k}}} = a\cdot\hat{\mathbf{k}} = \begin{bmatrix} \sin(\hat{\theta})\cos(\hat{\varphi}) \\ \sin(\hat{\theta})\sin(\hat{\varphi}) \\ \cos(\hat{\theta}) \end{bmatrix} \tag{9}$$

$\Delta\mathbf{d}$ is the difference between the projection of the position vector of microphone pair on the target direction and the sound path difference, where $\hat{\hat{\mathbf{k}}}$ is the target direction unit vector, which is composed of the angle estimation value, a is the coefficient, generally $a\neq 1$, and $\hat{\hat{\mathbf{k}}}^{\mathrm{T}}\hat{\hat{\mathbf{k}}}\equiv 1$. The threshold T_0 is set. When $\|\Delta\mathbf{d}\| < T_0$, the value of **k** obtained by equation (5) is considered to be valid; when $\|\Delta\mathbf{d}\|\geqslant T_0$, it is necessary to re-estimate **k** with the new matrix $\mathbf{R}_{\mathrm{nm}}$, where $\mathbf{R}_{\mathrm{nm}}$ is a sub-matrix formed by deleting n rows from **R**, m represents a different selection mode of n rows, and $\boldsymbol{\tau}_{\mathrm{nm}}$ is a sub-vector formed by deleting a corresponding rows from $\boldsymbol{\tau}$, $\Delta\mathbf{d}_{nm} = \mathbf{R}_{nm}\hat{\hat{\mathbf{k}}}_{nm} - c\cdot\boldsymbol{\tau}_{nm}$indicating that the difference between the projection of the position vector and the sound path difference that n microphone pairs are removed.

First remove 1 row to search. When $\min\{\|\Delta\mathbf{d}_{1\mathrm{m}}\|, \mathrm{m}\in[1, C_{\mathrm{Q}}^{1}]\}$ is less than the threshold T_1, stop searching, and the **k** corresponding to the minimum value is the best estimate. Otherwise, continue searching; remove 2 rows. When $\min\{\|\Delta\mathbf{d}_{2\mathrm{m}}\|, \mathrm{m}\in[1, C_{\mathrm{Q}}^{2}]\}$ is less than the threshold T_2, the search is stopped, otherwise the search is continued; and so on, until the Q-3 rows is removed, where Q is shown int equation (3). In the worst case, you need to search $C_Q^0 + C_Q^1 + C_Q^2 + \cdots + C_Q^{Q-3}$ times. When N is large, you need to search too many times. Under normal circumstances, a time delay error or poor performance of a certain microphone is more likely. Therefore, generally only consider removing a row from **R** or a few rows related to a certain microphone, so that the number of searches $C_Q^0 + C_Q^1 + N$ will be greatly reduced.

3.2 Threshold Determination

In the search algorithm, the accurate selection of the threshold T can effectively delete the error delay, and the selection of the threshold T is the key of the algorithm. According to the microphone array orientation error analysis in (4, 6), the error of the orientation angle is proportional to the delay error. When the position of the microphone array is relatively accurate, the main source of error is the time delay error, so $\mathbf{r}_{\mathrm{ij}}^{\mathrm{T}}\hat{\hat{\mathbf{k}}} - c\cdot\tau_{ij} < c\cdot\sigma_{\tau}$, where σ_{τ} is the time delay the estimation error. Therefore, the formula of the threshold value T_n can be given as

$$T_n = \sqrt{\mathrm{row}(\mathbf{R}_{nm})}c\cdot\sigma_t \tag{10}$$

In the equations (10), $\mathrm{row}(\mathbf{R}_{\mathrm{nm}})$ represents the number of rows of the matrix $\mathbf{R}_{\mathrm{nm}}$, and the rule for judging whether to continue searching is

$$\left\|(\mathbf{R}_{nm}\hat{\mathbf{k}} - c\cdot\boldsymbol{\tau}\right\| = \sqrt{(\mathbf{R}_{nm}\hat{\mathbf{k}} - c\cdot\boldsymbol{\tau})^{\mathrm{T}}(\mathbf{R}_{nm}\hat{\mathbf{k}} - c\cdot\boldsymbol{\tau})} < T_n \tag{11}$$

4. SIMULATION AND ANALYSIS

4.1 Simulation Parameter Determination

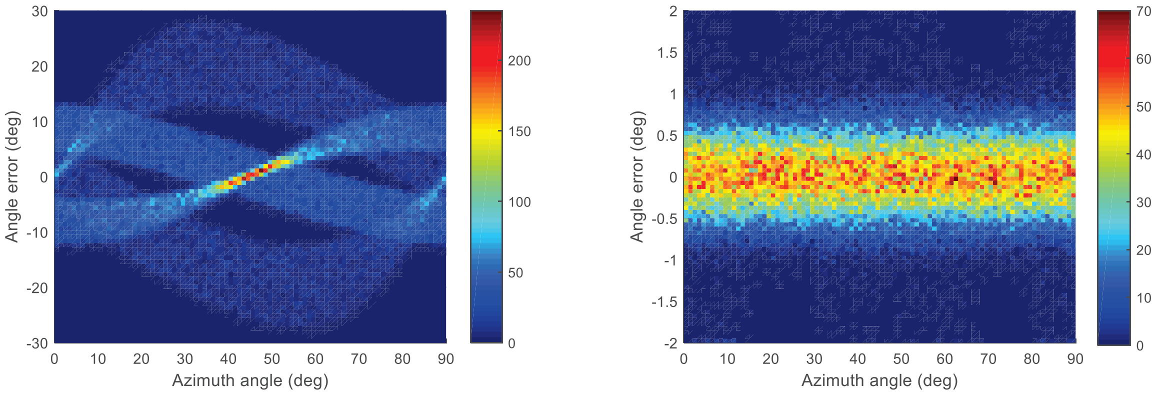
In this paper, the 2D model is selected for simulation. Only the azimuth angle is calculated. The 4-element cross-plane array is used, with an array radius of 1.5m. The sampling rate is 8000kSa, the corresponding time is 0.125ms, and there are 6 time delays for the microphone pair. The time delay used in the simulation is generated by the simulation method, as shown in equation (12).

$$\tau_{ij} = \mathbf{r}_{\text{ij}}^{\text{T}}\mathbf{k} / c + noise \tag{12}$$

The *noise* satisfies the average distribution, and the maximum amplitude is 0.125ms corresponding to 1 sampling point. Considering the symmetry of the 4-element cross-plane array, only the azimuth angle of 0~90° is selected. During the simulation, a certain time delay is randomly selected to set it to a random value, or a microphone is randomly selected, and the corresponding time delays are set to random values, wherein the random values satisfy the average distribution. The Monte Carlo method is used to simulate in these two ways. The preset angle interval is 1°. The azimuth algorithm uses the algorithms of equations (5)-(7) (hereinafter referred to as general algorithms) and angle search algorithms.

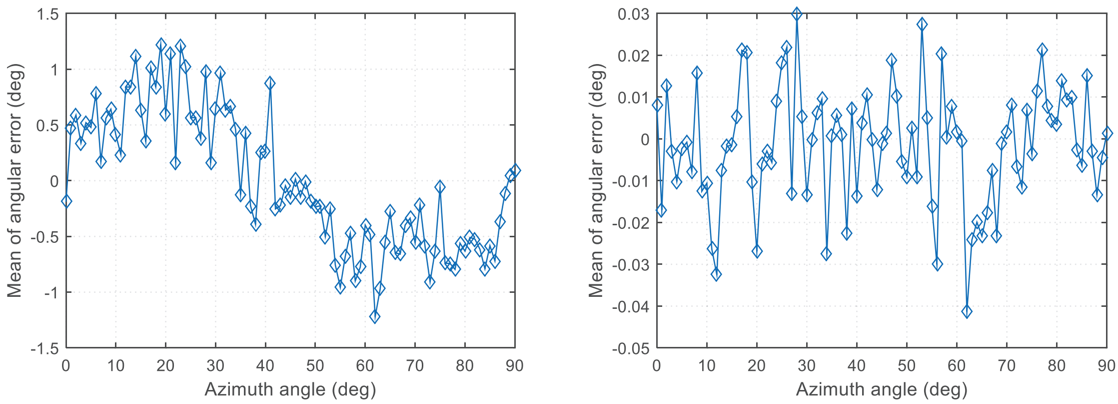
4.2 The Case of A Time Delay Error

The first method described in 4.1 is used for 1000 simulations. The angle error is defined as the difference between the calculated angle and the preset value. The error statistics of each angle are shown in Figure 2. The curves of mean and standard deviation of the angle error in the case of two algorithms are shown in Figure 3-4. The maximum mean and maximum standard deviations of the two algorithms are shown in Table 1. It can be seen from Fig. 2-4 that the angle error of the general algorithm is very large, and the worst case is greater than 30°, which far exceeds the cross-plane array error analyzed in the literature (5, 6). The maximum standard deviation of the angle error after using the search algorithm is 0.4713 < σ_{φ}, and the mean is randomly distributed around 0.



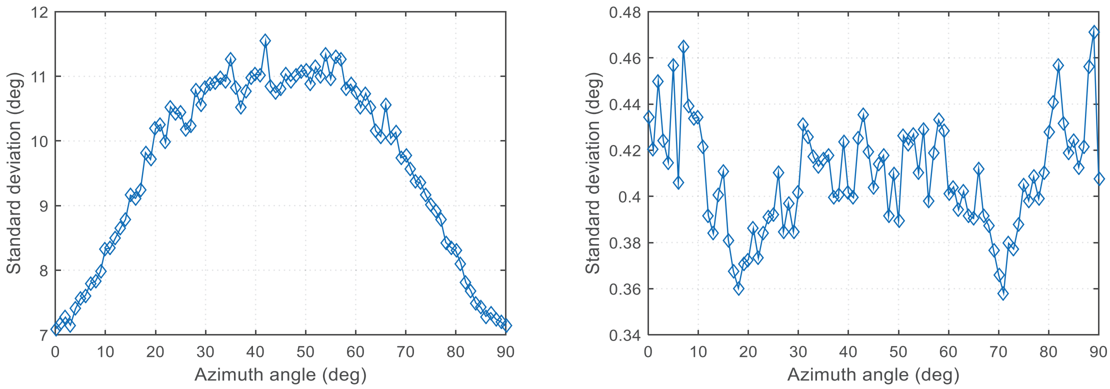
(a) General algorithm (b) Search algorithm

Figure 2 – Angle error statistics figure



(a) General algorithm (b) Search algorithm

Figure 3 – The mean curve of angle error at each angle

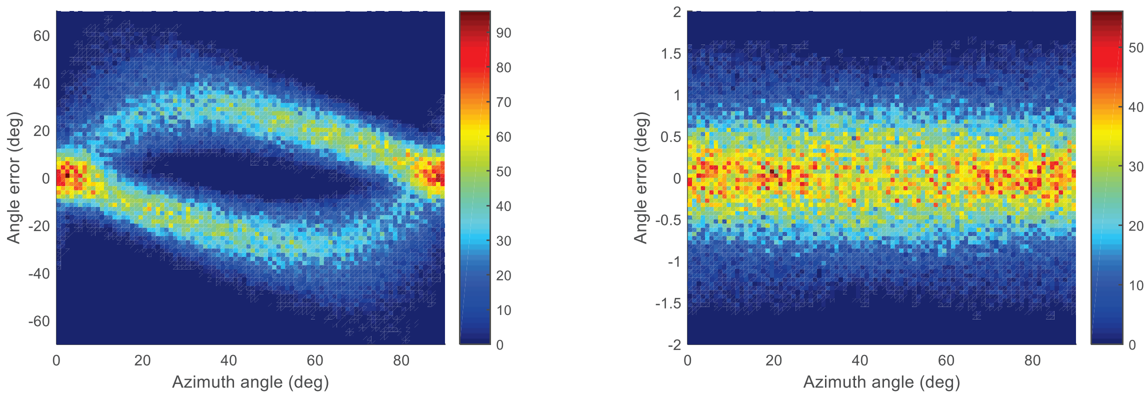


(a) General algorithm (b) Search algorithm

Figure 4 – The standard deviation curve of angle error at each angle

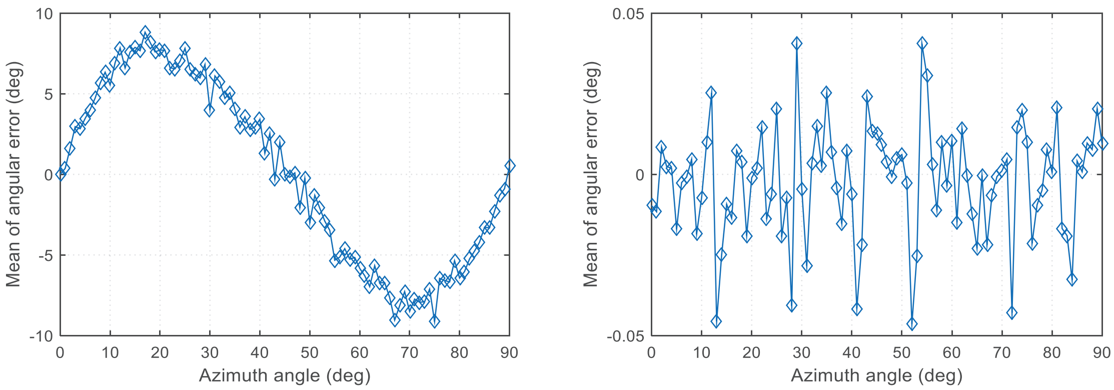
4.3 The Case of Several Time Delay Errors Related to A Microphone

The second method described in 4.1 was used for 1000 times of simulation to obtain the angle error. The error statistics of each angle are shown in Figure 5. The curves of the mean and standard deviation of the angle error in the two algorithms were shown in figure 6-7. The maximum mean and maximum standard deviations of the two algorithms are shown in Table 1. It can be seen from Fig. 5-7 that the angle error obtained by the general algorithm is very large, and in the worst case, it is larger than 60°, and it is impossible to use it for angle calculation; the maximum standard deviation of the angle error after using the search algorithm is 0.5764.$<\sigma_{\varphi}$, there is no significant change in the angle error mean value of 0.0276 and the standard deviation of 0.3499 when the time delay is normal, and the mean value is also randomly distributed around 0.



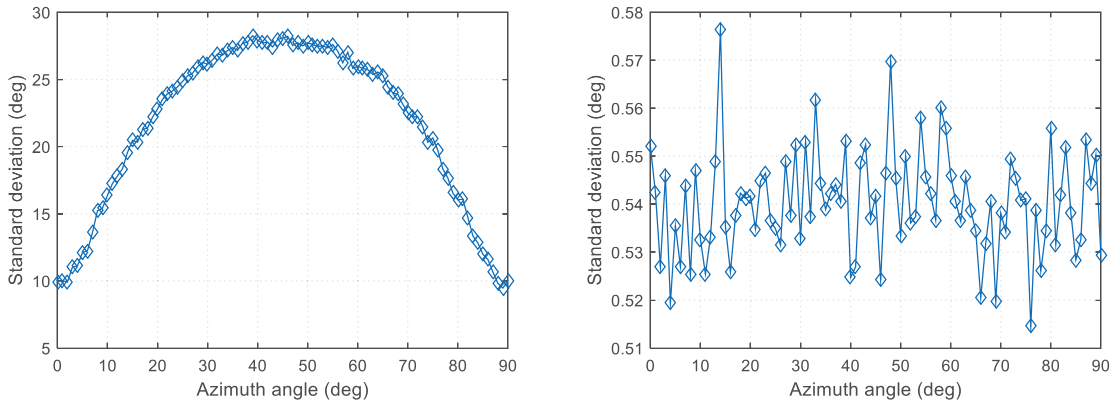
(a) General algorithm (b) Search algorithm

Figure 5 – Angle error statistics figure



(a) General algorithm (b) Search algorithm

Figure 6 – The mean curve of angle error at each angle



(a) General algorithm (b) Search algorithm

Figure 7 – The standard deviation curve of angle error at each angle

4.4 Analysis of Simulation Results

According to the analysis of 3.1, the general algorithm is a special case of the proposed algorithm in this paper. The simulation was also carried out for such cases. The simulation results are shown in Table 1. The results are consistent with the analysis. The simulation results of the search algorithm used in 4.2 and 4.3 are subjected to probability density analysis, and then compared with the normal distribution probability density function, wherein the mean μ of the normal distribution is the average of the error mean, and the standard deviation σ is the average of the error standard deviation. As can be seen in Figure 8-9, it is consistent with the normal distribution function

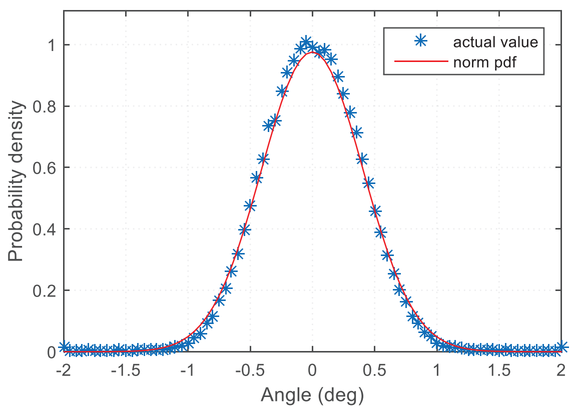


Figure 8 – Probability density curve with a delay error

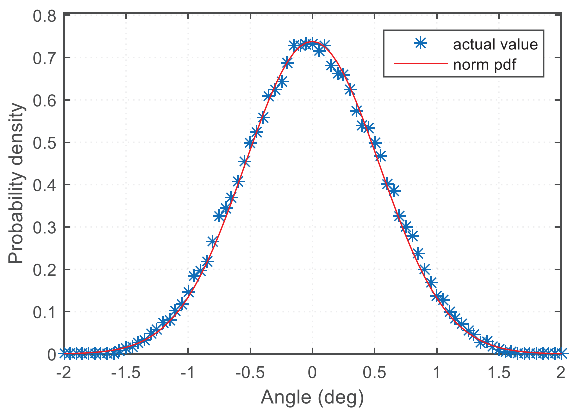


Figure 9 – Probability density curve for several delay errors

In this paper, the simulation results of the general algorithm and the search algorithm in three cases are listed, as shown in Table 1.

Table 1 – Simulation results

Condition	General algorithm, deg		Search algorithm, deg	
	Max of mean	Max of stddev	Max of mean	Max of stddev
Normal	0.0294	0.3493	0.0276	0.3499
One time-delay error	1.2203	11.5431	0.0413	0.4713
Mult time-delay errors	9.1309	28.2536	0.0463	0.5764

From the above simulation results, we can conclude:

1) In the case that all time delays are normal, both the general algorithm and the search algorithm in this paper can obtain correct orientation results and there is no difference between the two. The search algorithm in this paper is degraded to the general algorithm, which verifies the analysis of 3.1;

2) A time delay error, or several time delay errors caused by poor performance of individual microphones, will directly lead to the failure of the general algorithm, unable to obtain the correct orientation results. However, the search algorithm of this paper is not affected and can still obtain the correct orientation result;

3) Using the search algorithm of this paper, the positioning accuracy is high, the angle error conforms to the normal distribution, and the algorithm has strong stability and high reliability.

5. CONCLUSIONS

In this paper, the method of time delay orientation for arbitrary arrays is analyzed. Based on this, a search algorithm is proposed, which can eliminate the incorrect time delay and retain the correct delay participation angle calculation, avoiding the error of angle estimation caused by individual incorrect time delay. Through the simulation comparison of approximate real environment, it is proved that the proposed algorithm not only has high precision, but also has strong stability and high reliability. It has important reference significance for the practical engineering application of using time-delay orientation such as the target impacting-point measurement.

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