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A monostable acoustic metamaterial for broadband low frequency sound absorption *

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Abstract

This paper presents a monostable acoustic metamaterial for achieving a broadband sound absorption in low frequencies. The proposed system is realized by placing a flexible panel with a magnetic proof mass in a symmetric magnetic field. An equivalent circuit model of such a system is presented. It is shown that the sound absorption peak significantly shifts downwards with the increasing magnetic field. In the nonlinear regime, the jump phenomenon is observed in the frequency sweeping test. Numerical simulation demonstrates that the proposed design can realize a broadband low frequency sound absorption in the deep subwavelength scale.

Keywords: Sound absorption, acoustic metamaterial, monostable

1 INTRODUCTION

Sound absorption is an important topic in noise control engineering. Traditional sound absorption materials include porous materials and fiber materials. They are mostly effective in the middle and high frequency ranges. While in the low frequency range, resonant structures are often used to gain better sound absorption performances. However, their applications in real practice are quite limited due to the large back cavities involved in very low frequencies. Some researchers proposed to resolve the problem by place a flexible back wall or adding flexible structures in the absorbers (1, 2). Others designed compound absorbers using multiple layers of microperforated panels or membranes to broaden the absorption bandwidth in the low frequencies (3-5). However, these complex structures have only limited sound absorption capability in the deep subwavelength scale.

In recent years, acoustic metamaterials (AMMs) have drawn a lot of attention in the noise control community due to their unusual properties. Among various implementations of AMMs, membrane- and plate-type AMMs are particularly important for noise control (6, 7). They are relatively simple and light-weighted, having the ability to actively tune the absorption characteristics. When properly designed, this type of AMMs can achieve a high sound absorption in the deep subwavelength scale. Unfortunately, it is narrow-banded in nature due to the causality constraint on the minimal structural thickness (8).

In this paper, a monostable acoustic metamaterial is proposed and its equivalent circuit model is presented. It is shown that the sound absorption peak significantly shifts downwards with the increasing magnetic field. In the nonlinear regime, a jump phenomenon is observed in the frequency sweeping test. Numerical simulation demonstrates that the proposed design can realize a broadband low frequency sound absorption in the deep subwavelength scale.

2 MODELING

The proposed monostable absorber, illustrated in Figure 1, consists of a plate-type AMM with a centered magnetic proof mass. Two external magnets are symmetrically positioned in coaxial with the proof mass, resulting an attractive force on each side of it. For such a configuration, the proof mass and one of the two external magnets would snap together if their gap is small, which changes the overall boundary conditions of the system and disables the system. Care should be taken to adjust the external magnets away from the proof mass so that

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the elastic force of the plate is always larger than the magnetic attraction. Meanwhile, the gap should be small enough to ensure the strong coupling between the plate and the external magnets for better performances.

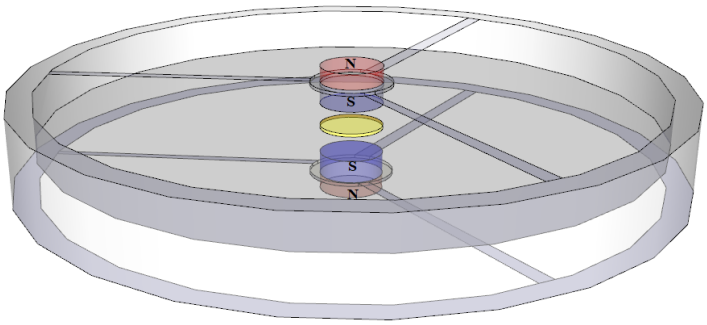

Figure 1. Diagram of the monostable absorber

The monostable absorber can be modeled by an equivalent circuit, as shown in Figure 2.

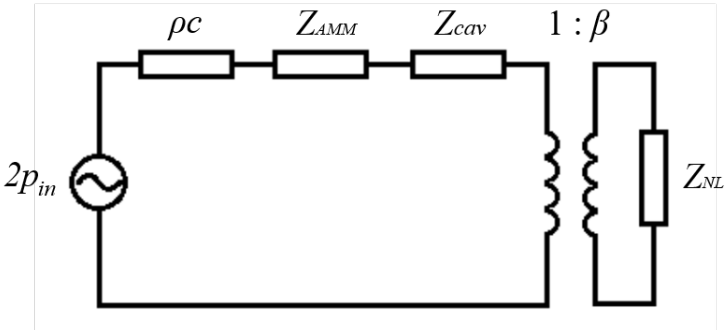

Figure 2. Equivalent circuit of the monostable absorber

Suppose a plane wave with an amplitude of p_{in} incident on the absorber from above, it drives the AMM with a blocked pressure $2p_{in}$, which vibrates with the air loadings from both sides, together with the loading due to the magnetic interaction between the proof mass and the external magnets. The impedance of the air above is ρc and that of the air in the cavity is $Z_{cav} = -j\rho c \mathrm{ctg}(\omega D/c)$, where ρ is the air density, c the air sound speed, ω the radian frequency, and D the depth of the cavity.

The impedance of the AMM is

$$Z_{AMM} = S\left(\sum_n \frac{j\omega(\int \phi_n(\mathbf{x})\,\mathrm{d}S)^2}{M_n(\omega_n^2 - \omega^2 + 2j\zeta_n\omega_n\omega)}\right)^{-1}, \tag{1}$$

where S is the surface area of the plate, M_n, ω_n, and ζ_n are the modal mass, natural frequency, and modal damping ratio of the nth mode of the AMM, respectively. Assuming that the dynamics of the AMM is dominated by a single mode ϕ_i, Equation 1 reduces to

$$Z_{AMM} \approx S\frac{M_i(\omega_i^2 - \omega^2 + 2j\zeta_i\omega_i\omega)}{j\omega(\int \phi_i(\mathbf{x})\,\mathrm{d}S)^2}. \tag{2}$$

The magnetic force between the proof mass and the external magnets is approximated by

$$f_{NL} \approx -k_1 x - k_3 x^3, \tag{3}$$

where x is the displacement of the proof mass, k_1 and k_3 are the linear and nonlinear stiffness coefficients derived from the magnetic force. The corresponding impedance is

$$Z_{NL} = \frac{k_1}{j\omega S} - \frac{k_3}{j\omega S}\left(\frac{\bar{v}}{\omega\beta}\right)^2, \tag{4}$$

where $\bar{v}$ its surface averaged velocity. The magnetic loading is connected to the circuit through a transformation element with the transformation ratio

$$\beta = \langle \phi_i \rangle / \phi_{i,0}, \tag{5}$$

where $\phi_{i,0}$ and $\langle \phi_i \rangle$ are the center and the surface averaged values of the ith modal shape, respectively.

When p_{in} is small enough, the monostable absorber can be considered as a linear resonator. The magnetic interaction works as a negative stiffness as in Reference (9). The effective impedance from the air cavity and the magnetic interaction is

$$Z_{eff} = Z_{cav} + k_1/(j\omega S\beta^2) \approx \frac{\rho c^2}{j\omega}\left(\frac{1}{D} + \frac{k_1}{\rho c^2 \beta^2 S}\right), \tag{6}$$

where the low frequency approximation $\omega D/c \ll 1$ is made. Note that $k_1 < 0$ due to the symmetrical setup of the external magnets, its effect on the monostable absorber is equivalent to a cavity with a larger depth $D_{eff} = \left(1/D + k_1/\rho c^2 \beta^2 S\right)^{-1}$. Hence the absorption peaks shift downwards to the lower frequencies.

To simulate the dynamics of the monostable absorber in the time domain, the above equations in the frequency domain are rewritten as the following equation

$$M\ddot{x}(t) + (2\zeta_i \omega_i M + \rho c \beta^2 S)\dot{x}(t) + \left(K + \rho c^2 \beta^2 S/D + k_1\right) x(t) + k_3 x^3(t) = 2\beta S p_{in}(t), \tag{7}$$

where the dot denotes the time derivative, $M = M_i/\phi_{i,0}^2$, and $K = M\omega_i^2$.

For the case of moderate acoustic and magnetic fields applying on the absorber, the system is weakly nonlinear and the dissipated acoustic power is approximated by

$$P_{diss} = 1/2(2\zeta_i \omega_i M)\,|\dot{x}|^2, \tag{8}$$

where $|\dot{x}|$ denotes the velocity amplitude of the proof mass.

The acoustic power incident on the absorber is

$$P_{in} = S p_{in}^2 / 2\rho c. \tag{9}$$

The sound absorption coefficient of the absorber is defined as

$$\alpha = P_{diss}/P_{in} = 2\rho c \zeta_i \omega_i M\,|\dot{x}/p_{in}|^2 / S. \tag{10}$$

3 VALIDATION

To validate the above model, an absorber is made up of a aluminum disk of radius 34 mm and thickness 0.28 mm, backed by a rigid cylindrical cavity of depth 49 mm. The material has a density of 2700 kg/m^3, Young's modulus 69 GPa, Poisson's ratio 0.33, and modal damping ratio 0.08. The density of the air is 1.21 kg/m^3 and its sound speed 340 m/s. On each side of the disk, a proof mass of weight 0.183 g is placed at the center. Two

Table 1. Absorption peaks of the monostable absorber

Gap [mm]	3.2	3.7	6.0	∞
k_1 [N/m]	-1540	-913	-170	0
Predicted [Hz]	274	297	320	327
Measured [Hz]	276	296	312	328

external magnets are symmetrically positioned in coaxial with the proof mass. The attractive force on each side is measured with a dynamometer and the effective stiffnesses are obtained by curve fitting.

Firstly, the absorber is installed in a impedance tube and tested according to ISO 10534-2 (10). In the linear regime, the interaction between the proof mass and the external magnets leads to a negative stiffness effect. The predicted peak frequencies are in good agreement with the measured data, which is shown in Table 1.

To investigate the nonlinear dynamics of the monostable absorber, a frequency sweeping test is conducted with a sweep rate of 18 Hz/s. The driving sound pressure is monitored with a microphone in front of the absorber, which is about 90 dB in the test. The back cavity is removed so that the velocity of the absorber can be measured by a laser vibrometer. The results from the upward and downward sweep are shown in Figure 3, which clearly shows a jump phenomenon due to a softening nonlinearity. In the numerical simulation, the following parameters $k_1 = -0.13$ N/mm, $k_3 = -290$ kN/mm^3, $\zeta_i = 0.01$, and $p_{in} = 0.048$ Pa are used. The simulation results are shown in Figure 4. It should be noted that the above parameters are not obtained from the experimental setup and used for illustration only. Further work is needed for the purpose of validation.

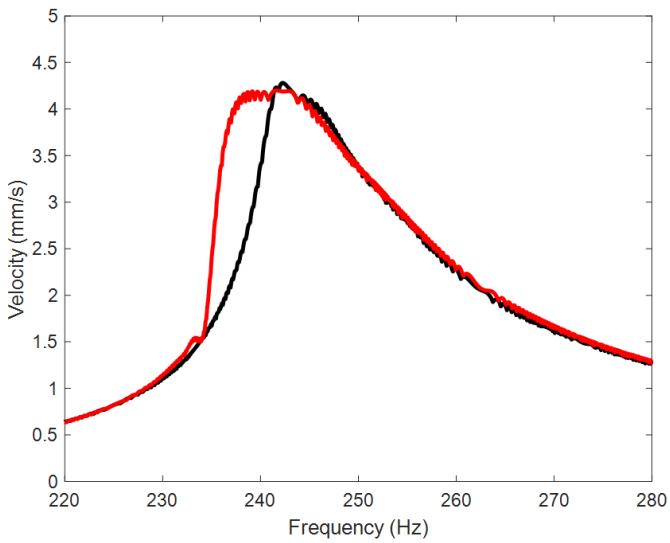

Figure 3. Frequency response of the velocity on a point near the proof mass: upward sweep, black line; downward sweep, red line

The last example illustrates a broadband low frequency absorber in a deep subwavelength scale. The structure parameters remains the same except that the back cavity depth is reduced to 5 mm. The gap between the external magnets and the proof mass is chosen to be 1.4 mm, which has the effective stiffnesses $k_1 = -23$ N/mm and $k_3 = -45.7$ N/mm^3. The nonlinearity is negligible for the current configuration. As shown in Figure 5, the peak absorption frequency drastically shifts downwards from 723 Hz to 161 Hz. The relative bandwidth is also broadened due to the symmetrically positioned magnets.

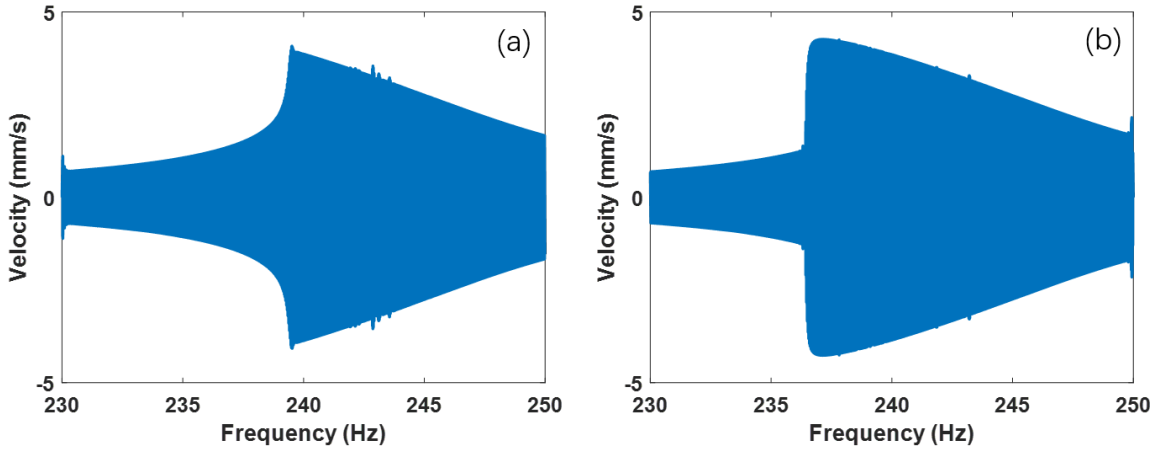

Figure 4. Frequency response of the velocity of the proof mass: (a) upward sweep, (b) downward sweep

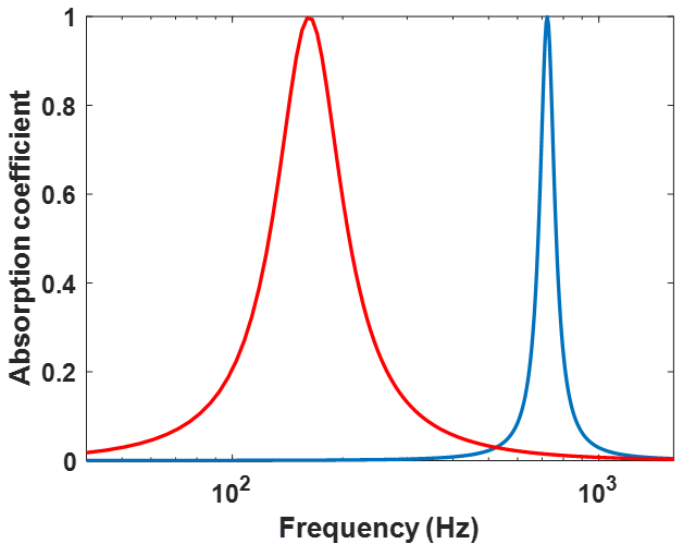

Figure 5. Absorption coefficients of the absorbers with (red line) and without (blue line) the external magnets

4 CONCLUSIONS

This paper proposed a monostable acoustic metamaterial which places a plate-type AMM with a magnetic proof mass in a symmetric magnetic field. Under the assumption that the absorber is dominated by a single mode, the equivalent circuit model is obtained by connecting the nonlinear impedance to that of the AMM through a transformation element. It is shown that the sound absorption peak significantly shifts downwards with the increasing magnetic field. In the nonlinear regime, a jump phenomenon due to a softening nonlinearity is observed in the frequency sweeping test. Numerical simulation demonstrates that the proposed design can realize a broadband low frequency sound absorption in a deep subwavelength scale.

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