

Research on Structural Vibration Control Based on Local Stiffness Reinforcement and Local Constrained Damping

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ABSTRACT

Structural strengthening and restrained damping have been widely used for vibration control, but some negative effects was brought such as weight increase. Stiffness and damping are both important measures for structural vibration control. Based on harmonic response analysis, the technology of design with stiffness and damping for vibration control is studied in this paper. Under the condition of little weight gain, the wonderful performance of vibration control was obtained. This method has significance for structures with high requirements for weight and vibration control in engineering.

Keywords: Constrained damping, Stiffness enhancement, Vibration control

1. INTRODUCTION

From the point of view of vibration reduction, the energy of damper loss system is to transform the energy of mechanical vibration into heat energy, electric energy, magnetic energy or other forms of energy loss, so as to achieve the purpose of vibration reduction. In order to control the vibration in various thin plate structures, additional damping is a common way to reduce the vibration of thin shell structures(1-6). Additional damped structures mainly include free damped structures and constrained damped structures. When strained, the damped structures consume vibration energy by bending or shearing deformation. Stiffness reinforcement is to lay reinforcement bars on the surface of thin plate structure, to exchange smaller mass for greater stiffness and strength of the whole structure, so as to control its vibration. Reinforcement tendons are widely used in ship, construction and machinery. According to statistics, various types of reinforcement tendons can account for more than 35% of large ship structures, and play a very important role in lightweight hull structures.

In a large number of literatures, scholars have only studied the effect of damping or stiffness on vibration reduction. Few literatures have combined the damping and stiffness. Damping and stiffness are both important measures for structural vibration control. If the combined effect of them can be optimized, it will bring better vibration reduction effect. In this paper, the effect of damp and stiffness on structural vibration control is studied, and the design method of damp-stiffness combined vibration reduction is proposed.

2. DAMPING STRUCTURE VIBRATION REDUCTION TECHNOLOGY

Viscoelastic material is the most commonly used damping layer material in the additional damping structure. From the macroscopic point of view, it has two characteristics: solid elasticity and fluid viscosity. Microscopically, it is a kind of macromolecule polymer, and its molecules are easy to produce relative movement. The viscoelastic damping materials need to overcome the internal friction caused by the slip and torsion between molecular chains when strain occurs. The energy dissipation of viscoelastic damping material per unit volume in a vibration period can be expressed as follows:

$$\Delta W = \pi \varepsilon^2 E' \beta \quad (1)$$

Where ε was strain of viscoelastic material, β was viscoelastic material loss factor, and the E' was the real part of the complex modulus of viscoelastic damping material, also known as the storage modulus.

When the vibration amplitude is fixed, the greater the value of $\beta E'$, the more vibration energy

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consumed by viscoelastic damping materials. So loss factor and real modulus of elasticity are generally used as the main indicators of vibration energy dissipation, but they are greatly affected by environmental factors such as frequency and temperature. Many constitutive models have been proposed by scholars to describe the dynamic characteristics of viscoelastic materials more accurately. Maxwell model and Kelvin-Voigt model simulate the dynamic characteristics of viscoelastic materials by the combination of dampers and springs, but they only represent a part of creep and relaxation processes. Rational fractional derivative model has just been developed in recent decades. The constitutive relationship established by fractional derivative theory can accurately describe the dynamic characteristics of viscoelastic materials, but it is necessary to apply it to practical applications, and multiple parameters are fitted. As one of the most commonly used models, the complex modulus model can accurately represent the vibration energy dissipation under harmonic excitation, avoiding the mechanical characteristics of viscoelastic materials when strain occurs.

Although viscoelastic materials have a history of several decades, there is no unified constitutive equation to describe their dynamic characteristics accurately and simply. The analytical method can only deal with the dynamic problems of simple viscoelastic damped structures. At present, the most effective method to solve complex engineering problems is to establish the model of viscoelastic composite structure based on finite element method, and carry out finite element analysis of damped structure. The main analysis methods are complex stiffness method and modal strain energy method. The above two finite element methods have higher accuracy for constrained damping analysis.

2.1 Complex stiffness method

The dynamic equation of elastic-viscoelastic composite structure is transformed into free vibration state, that is:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = 0 \quad (2)$$

According to the dynamic properties of viscoelastic damping materials, the elastic modulus is expressed in the form of complex modulus:

$$E = E' + iE'' = E'(1 + i\beta) \quad (3)$$

Where E was complex elastic modulus of viscoelastic damping materials, E' was the imaginary part of the complex modulus of viscoelastic damping materials is also called the dissipative modulus.

At this time, (2) can be simplified as:

$$[M]\{\ddot{u}\} + [K^*]\{u\} = 0 \quad (4)$$

Where $|K^*|$ is a complex stiffness matrix of viscoelastic composite structure, which is expressed as:

$$[K^*] = [K'] + i[K''] \quad (5)$$

Where $[K']$ was real part of complex stiffness matrix, $[K'']$ was imaginary part of complex stiffness matrix. Under harmonic excitation, the form of solution of equation (4) is as follows:

$$\{x\} = \{\phi^*\} e^{i\omega t} \quad (6)$$

Then the complex eigenvalue of the system is:

$$\omega^* = \omega \sqrt{1 + i\xi} \quad (7)$$

The real and imaginary parts of the complex eigenvalues of viscoelastic damped composite structures can be obtained by substituting formula (7) into (4):

$$\omega = \frac{\{\phi^*\}^T [K'] \{\phi^*\}}{\{\phi^*\}^T [M] \{\phi^*\}} \quad (8)$$

$$\xi \omega^2 = \frac{\{\phi^*\}^T [K''] \{\phi^*\}}{\{\phi^*\}^T [M] \{\phi^*\}} \quad (9)$$

The characteristic values and modal loss factors of viscoelastic damped composite structures can be obtained. Under the excitation of harmonic response, the dynamic characteristics of damped structures could be described by the complex stiffness method, but the solving process takes a long time.

2.2 Modal strain energy method

The modal strain energy method is used to calculate the loss factor of viscoelastic composite structures from the point of view of energy dissipation of viscoelastic damped structures. The idea of this method is to

assume that the damped modes are similar to the non-damped modes in the process of vibration, that is, the modal shapes of the structures with additional dampers remain unchanged. The eigenvectors of the dynamic equation of the undamped structure obtained from the modal analysis of the viscoelastic composite structure are used to calculate the strain energy of each element, and then the viscoelastic damping treatment is brought into the dynamic equation to obtain the damping characteristics of the structure.

The overall stiffness of the elastic-viscoelastic composite structure includes the stiffness of the elastic layer element and the damping layer element, so the total modal strain energy can be expressed as:

$$U = \frac{1}{2} \{\phi\}^T [K] \{\phi\} = \frac{1}{2} \{\phi\}^T [K]_b \{\phi\} + \frac{1}{2} \{\phi\}^T [K]_v \{\phi\} \quad (10)$$

Where $[K]$ was stiffness matrix of viscoelastic damping composite structures, $[K]$, was stiffness matrix of elastic layer metal plate in viscoelastic damping composite structures, $[K]$ was stiffness matrix of damping layer of viscoelastic damped composite structure.

The dissipated energy ΔU_v of the damping layer element in a unit period can be expressed by the dissipated energy of the damping material.

$$\Delta U_v = \frac{1}{2} \{\phi\}^T [K]_v \{\phi\} \quad (11)$$

The loss factors of viscoelastic damped composite structures are expressed as follows:

$$\eta = \frac{\Delta U_v}{U} = \beta \frac{\{\phi\}^T [K]_v \{\phi\}}{\{\phi\}^T [K]_b \{\phi\} + \{\phi\}^T [K]_v \{\phi\}} \quad (12)$$

Modal strain energy method avoids the calculation of complex eigenvalues and reduces the amount of calculation. Using many finite element analysis software, the distribution and magnitude of structural modal strain energy can be calculated directly, and the dynamic analysis of complex structures can be carried out quickly.

3. VIBRATION REDUCTION TECHNOLOGY OF STIFFNESS-REINFORCED STRUCTURES

Increasing stiffness is one of the important measures to reinforce structures. In the reinforcement of buildings, aircraft and ships, reinforcement tendons are often used to consolidate the strength of structures. The stiffness of the structure is closely related to its vibration response, which in essence directly determines the vibration response of the structure. Appropriate increase of stiffness can not only reinforce the structure, but also increase the lower limit of its modal frequency and control its vibration.

The finite element dynamic equations of elastic structures can be expressed as follows:

$$[M] \{\ddot{u}\} + [C] \{\dot{u}\} + [K] \{u\} = \{P(t)\} \quad (13)$$

Where $[M]$ was mass matrix, $[K]$ was stiffness matrix, $[C]$ was damping matrix, $\{u\}$ was node displacement vector, and $\{P(t)\}$ was the external force vector of the node;

When the damping and external forces are not considered, the structural dynamic equation of equation (13) can be simplified as follows:

$$[M] \{\ddot{u}\} + [K] \{u\} = 0 \quad (14)$$

When the structure is in simple harmonic motion, the equation is as follows:

$$\{u\} = \{\phi\} \{\xi(\omega)\} e^{j\omega t} \quad (15)$$

Where $\{\phi\}$ was amplitude matrix, $\{\xi(\omega)\}$ was modal function.

The simplified formula (15) could be obtained by introduced into formula (14):

$$\{[K] - \omega^2 [M]\} \{\phi\} = 0 \quad (16)$$

Only when the coefficient determinant of equation (16) is zero, the equation have a non-zero solution, that is:

$$\det([K] - \omega^2 [M]) = 0 \quad (17)$$

The eigenvalues and eigenvectors of the equation can be obtained by solving formula (17). The eigenvalues are the square of the natural frequency of the structure ω^2 , and the eigenvectors are the mode shapes of the structure $\{\phi\}$. The modal parameters of the structure can be obtained from the stiffness matrix and the mass matrix of the structure by the finite element method.

4. DESIGN METHOD OF VIBRATION REDUCTION WITH LOCAL DAMPING AND STIFFNESS REINFORCEMENT

The location of laying local damping is near the excitation point to ensure that the vibration of the structure is controlled within a local range. At present, constant damping factor is generally used to describe the damping characteristics of materials in the acoustic study of damped composite plate structures.

The most commonly used method of stiffness structure strengthening is to arrange reinforcing bars to support the thin plate in the way of meridians, so as to strengthen the structure. The most common reinforcing bars structure is orthogonal regular layout. This structure is widely used in navigation, aviation and so on.

In order to combine local strengthening with local constrained damping and obtain wonderful in structural vibration reduction, the design method is as follows:

- (1) Determining the location of load excitation;
- (2) Isolate the excitation position with stiffeners;
- (3) Damping materials are coated in the isolation area.

5. ANALYSIS OF SIMULATION RESULTS

The simulation model is a steel sheet with a length and width of 1000 mm and a thickness of 2 mm. The parameters of local restraint damping are as follows: the thickness of the damping layer is 10 mm, the thickness of the restraint layer is 2 mm, and the edge length is 400 mm, covering around the excitation point. On the other side of the sheet, reinforcement bars are arranged with height of 20 mm, width of 4 mm and orthogonal staggered arrangement of 4 reinforcement bars. Four stiffeners surround the excitation point in a region with damping measures at the same time. As shown in figure 1.

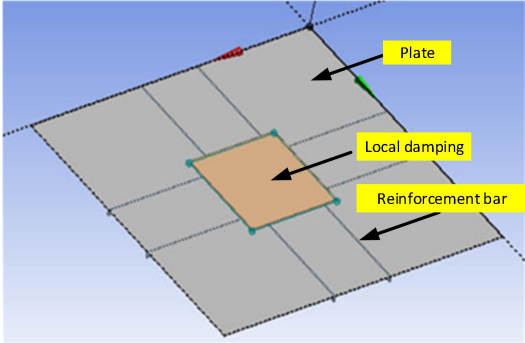


Figure 1 - Models of Local Stiffness Reinforcement and Local Constrained Damping

The thin plate structure is constrained by simply supported four ends, and a simple harmonic excitation force of 10N is applied at the center of the thin plate. Table 1: Physical parameters of steel plate and damping material. Usually, the damping rate of material varies with frequency, for simplified model, the damping rate is 0.9.

Table 1 - Material parameters

Materials	Density/kg m ⁻³	Modulus of Elasticity/pa	Poisson's Ratio
steel	7780	2.1E11	0.27
Damping material	1980	5.8E8	0.47

The following three models are analyzed:

- Model I: Thin plate;
- Model II: A thin plate covered with a free damping layer;
- Model III: Thin plates with locally stiffened and locally constrained dampers, as shown in Figure 1.

The frequency range of the analysis is 10 ~ 500 Hz. In the three models, the frequency response of the excitation position is shown in Figures 2 and 3.

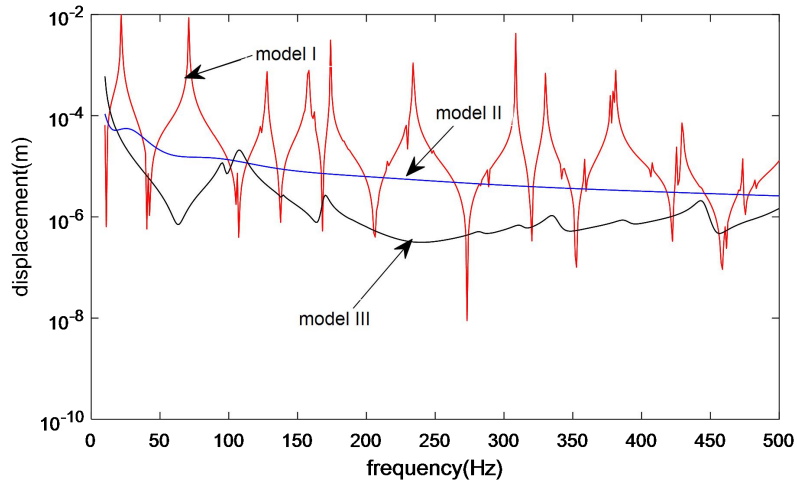


Figure 2 - Comparison of central point amplitudes of three models

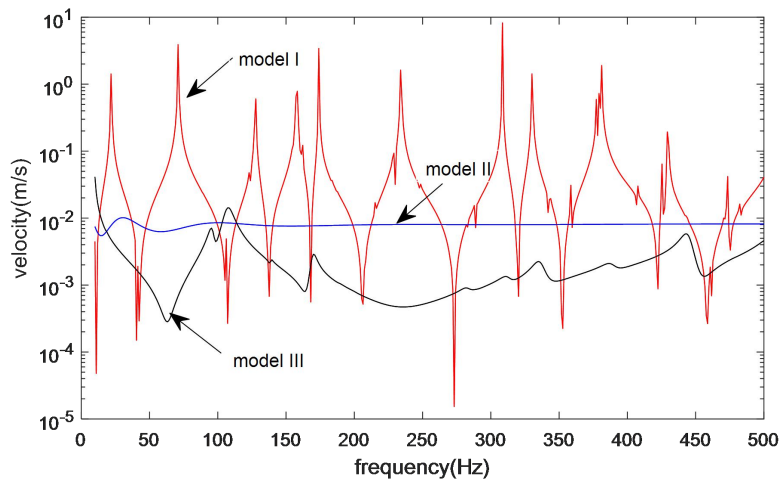


Figure 3 - Comparison of center point velocity of three models

According to Figure 3 and Figure 4, the displacement and velocity of the thin plate in the frequency range of 10-500 Hz are much higher than those of model II and model III when there is no vibration controlled measure. The vibration of model II is reduced by about 21.6 dB compared with model I. Model III is further reduced than model II. At 120 Hz, the vibration of model III and model II is basically the same. In the wide frequency range, the vibration of model III is reduced by 10.3 dB compared with that of model II. It shows that under the simultaneous action of stiffness and damping, the vibration reduction effect of model III is better than that of simple damping.

6. CONCLUSIONS

In this paper, based on the way of local constrained damping and stiffness strengthening, the local constrained damping layer is laid on the area where vibration reduction is needed. Because the damping is only partially coated, the additional weight is reduced. From the results of numerical simulation, it can be seen that both local damping and stiffness are effective methods to control vibration. This method has a good guiding significance for the design of structural vibration control.

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