

Robust virtual sensing of the exterior noise radiation from a complex structure in different acoustic environments

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ABSTRACT

By using a numerical model of a vibro-acoustic system in combination with a small set of measurements in a Kalman filter, it is possible to estimate the sound pressure at locations where no microphones are present. This can be achieved with increased accuracy as compared to using only the numerical model due to the inclusion of the expected process and measurement noise. This procedure is also known as virtual sensing. In this paper, a model-based virtual sensor is built for a complex, deep drawn structure that radiates sound into the free field. A finite element model is used that is extended with infinite elements to approximate the Sommerfeld radiation condition. Since the original model is too large to be used in a Kalman filter, it is reduced by a stable Krylov based model order reduction technique. The performance of this virtual sensor is tested in a semi-anechoic chamber, and additionally in several acoustical environments that do not conform to the Sommerfeld condition in order to validate whether the Kalman filter can handle the additional uncertainties. The obtained results show that the filter performs well, which means that the required modeling effort can be reduced significantly by assuming free-field conditions.

Keywords: Kalman filter, Model order reduction, Finite elements

1. INTRODUCTION

To optimize production processes and reduce downtime of machines in modern factories, cyber-physical systems start playing a more important role (1). Cyber-physical systems are computational entities that interact both with each other and the physical system to which they are attached to through sensors. The exact shape of these systems can vary, depending on the available data and knowledge of the system under investigation.

It is possible to create a physics based model when the equations of the underlying physics are known. An accurate physics-based model allows for a detailed physical analysis of the model, thus giving a high degree of physical insight in what is happening. Unfortunately, it is hard to construct an accurate physics based model due to geometrical complexities, uncertainty on material properties and boundary conditions, lack of insight in what type of physics have to be modeled, etc. Thus, a high degree of modeling expertise is required (2).

Also a purely data driven approach can be taken to construct the cyber-physical system. This approach does not require any knowledge about the physics of the system and thus leads to a reduced modeling effort. The disadvantages of this approach are that the results do not offer physical insight and that usually a large amount of data is required to train the system. Also, results are only obtained on the locations where the sensors are placed. In many dynamic systems of interest it is desired to place the sensors at non-intrusive locations, thus it would only be possible to measure the phenomena happening at intrusive locations indirectly.

A third option is to utilize a hybrid approach that combines a reasonably accurate physics based model with a small amount of measurements. Specifically, they are combined in a Kalman filter, which is a state estimation algorithm that takes process noise (which represents the modeling error) and measurement noise (representing noise on the sensors) statistics into account to estimate the field variables in the full domain (3), thus also at locations where it might be difficult to place sensors. Since the combination of only a small amount of measurements with the numerical modeling leads to an

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estimation of all the field variables, this approach is called virtual sensing.

In this paper it is chosen to investigate the effectiveness of this virtual sensing approach for exterior vibro-acoustic problems. In the field of active noise control already several data-driven virtual sensing approaches have been investigated (4,5). Conventional active noise control algorithms try to reduce the sound pressure at a certain microphone location by destructive interference. However, in many instances it is impractical to place the microphone at the location where it is desired to lower the sound pressure level. For example, at the ears of a person. Therefore, virtual sensing is used to estimate the pressure at this location, usually by means of a-priori transfer function identification. Since these approaches are data-driven, they only work on the pre-measured locations and do not offer flexibility.

An alternative is to use a numerical model that describes the physics. In (6) such an approach is considered for interior vibro-acoustic problems. They show that the combination of a Finite Element (FE) model with a small amount of measurements in a Kalman filter leads to accurate pressure estimates in the full cavity. To reduce the calculation complexity of the FE model, Model Order Reduction (MOR) is used to obtain compact models (7,8).

To be able to simulate exterior vibro-acoustic models with the Finite Element Method (FEM) instead, the model has to adhere to the Sommerfeld radiation condition. In (9) it is explained how a combined FE model with conjugated Astley-Leis infinite elements (10,11) on the boundary leads to a model that approximately adheres to the Sommerfeld radiation condition and can be reduced in a stable manner by using Krylov based MOR techniques. We utilize this technique in this paper to build a virtual sensor scheme for exterior vibro-acoustics.

The initial verification of the performance of the virtual sensor is done in an acoustic environment that closely represents the assumptions that are done in the numerical model (semi-anechoic conditions). From a modelling perspective this would be a convenient assumption, even when the environment is not strictly anechoic, since it does not require detailed modelling of the potentially complex acoustic environment, for example the modelling of the radiated sound from a structure in a factory. Therefore, the robustness of the Sommerfeld assumption on changing acoustic conditions is investigated by comparing the performance of the virtual sensor under correct assumptions (measurements in a semi-anechoic room) with incorrect assumptions (measurements in an office and in an entrance hall).

2. System formulation

The considered structure is an oil-pan, which is shown in Figure 1. The structure was produced by a deep-drawing process, leading to uncertainty on the final shape, the thickness and the material properties (12). The oil-pan is modeled with a structural FE model consisting of quadratic shell elements, that is divided into 12 patches, as is shown in Figure 1. Each of these patches has a separate Young's modulus and thickness. Before this model is coupled to an acoustic mesh, a model updating procedure is run to update the Young's modulus of the patches so that the structural frequency response functions at several locations match approximately until 500 Hz, see Figure 2. The final thicknesses and Young's moduli are shown in Table 1. These values do not correspond to real physical values, since in the model updating step other uncertain parameters are not updated, such as geometrical deviations between the real geometry and the used numerical model.

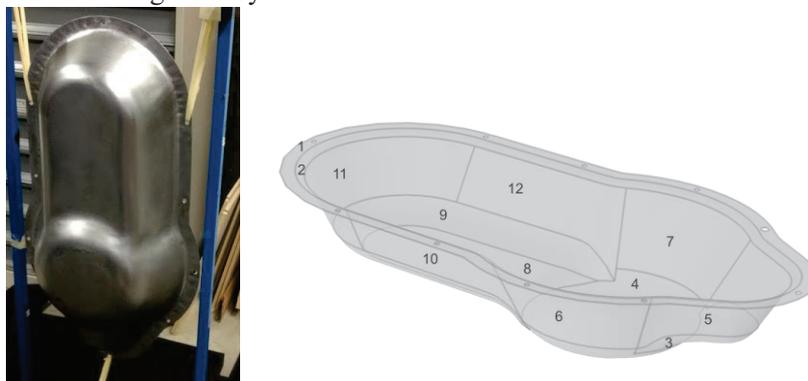


Figure 1 - Structure under investigation (left). Division into patches (right).

An acoustic mesh finite element is added that consists of a half sphere ($R = 0.75$ m) with

conjugated Astley-Leis infinite elements of 10th order attached on the surface of the sphere to model the acoustic radiation to infinity. On the bottom surface a perfectly reflecting boundary condition is added. Both the structural and acoustic mesh have at least 6 elements per wavelength, leading to a model with a total of 71259 Degrees Of Freedom (DOFs). A small amount of proportional damping is added to the structure ($\alpha_s = 1, \beta_s = 1e - 9$) to make sure that the response as function of time decays to zero. The resulting system of equations can be written as follows:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t), \quad (1)$$

$$y(t) = Lx(t), \quad (2)$$

in which $M, C, K \in \mathbb{R}^{n \times n}$ are the mass, stiffness and damping matrices respectively that follow directly from the finite element model, $F(t) \in \mathbb{R}^{n \times 1}$ is the time dependent input, $L \in \mathbb{R}^{n_o \times n}$ is the output matrix with n_o outputs and $x(t) \in \mathbb{R}^{n \times 1}$ is the vector of field variables to be solved for.

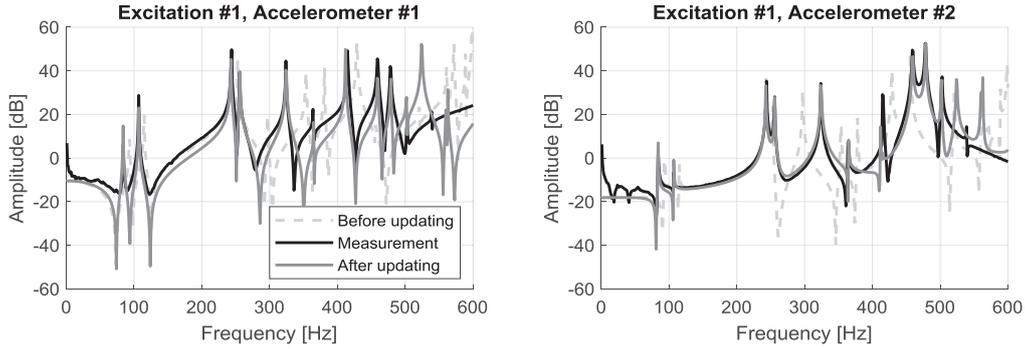


Figure 2 – Structural FRFs before and after model updating. The location of the excitation point is indicated in Figure 4.

Table 1 – Structural parameters after model updating.

Patch	1	2	3	4	5	6	7	8	9	10	11	12
Thickness (mm)	1.7	1.5	1.7	1.4	1.5	1.5	1.5	1.4	1.3	1.3	1.3	1.4
Young's modulus (GPa)	260	78	17	230	1023	249	85	340	219	379	147	350

2.1 Model order reduction

Since the system size resulting from FE discretization is far too large to be solved in (near-) realtime, MOR techniques are used to reduce the system size. The aim of the MOR scheme is to reduce the size to a size $k \ll n$, while retaining the important dynamics. This is done by calculating a reduced basis $V \in \mathbb{R}^{n \times k}$ that projects the matrices on a subspace of the system matrices as follows:

$$M_r = V^T M V, \quad K_r = V^T K V, \quad (3)$$

$$C_r = V^T C V, \quad F_r = V^T F, \quad (4)$$

$$L_r = L V, \quad x = V x_r, \quad (5)$$

which leads to the following system of reduced equations:

$$M_r \ddot{x}_r(t) + C_r \dot{x}_r(t) + K_r x_r(t) = F_r(t), \quad (6)$$

$$y_r = L_r x_r. \quad (7)$$

The reduced basis can be obtained in several ways. In this paper a Krylov based MOR technique is used to calculate V . This method aims to approximate the transfer function of the system around a set of expansion points by an implicit moment matching procedure. The moments are equivalent to the derivatives of the transfer function at the chosen expansion point. This method works well for large sparse matrices, such as the matrices that result from FE models. Details about the used algorithm, the

second order Arnoldi procedure, can be found in Refs. (8,13). To retain stability under reduction, the strategy described in (6,9) is followed. The application of this procedure to the full order model leads to a ROM of 42 DOFs that is accurate until 600 Hz (meaning a relative error below 0.1% for all the considered frequencies), thus a very significant reduction.

2.2 Conversion to first order form

To implement the derived model in a Kalman filter (see Section 3), the system has to be rewritten to first order form. This is done by rewriting the system to an equivalent system that has twice the amount of DOFs.

$$\begin{bmatrix} \dot{x}_r(t) \\ \dot{\ddot{x}}_r(t) \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M_r^{-1}K_r & -M_r^{-1}C_r \end{bmatrix} \begin{bmatrix} x_r(t) \\ \dot{x}_r(t) \end{bmatrix} + \begin{bmatrix} 0 \\ M_r^{-1}F_r \end{bmatrix} u(t), \quad (8)$$

$$y_r(t) = \begin{bmatrix} L_r^{pos} & L_r^{vel} \end{bmatrix} \begin{bmatrix} x_r(t) \\ \dot{x}_r(t) \end{bmatrix}. \quad (9)$$

We will rewrite this using the following shorthand notation:

$$A_r^c = \begin{bmatrix} 0 & I \\ -M_r^{-1}K_r & -M_r^{-1}C_r \end{bmatrix}, B_r^c = \begin{bmatrix} 0 \\ M_r^{-1}F_r \end{bmatrix}, L_r^c = \begin{bmatrix} L_r^{pos} & L_r^{vel} \end{bmatrix}, \quad (10)$$

in which the superscript c indicates that the matrices are defined in continuous time. To discretize these matrices several strategies can be used. In this paper it is chosen to use an exponential integration scheme which leads to the following system of equations (for a sampling rate $f_s = 1/T_s$):

$$x_{i+1}^d = A^d x_i^d + B^d u_i^d, \quad (11)$$

$$y_i^d = L x_i^d, \quad (12)$$

with

$$A^d = e^{A_r^c T_s}, B^d = A_r^{c-1} (A^d - I) B_r^c. \quad (13)$$

The current sample is indicated with subscript i . Note that the exponential in Eq. (13) is a matrix exponential. While the matrix exponential is far too expensive to calculate for the full order model, the calculation can be easily done for the ROM.

3. Kalman filter

The virtual sensor works by using a state estimation algorithm. This algorithm has the aim to estimate the internal states, such as pressure, acceleration, displacement, etc., by using the derived numerical model in combination with a limited set of measurements to get an estimation that is better than the prediction of merely the numerical model. Since the used numerical model is linear, it is chosen to use a Kalman filter as state observer. The Kalman filter explicitly takes measurement and process (model) noise into account as follows: They are both assumed to have a Gaussian distribution \mathcal{N} with a zero mean, leading to covariance matrices $Q_i: w_i \sim \mathcal{N}(0, Q_i)$ and $R_i: v_i \sim \mathcal{N}(0, R_i)$. The Kalman performs a two-step procedure. Firstly, a prediction step is done:

$$\text{Predicted state estimate:} \quad \hat{x}_{i|i-1}^d = A^d \hat{x}_{i-1|i-1}^d + B^d u_i, \quad (14)$$

$$\text{Predicted error covariance:} \quad P_{i|i-1} = A^d P_{i-1|i-1} A^{dT} + Q_i. \quad (15)$$

In this step the estimate from the previous step is used to get an estimate of the current time step. The second step corrects this estimate by using the measurement z_i :

$$\text{Innovation:} \quad \tilde{y}_i = z_i - L \hat{x}_{i|i-1}^d, \quad (16)$$

$$\text{Innovation covariance:} \quad S_i = R_i + L P_{i|i-1} L^T, \quad (17)$$

$$\text{Kalman gain:} \quad K_i = P_{i|i-1} L^T S_i^{-1}, \quad (18)$$

$$\text{Updated state estimate:} \quad \hat{x}_{i|i}^d = \hat{x}_{i|i-1}^d + K_i \tilde{y}_i, \quad (19)$$

$$\text{Updated error covariance:} \quad P_{i|i} = (I - K_i L) P_{i|i-1}, \quad (20)$$

$$\text{Measurement post-fit residual:} \quad \tilde{y}_{i|i} = z_i - L \hat{x}_{i|i}^d. \quad (21)$$

To make the Kalman filter stable, the system has to be fully detectable (14). Detectability of the system can be guaranteed by choosing the right sensors. Furthermore, the sensor locations for the best virtual sensor performance can be calculated by using this metric, as has been explained in detail in Refs. (15–17).

4. Virtual sensing performance

4.1 Performance under ideal acoustic conditions



Figure 3 - Oil-pan setup in semi-anechoic room.

Before the performance of the filter under non-ideal conditions is assessed, firstly the performance of the filter is evaluated under ideal conditions. The oil-pan was placed in a semi-anechoic room at the KU Leuven Sound and Vibration Lab, see Figure 3. Thus, the setting of the real test setup has the same boundary conditions as the numerical model described in Section 2. Six microphones were placed around the structure and act as measurements that are used by the Kalman filter (their locations appear in matrix L). Furthermore, 4 microphones are used as reference microphones. These microphones are not used in the filter, but are only used to assess the estimation performance of the virtual sensor. The microphone locations are indicated in Figure 4.

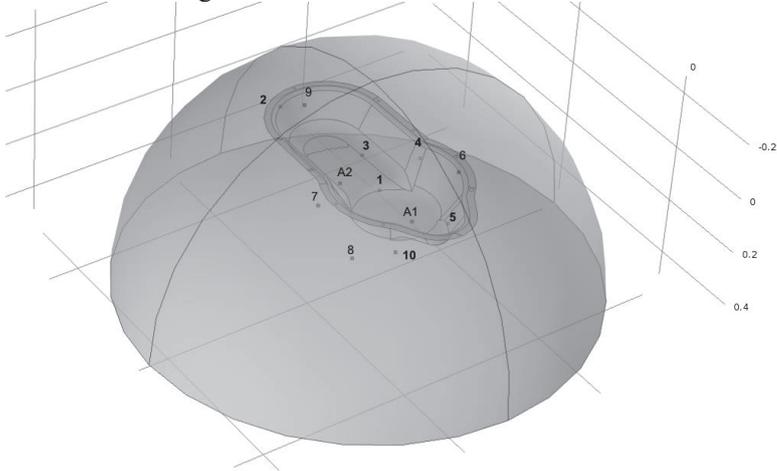


Figure 4 – Microphone and excitation locations (indicated with A1, A2). The microphones that are used in the Kalman filter are indicated in a bold font. The other microphones only function as verification of the results.

The structure is excited with a modal hammer and both the excitation force and acoustic response is used in the Kalman filter. A sample rate of $f_s = 4096$ Hz is used throughout the experiment and a low-pass filter is applied on the signal with a cut-off frequency at 500 Hz, since in that frequency range the structural model gives reasonably accurate results. The covariance of the model uncertainty is set to $3e-6$ and the covariance of the measurement uncertainty is set to $4.2e-2$, since these values lead to

the best performance. The results are shown in Figure 5, where the measured acoustic pressure signals at the reference microphones are compared to the estimated pressure signals. In the figure only microphone 6 and 8 are shown due to space constraints in this paper, but a more in-depth analysis of a similar analysis can be found in (16). It can be observed that the virtual sensor performance is significantly better than the prediction from the numerical model alone, thus the virtual sensor can successfully estimate the pressure signal at locations where no microphone is placed. Although not detailed in this paper, the same virtual sensing scheme could also be used for the estimation of other acoustic quantities, such as the particle velocity, the acoustic intensity and the acoustic power. Details can be found in Refs. (16,17).

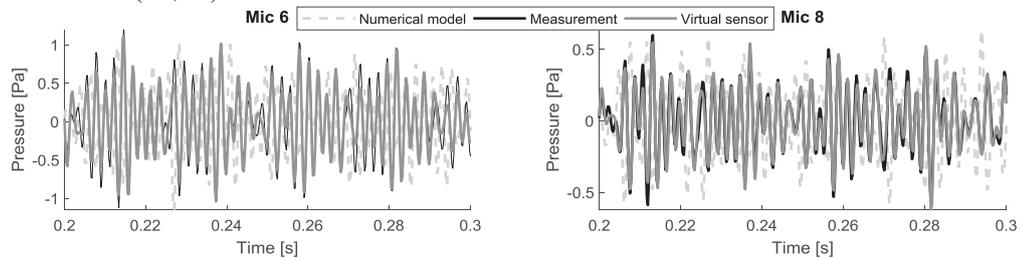


Figure 5 – Virtual sensor estimation under ideal conditions.

4.2 Performance under non-ideal acoustic conditions

To assess the robustness of the above procedure for other acoustic environments, which are only approximately behaving as a semi-infinite free field, the above experiment with the same sensor placement is repeated in two different acoustic environments, namely an office (around 3x6x3 m (length, width, height)) and an entrance hall (around 10x20x10 m (length, width, height)) of a university building. The setup at these locations is shown in Figure 6.



Figure 6 – Oil pan setup in office (left) and entrance hall (right).

Both of the setups are placed on a table, instead of on the floor, which is modelled as a perfectly reflecting surface. The main assumption that is done here is that the Kalman filter is able to capture the uncertainties in the acoustic domain, due to the noise assumptions of the filter. The possibility of using the Kalman filter for these non-ideal environments would increase the potential applications for these filter, since it would allow for the estimation of the acoustic pressure in many situations (for example structures in large halls) that would otherwise need a complex acoustic model. The results are shown in Figure 7.

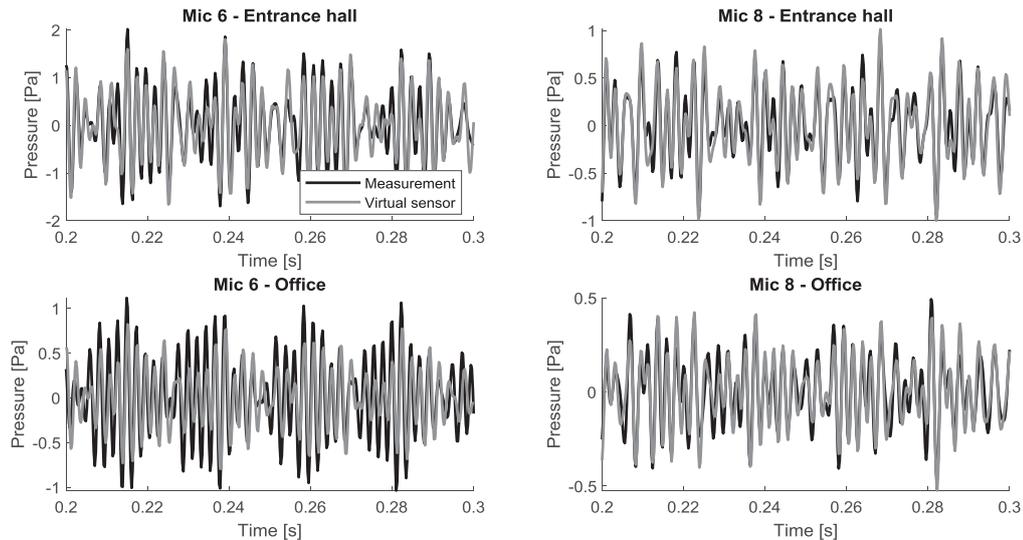


Figure 7 – Virtual sensing performance under non-ideal conditions.

It can be observed that the virtual sensor still manages to capture the global dynamic behavior, although the accuracy of the estimation is less than for the ideal case, which is expected.

5. Conclusions

In this paper it is shown how an exterior vibro-acoustic virtual sensor can be created by combining a numerical model with a small amount of microphone measurements. The numerical model consists of finite elements, combined with infinite elements to approximate the Sommerfeld radiation condition. To allow for (near-)real time virtual sensing, the numerical model is reduced in size with state-of-the-art model order reduction techniques. The measurements and the reduced numerical model are combined in a Kalman filter for acoustic pressure estimation in the full domain.

The effectiveness of the proposed virtual sensor is shown with several experiments. Firstly, the geometry under investigation, which is an oil-pan-like structure, was placed in a semi-anechoic room. This is considered to be the best case scenario, since the acoustic boundary conditions of the numerical model correspond to the experimental setup. It is shown that the virtual sensor gives good correspondence between the estimated pressure and the measured pressure.

Furthermore, the same setup was placed in two non-ideal acoustic scenarios that do not behave like semi-infinite free spaces, while the numerical model was kept the same, thus with the Sommerfeld assumption, to see if this additional model uncertainty can be captured with the noise assumption of the Kalman filter. It is shown that the virtual sensing performance for both scenarios is still acceptable, although the estimation quality is less than for the ideal case. Further research will be devoted to a more in-depth investigation of how this uncertainty can be captured, for example by utilizing a Kalman filter with colored process noise, instead of white noise, so that the performance of the filter can be improved when it is used in a non-ideal acoustic environment.

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