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Air flow computations for Helmholtz resonators in a sound field

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Abstract

We present computations of the air flow near the neck of a Helmholtz resonator which is located in a time-harmonic sound field. Of particular interest are sound frequencies near the resonance frequency of the resonator, where the flow speed is maximal. Besides BEM and FEM calculations for the resulting boundary value problem for the Helmholtz equation we have performed also fluid dynamical FEM computations based on the Navier-Stokes equations and the Bernoulli equation, respectively. In the fluid dynamical case, small amplitudes of the initial values cause only weak nonlinear effects. Thus, if such small initial values are taken from the corresponding solution of the Helmholtz equation, this time-harmonic solution should be similar to the fluid dynamical solutions. This can be used to validate the fluid dynamical solutions. Basically we are interested in excitations of the Helmholtz resonator with greater amplitudes and the resulting different behaviour between the inflow and the outflow at the resonator neck.

Keywords: Helmholtz resonators, Air flow, Computational fluid dynamics

1 INTRODUCTION

This paper describes the present state of mathematical modelling within a project which aims at the development of cooling devices based on tiny Helmholtz resonators. These resonators shall be excited by ultrasound with a frequency near their resonance frequency. For great amplitude of the irradiating ultrasound wave each resonator produces a directed flow of air starting at its neck.

For the adequate computation of the air flow caused by a resonator that at its resonance frequency is excited with great amplitude, linear wave acoustics is no longer sufficient. Therefore fluid dynamical computations based on the Navier-Stokes equations and the Bernoulli equation, respectively, have been performed. The fluid dynamical solutions have been verified by comparing them with corresponding solutions obtained by solving the Helmholtz equation in the case that the amplitude of the exciting wave is small. This is also a nice illustration of the adequateness of the linear theory for small wave amplitudes. The examination of the air flow for great amplitudes together with suitable extensions of the mathematical model is planned for the future. All calculations presented here have been done with the software “COMSOL Multiphysics” [1, 2]. The fluid dynamical computations have all been done by the FEM method. The Helmholtz equation has also been solved by the FEM method but, for checking purposes, additionally by the boundary element method (BEM).

In practice, the cavities of the resonators are placed inside a metal plate with a plane surface [3]. Thus, in the mathematical simulations the resonator is treated together with a plane, see Figures 1 and 2.

The presence as well as the position of the plane affects the resonance frequency. Compared to a resonator in free space, the insertion of a plane also reduces the effort of a FEM computation since besides the resonator only the space above the plane has to be meshed.

As irradiating wave we have taken a plane wave. By using the symmetry of the problem, the computational cost could be lowered further by considering just the half configuration. This is indicated in Figure 1.

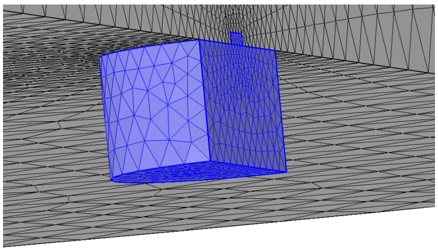

Figure 1. Resonator with a plane which separates the resonator's cavity and neck. A FEM mesh is also shown. By symmetry property it suffices to consider only one half of the configuration.

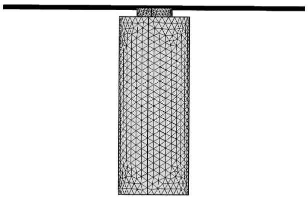

Figure 2. Resonator with a plane at the top of the resonator neck. Here a BEM mesh is shown.

2 COMPUTATIONAL ISSUES

In contrast to resonance frequencies for totally enclosed domains which are defined by eigenvalues, there is no exact definition for the resonance frequency of a Helmholtz resonator. For determining the resonance frequency, for incident waves of the same amplitude but different frequency we have computed the corresponding wavefields due to scattering at the resonator by solving the Helmholtz equation. Then for each such wavefield we have considered the root mean square of the normal velocity, taken over a circular disk above the resonator neck and one period of oscillation (cf. Figure 3). As resonance frequency we have taken the frequency at which this root mean square had attained its maximum peak. Regarding the computation of the root mean square we want to note that, because of the time harmonicity, the integral over one period of oscillation simply is given by half of the period times the square of the absolute value of the complex normal velocity. This expression has yet to be integrated over the disk.

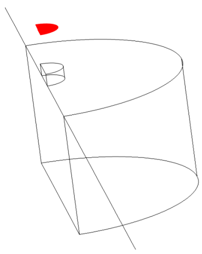

Figure 3. The resonance frequency is computed by considering the air flow through a circular disk (red).

Now we specify the fluid dynamical equations which have been solved. In what follows, $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x},t)$ denotes the flow velocity of the air, $\rho = \rho(\boldsymbol{x},t)$ its density, $p = p(\boldsymbol{x},t)$ its pressure and $\gamma \approx 1.4$ its adiabatic index (ratio of specific heats). μ is the dynamic viscosity of the air; at 293.15 K one has $\mu \approx 1.814$e-5 Pa s. The dynamic viscosity has been neglected in the validation process. Regarding the volume viscosity, we assume throughout that this quantity vanishes.

The Navier-Stokes equations used by us are the equations (written in vector form)

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho\,(D\boldsymbol{u})\boldsymbol{u} + \operatorname{grad} p = \mu\,\Delta \boldsymbol{u} + \frac{\mu}{3}\operatorname{grad}\operatorname{div}\boldsymbol{u}\,. \tag{1}$$

Here $D\boldsymbol{u}$ denotes the Jacobian matrix of $\boldsymbol{u}$ (with respect to $\boldsymbol{x}$).
The Navier-Stokes equations are solved together with the continuity equation

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho\,\boldsymbol{u}) = 0 \tag{2}$$

and the adiabatic equation of state

$$p = p_{ref}\left(\frac{\rho}{\rho_{ref}}\right)^{\gamma}. \tag{3}$$

Here p_{ref} and ρ_{ref} are reference values for the pressure and the density, e.g. pressure and density of the air when there is no flow (and thus no sound) at all ($p_{ref} \approx 101325$ Pa and $\rho_{ref} \approx 1.2043$ kg/m^3).
If the flow $\boldsymbol{u}$ is irrotational, i.e. $\operatorname{curl}\boldsymbol{u} = 0$, then $\boldsymbol{u}$ can be represented by a velocity potential ϕ in the form $\boldsymbol{u} = \operatorname{grad}\phi$. If the flow is irrotational and there is no viscosity, i.e. $\mu = 0$, then from Equations (1) and (3) one obtains the Bernoulli equation

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}|\operatorname{grad}\phi|^2 + \frac{\gamma}{\gamma - 1}\frac{p_{ref}}{\rho}\left(\frac{\rho}{\rho_{ref}}\right)^{\gamma} = a; \tag{4}$$

here the right-hand side a is a function of only the time variable: $a = a(t)$.
Namely, since the right-hand side of Equation (4) does not depend on the variable $\boldsymbol{x}$, the gradient of the left-hand side of Equation (4) is equal to zero. So, after multiplying this gradient by ρ, with Equation (3) one just obtains the Navier-Stokes equations (1).
The Bernoulli equation has to be solved together with the continuity equation (2), where $\boldsymbol{u} = \operatorname{grad}\phi$.
By adding an appropriate function of only t to the velocity potential ϕ, one sees that the right-hand side a of the Bernoulli equation can be chosen as an arbitrary function of t; especially a can be a constant or equal to zero.
Vice versa, if one starts with Equations (4) and (2), none of which includes the pressure p, and deduces the Navier-Stokes equations (1) (with $\mu = 0$) together with Equations (2) and (3), the pressure p takes on the general form

$$p(\boldsymbol{x},t) = p_{ref}\left(\frac{\rho(\boldsymbol{x},t)}{\rho_{ref}}\right)^{\gamma} + b(t)\,. \tag{5}$$

The physical pressure, which fulfills the equation of state, then is obtained by setting the function b equal to zero.
Regarding the boundary conditions for the fluid dynamical equations, throughout we assume that at all walls the flux is purely tangential: $\boldsymbol{u}\cdot\boldsymbol{n} = 0$ for a unit normal vector $\boldsymbol{n}$. In the case of wave acoustics this corresponds to sound hard walls. In the case of the Navier-Stokes equations with non-vanishing viscosity $\mu > 0$ the Equations (1) are of the second order and thus an additional condition is required. Here besides $\boldsymbol{u}\cdot\boldsymbol{n} = 0$ we take the Navier boundary condition $\boldsymbol{S}\boldsymbol{n} - (\boldsymbol{S}\boldsymbol{n}\cdot\boldsymbol{n})\boldsymbol{n} = 0$ with the viscous stress tensor $\boldsymbol{S}$,

$$\boldsymbol{S} = \mu\left(D\boldsymbol{u} + (D\boldsymbol{u})^T\right) - \frac{2}{3}\mu\,(\operatorname{div}\boldsymbol{u})\boldsymbol{I}\,. \tag{6}$$

Here $\boldsymbol{I}$ denotes the 3×3 identity matrix.
We have computed the air flow under the assumption that the resonator together with the plane is located in the free space and irradiated by a plane wave. For the FEM computations, a sphere around the resonator has

been taken and the resonator and the part of the sphere above the plane have been meshed. For reasons of symmetry it sufficed to consider only one half of the configuration, here the direction of the incident plane wave is parallel to the cut surface. In the case of the Helmholtz equation, for the boundary of the sphere in COMSOL there has been chosen a boundary condition which corresponds to an outgoing wave, that is, which simulates the Sommerfeld radiation condition; see Figure 4.

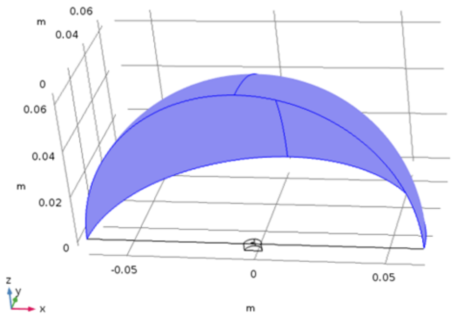

Figure 4. Boundary part of the sphere (blue) on which the Sommerfeld radiation condition is simulated.

For the fluid dynamical calculations, in COMSOL there is no such outgoing wave condition. Here we have taken the boundary condition $\boldsymbol{u} \cdot \boldsymbol{n} = 0$ also for the boundary of the sphere, for $\mu > 0$ in addition we have considered the Navier boundary condition given above. Thus, for computing the air flow up to a certain instant of time, the sphere had to be taken sufficiently large. We have made the sphere so large that no sound originating from the boundary of the sphere could have arrived near the resonator during the time period for which the flow has been calculated.

All of the FEM computations presented here have been performed by using quadratic basis functions. The process of solving the Navier-Stokes equations has been stabilized by the method "streamline diffusion". The time stepping for the Bernoulli equation and the Navier-Stokes equations has been done with the BDF method (backward differentiation formula). Here we have used the maximum order 2, since with high-order BDF methods the pressure is not computed accurately.

3 VALIDATION OF THE FLUID DYNAMICAL SOLUTIONS

For the purpose of checking the fluid dynamical solutions we have neglected the viscosity of the air, i.e. for the Navier-Stokes equations (1) we have assumed that $\mu = 0$.

As initial values for the Navier-Stokes equations and the Bernoulli equation we have taken values obtained by solving the corresponding boundary value problem for the Helmholtz equation. We illustrate this by the following example.

Example: Initial values for the velocity potential ϕ for the Bernoulli equation:

$$\mathrm{Re}\left(e^{\frac{4}{20}\cdot 2i\pi}\,\frac{i}{\omega\rho_0}\,p(\boldsymbol{x})\right). \tag{7}$$

Here Re denotes the real part of the subsequent expression, i the imaginary unit, ω the angular frequency of the exciting wave, $\rho_0 \approx 1.2043$ kg/m^3 the density of air at rest and p the complex sound pressure obtained by solving the Helmholtz equation. The expression given in (7) is the wave acoustical velocity potential after 4/20 oscillations.

If the amplitude of the exciting wave is small, then the nonlinearities of the fluid dynamical equations (Equations (1), (2) and (3) on the one hand and Equations (4) and (2) on the other) cause only weak effects. In this case the exact fluid dynamical and wave acoustical solutions are nearly the same. Therefore, if one has a reliable wave acoustical solution, the quality of the fluid dynamical solutions can be assessed by comparison.

The resonator which we have considered is that from the Figures 1 and 3. It has been considered together with a plane at the bottom of its neck (cf. Figure 1). The resonator's cavity is a cylinder of height 3 mm and radius 3.4 mm. The neck is a cylinder of height 0.3 mm and radius 0.5 mm and is placed centrally on the cavity.

We have plotted the values of our solutions on a line segment above the resonator neck, cf. Figure 5.

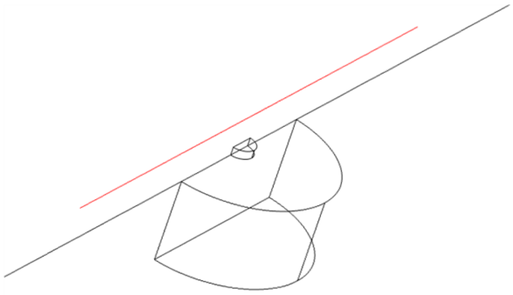

Figure 5. Line segment (red) on which the solutions are considered.

The line segment is of the length 2 cm and lies parallel to the plane. Its midpoint is exactly above the midpoint of the top of the neck, the distance between these midpoints is 1 mm.

The amplitude of the irradiating plane wave is 1 Pa and its frequency is 4700 Hz, this is near the resonance frequency of the configuration. The wave is incident perpendicular to the plane from above. Although this axially symmetric case can be treated by calculations in only two space dimensions, the following results have been obtained by three-dimensional computations which also can be used for other angles of incidence.

In our setting the plane between the resonator's cavity and its neck lies parallel to the x-y-plane and the neck raises towards increasing values of the z-coordinate (cf. Figure 4). The Figures 6 to 8 show the third component u_z of the air flow $\boldsymbol{u}$, i.e. the part of the flow perpendicular to the plane.

The starting time for the fluid dynamical calculations has been chosen as $t = 0$. The corresponding initial values have been computed by solving the Helmholtz equation, where the exponential factor in Equation (7) has been chosen such that the air is streaming into the resonator. Figure 6 shows the third component of $\boldsymbol{u}$ on the line segment above the resonator neck at $t = 0$.

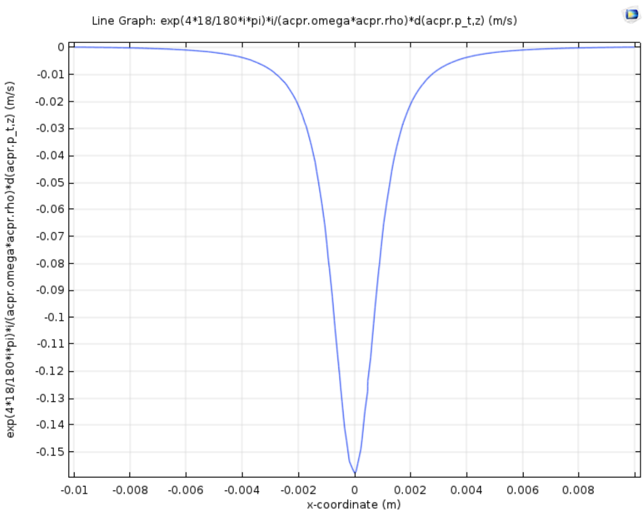

Figure 6. Velocity component u_z at time $t = 0$, obtained by the use of the Helmholtz equation.

The Figures 7 and 8 show the velocity component u_z for $t = 0$ and half an oscillation later, obtained by the use of the Bernoulli equation and the Navier-Stokes equations, respectively. The change of sign of the time-harmonic solution obtained by solving the Helmholtz equation is well reproduced here.

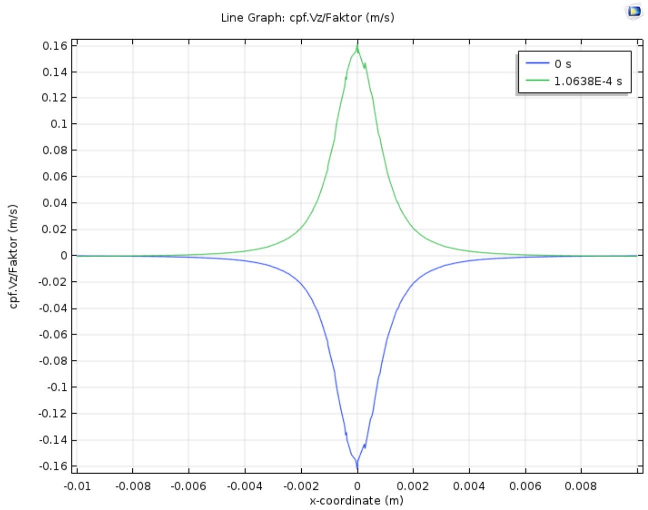

Figure 7. Velocity component u_z at time $t = 0$ (blue) and half an oscillation later (green), obtained by the use of the Bernoulli equation.

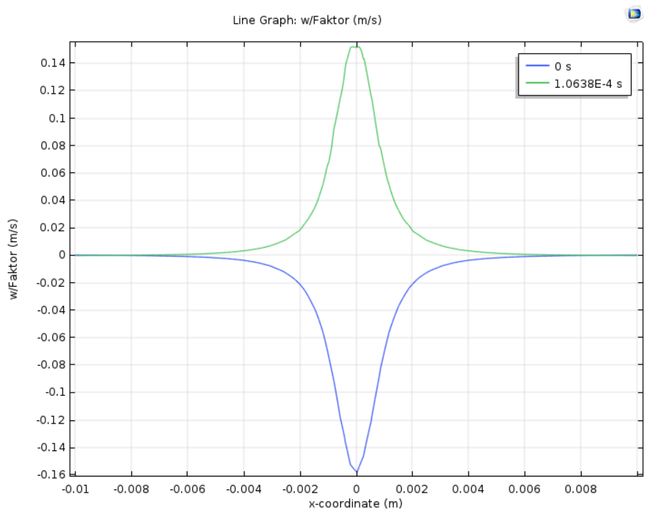

Figure 8. Velocity component u_z at time $t = 0$ (blue) and half an oscillation later (green), obtained by the use of the Navier-Stokes equations.

4 SOME FURTHER RESULTS

Supplemental to the results in the foregoing section, Figure 9 shows the analogue of Figure 8 for the case that the viscosity of the air has been incorporated.

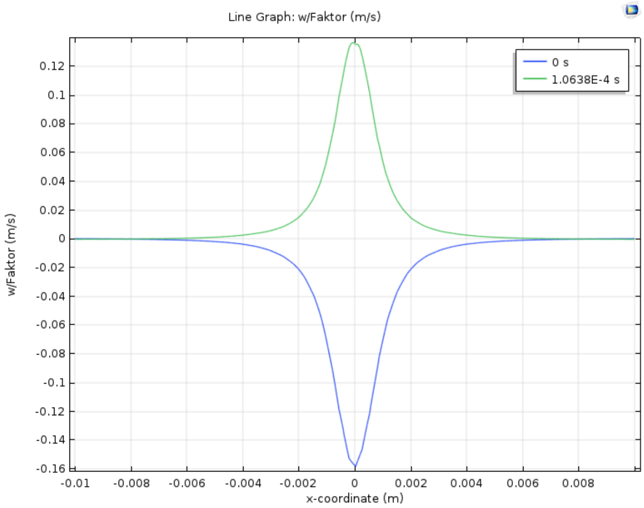

Figure 9. Velocity component u_z at time $t = 0$ (blue) and half an oscillation later (green), obtained by the use of the Navier-Stokes equations taking account of the viscosity of the air.

We yet show the flow through the resonator neck for an excitation with great amplitude, computed by using the Bernoulli equation. The resonator is the same as above but now the plane is placed at the top of the neck (analogous to the configuration shown in Figure 2). Again the exciting wave is incident perpendicular to the plane from above. Its amplitude is 42 Pa and its frequency 4500 Hz; this is near the resonance frequency, which has changed by changing the position of the plane. Contrary to the calculations presented in Section 3, the axially symmetric problem now has been solved by a two-dimensional computation. Figure 10 shows the velocity field and the pressure at a moment of the outflow phase and Figure 11 shows the corresponding values at a later moment of this phase. At this later moment the velocity is maximal. In accordance with Bernoulli's principle, at the later moment the pressure is smaller than at the first moment.

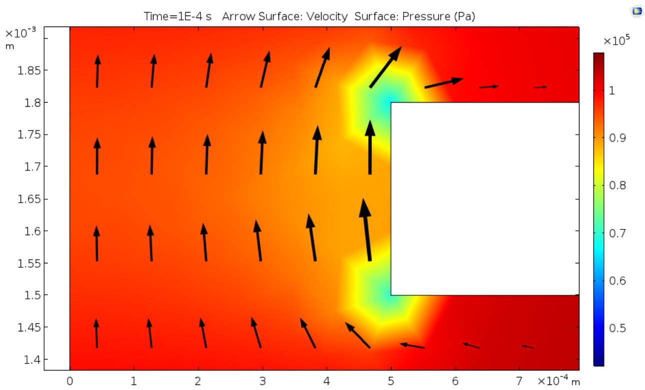

Figure 10. Velocity vectors and pressure (coloured) of the air at a moment of the outflow phase.

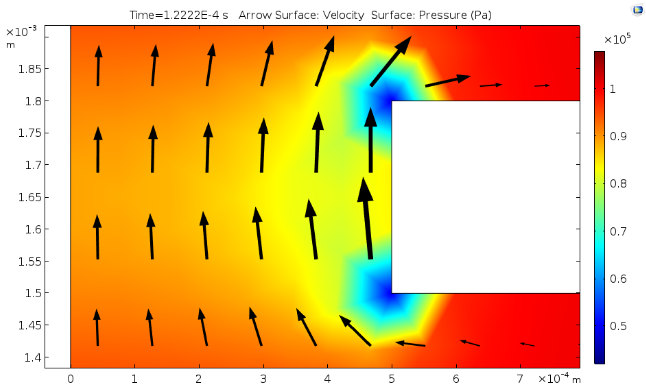

Figure 11. At this later moment of the outflow phase the velocity is maximal. With respect to the situation shown in Figure 10 the pressure has decreased. This demonstrates Bernoulli's principle.

5 SUMMARY

We have considered the air flow near the neck of a Helmholtz resonator that is excited by a time-harmonic sound field with a frequency near the resonance frequency of the resonator. For practical reasons the resonator has been supplemented with a plane above its cavity. We have computed the air flow in the context of linear wave acoustics by solving the Helmholtz equation as well as in the context of nonlinear fluid dynamics by using the Navier-Stokes equations and the Bernoulli equation, respectively. When the viscosity of the air was neglected then for small amplitudes of the irradiating sound wave the wave acoustical and fluid dynamical solutions were found to be practically identical. This shows that the fluid dynamical implementations for the problem are working well. In the future we want to examine the air flow for great amplitudes of the incident wave and find suitable extensions of our mathematical model which allow the modelling of a directed stream of air starting at the resonator neck.

ACKNOWLEDGEMENTS

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