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Fluttering Reverberance: Real Life Examples of Chaotic Billiards with Convex Sections

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ABSTRACT

Billiards are used to explore dynamical systems. The mathematical literature focusses on idealised physics and simple shapes like the Bunimovich stadium. Real enclosures where sound waves created by such shapes exhibit strong audible artefacts are uncommon. Also, billiard modelling is a high frequency approximation and diffraction is often important in rooms. Notwithstanding, some rare real-life acoustic billiards where the effects of closed-orbits are clearly audible have been found and examined with simulations and measurements. The abandoned Thurgoland railway tunnel in Yorkshire, UK has an extraordinary metallic flutter in its long reverberant decay. A booking hall in the National Theatre railway station in Oslo has a very strong warble, and sounds different to the flutter echo heard in a simple cylindrical space. Both spaces feature concave geometries: the Thurgoland tunnel has a horseshoe-shaped cross-section, whereas the Oslo station is a distorted cylinder in plan with a corrugated domed roof. Ray-tracing reveals closed orbits, which lead to repeated reflections and marked flutter echoes in both spaces. Examining the angles of the rays arriving at the receiver, and the autocorrelation of the impulse response envelope, provides further evidence of non-diffuseness. A Boundary Element Method is used to explore diffraction within the Thurgoland tunnel.

Keywords: Billiards, Flutter echo

1. INTRODUCTION

Flutter echoes caused by repetitive reflection paths are commonly heard in spaces such as stairwells or lecture theatres with parallel, smooth, large reflective surfaces.[1] Probably the most common audible artefact is colouration caused by the amplification of selected mid-high frequency components.[2] These are usually harmonics of the repetition pitch fundamental.[3] This paper explores spaces where the flutter echoes are unusually marked with distinct temporal fluctuations.

The spaces are considered as mathematical billiards.[4] These are systems that can be used to explore aspects of dynamic systems such as chaotic behavior. For acoustic waves, an enclosed space is modelled as a billiard with perfectly reflective surfaces. The sound is modelled in the high frequency limit as billiard balls. By following the path of the balls and applying the Law of Reflection (angle of incidence = angle of reflection), systems display different behaviour depending on the shape of the boundary. A classic example is the Bunimovich stadium, which has two parallel sides capped at each end by semi-circular boundaries. This geometry displays ergodic behaviour - almost all paths explore almost all positions on perimeter and angles of reflection. Most of the billiard geometries considered in the literature are far removed from everyday architectural acoustic spaces.

This paper brings together mathematical billiards, psychoacoustics and room acoustic models to examine real enclosures that have unusual temporal artefacts. The enclosures are explored by measurement, ray-tracing and boundary element method (BEM). The ray-tracing is equivalent to the classic billiard ball modelling used within the mathematical literature. BEM allows an exploration of what happens when sound is properly modelled as a wave. The interest here is in audible artefacts, so bringing in knowledge from psychoacoustics is vital.

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2. THURGOLAND TUNNEL

2.1 History

This tunnel was part of the Manchester, Sheffield and Lincolnshire railway. Constructed in 1946-8, the 275m tunnel has a horseshoe shape cross-section - see Figure 1.[5] The tunnel was blasted and excavated, before being lined with concrete 23-46 cm thick. This was done by pumping concrete between the bedrock and steel shuttering, resulting in a smooth concave internal surface. It is now part of the National Cycle Network in the UK. The floor has a central tarmac path with aggregate to either side. In the modelling the aggregate was treated as a hard surface, because the floor plays only a secondary role in creating the temporal flutter.

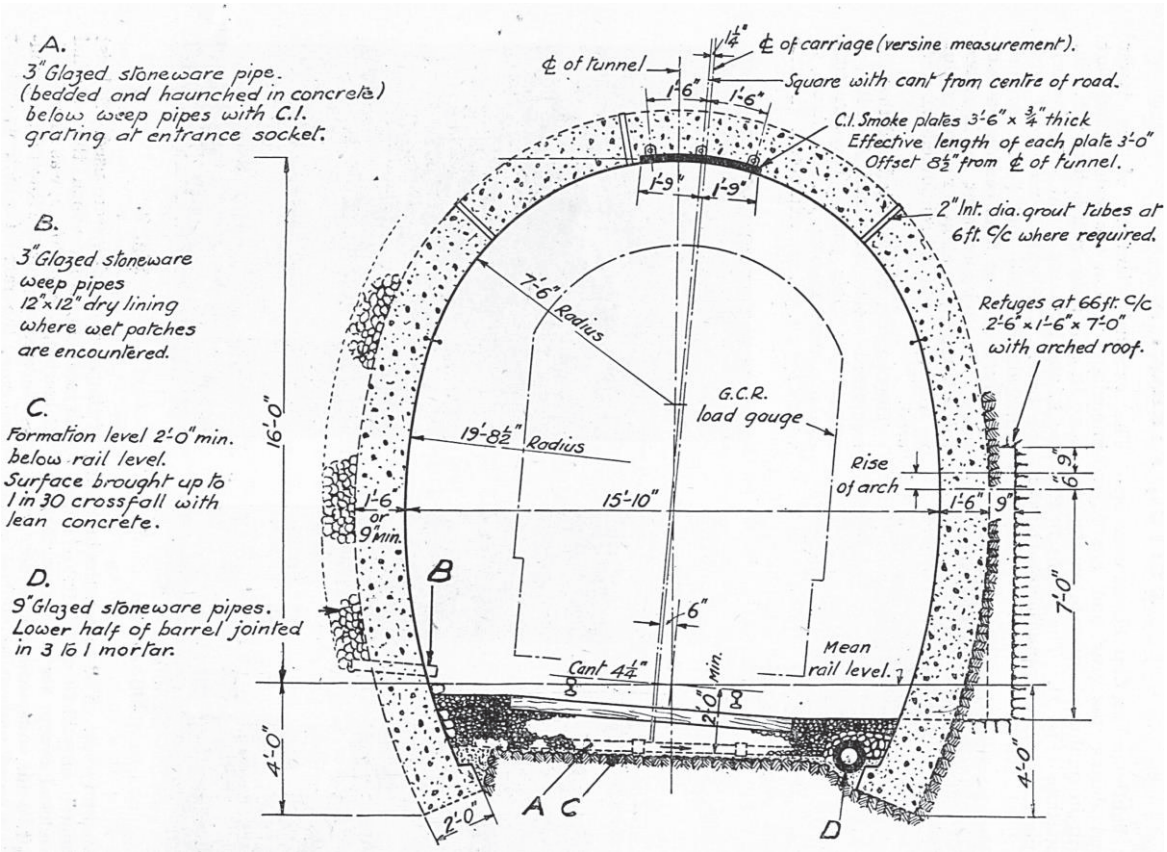

Figure 1 Cross-section through Thurgoland railway tunnel (After Concrete[5])

2.2 Measurements

Impulse responses were measured using thirty-second swept sine waves.[6] The loudspeaker was a B&O Beolit; the microphone a NTI Audio M4261 and the interface to the computer was via a Focusrite Scarlett 2i2 soundcard. See Table 1 for source and receiver positions. The tunnel is continuously used as a cycle way, and so this limited the sound levels that could be used for measurement. Furthermore, natural sounds from outside the tunnel also increased the background noise. Table 2 shows the reverberation times measured in the tunnel based on 8 measurement positions. Other results will be presented just for receiver 2, because this position is where the temporal fluctuation is most pronounced for the positions measured. Note, the space is highly non-diffuse, and consequently the reverberant decays are distinctly non-linear. Given the heavy wall-construction, at mid-high frequency air absorption plays an important role in determining the reverberation time. The smooth surfaces are also important, because scattering would promote more reflections perpendicular to the tunnel length, reducing the mean free path and so reducing the decay time.[7]

Table 1 – (x,y) source and receiver positions, where (0,0) is in the middle of the floor. The source and receiver were displaced by 1, 2 or 4m in the z-direction (along length of tunnel)

Position	x (horizontal)	y (vertical)
Source	0	1.2
Receiver 1	0	1.2
Receiver 2	-1.2	1.2
Receiver 3	0	2
Receiver 4	-1.2	2

Table 2 – Average reverberation time and 95% confidence intervals in Thurgoland Tunnel

Frequency (Hz)	125	250	500	1000	2000	4000	8000
T_{25} (s)	13 ± 1	26 ± 4	22 ± 1	10 ± 1	6.0 ± 0.5	2.8 ± 0.2	0.90± 0.04

Figure 2 shows a smoothed envelope of the broadband impulse response for receiver 2. This was formed by calculating the envelope of the impulse response using a Hilbert transform, and applying an exponential filter with a time constant of 10 ms.[1] The regular bumps in the decaying sound give rise to the temporal warble. The literature[1,2] reports that reflections with delays between roughly 25 and 50 ms produce temporal flutter, with delays much longer having potential to create audible discrete echoes. In this case the temporal modulation is spaced at 63 ms.

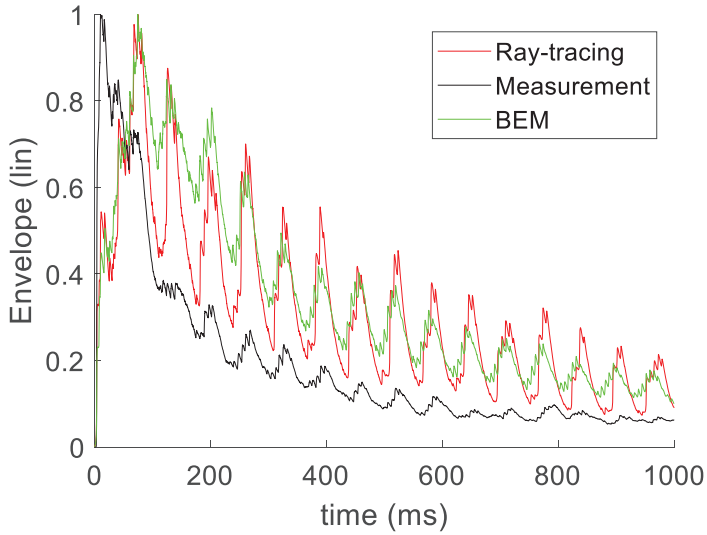

Figure 2, Broadband smoothed envelope of measured and modelled impulse responses. Each envelope is normalised to its maximum value.

Figure 3 shows the autocorrelation of the impulse response envelopes. This is done to explore repetitive temporal structure in the envelope. Before calculating the autocorrelation function, the impulse response envelopes were high pass filtered (1st order Butterworth, -3dB point = 50 Hz) to remove the general decay of sound so the envelope fluctuations are clear. The broad peak at ≈60 ms is present because of the flutter.

2.3 Ray-tracing

As the tunnel is long and has constant profile, it was assumed that the temporal flutter is caused by the cross-section geometry and therefore 2D ray-tracing would be sufficient. The distance and angles to 43 positions on the cross-section were obtained using a laser measurer (Bosch GLM 80). A curve fit was then used to create the geometry used in the predictions. Pure ray-tracing was implemented without any scattering. The absorption coefficient of the walls was set so the decay time of the prediction matched the average measured reverberation time in each octave band.

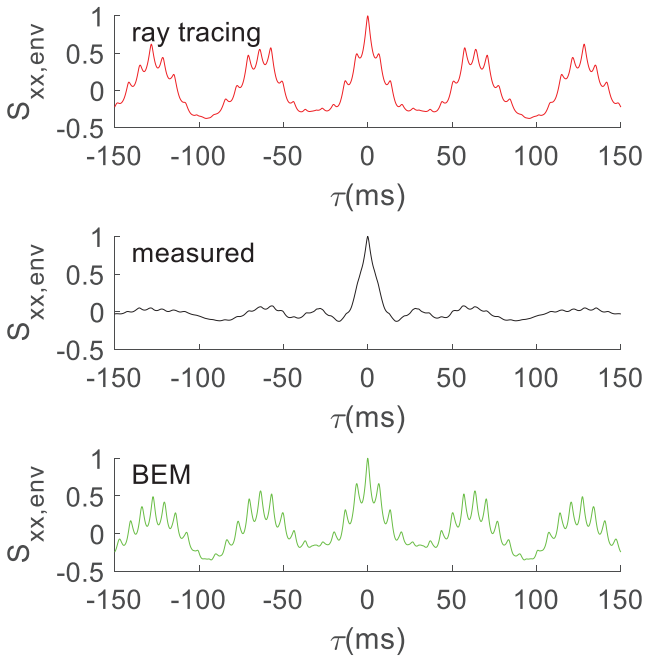

Figure 3 Broadband autocorrelation of high-pass filtered impulse response envelopes.

Figures 2 and 3 included ray tracing results. While the model correctly predicts the rate of the temporal flutter, the modulation is stronger than in the measurement. This is to be expected as propagation along the length of the tunnel is being neglected in the model. In the real tunnel, sound will return to the microphone from different positions along the length of the tunnel due to diffraction, leading to the reflections being more smeared overtime, and therefore weakening the temporal flutter. Note, the measurements had a stronger direct sound due to the source not being omni-directional.

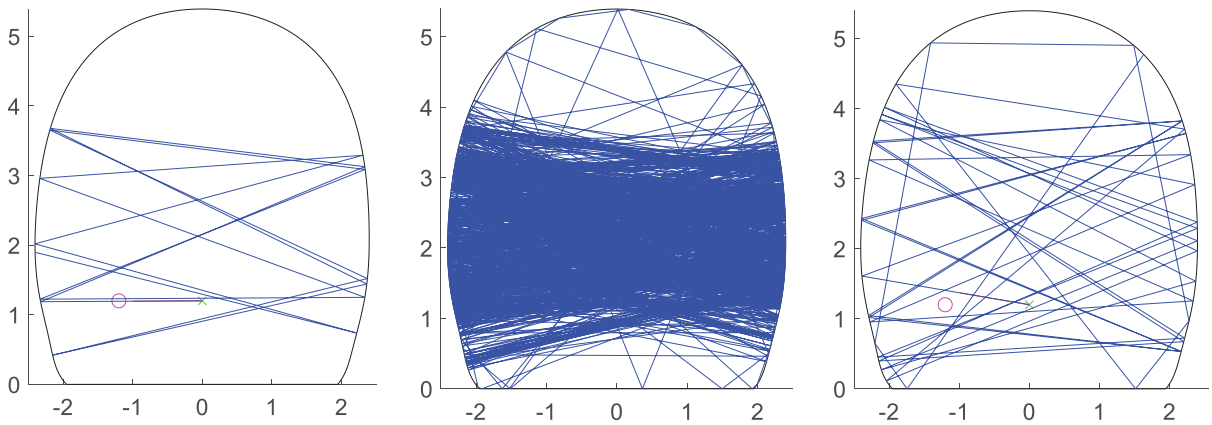

Figure 4. The path of one ray. Source: green cross. Receiver: magenta circle. Left: 15 reflections, initial angle 180.3°; Middle, 1000 reflection; 180.3°; Right initial angle 170.3° 40 reflections.

Figure 4 shows the ray-tracing path for one ray with different reflection orders and different initial angles. The left plot illustrates how the geometry creates closed-orbit paths where the ray arrives at the receiver at regular intervals creating a flutter echo. As the middle plot shows, the number of reflections modelled has to be extended to one-thousand to get the ray breaking out from the predominantly horizontal orbits and reaching the ceiling. The right plot has a ray starting at an angle 10° different from the other plots. For this path, the ray visits different parts of the surface after only order 40.

These plots illustrate some important features of the geometry that create the temporal flutter. First, an audible temporal effect needs repeated delays of greater than roughly 25ms (path length of 8.5m). If the sound was being focussed to the receiver every reflection, the delay would be too short for the perceived effect. The bowed shape of the tunnel creates paths that traverse the width more than once before returning to the receiver. A second important feature is there is a relatively broad range of angles over which these closed-orbit paths exist. This means that for an omnidirectonal sound source,

there is sufficient energy traversing closed-orbit paths to make the flutter audible above other reflections that are following more random paths. Finally, these closed-orbit paths last to a high reflection order, which is one reason that the temporal flutter is heard within the reverberant sound field. The behaviour is similar to that seen in rooms with domed roofs, where the focal point of the dome is below the floor.[8]

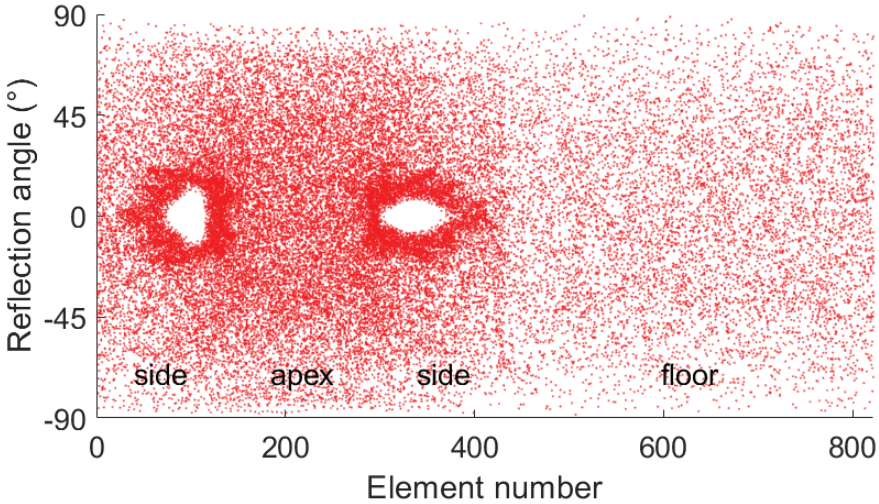

Figure 5 Reflection angle to surface normal vs element number. The elements are numbered in a clockwise direction starting from the bottom left corner of Figure 4. 200 rays traced up to order 400

In mathematical billiards,[4,9] a common way of analysing a space is to plot the angle of reflection of the billiard balls (or rays) vs the position on the surface (ignoring the strength of the reflections). Such a plot is shown in Figure 5. The x-axis is element number of the discretized surface. The reflections from the floor have random angle. The two 'holes' correspond to elements roughly two meters above the floor on either side. These are evidence of closed-orbits. If the ray-tracing was continued to a higher order, then these holes would disappear.

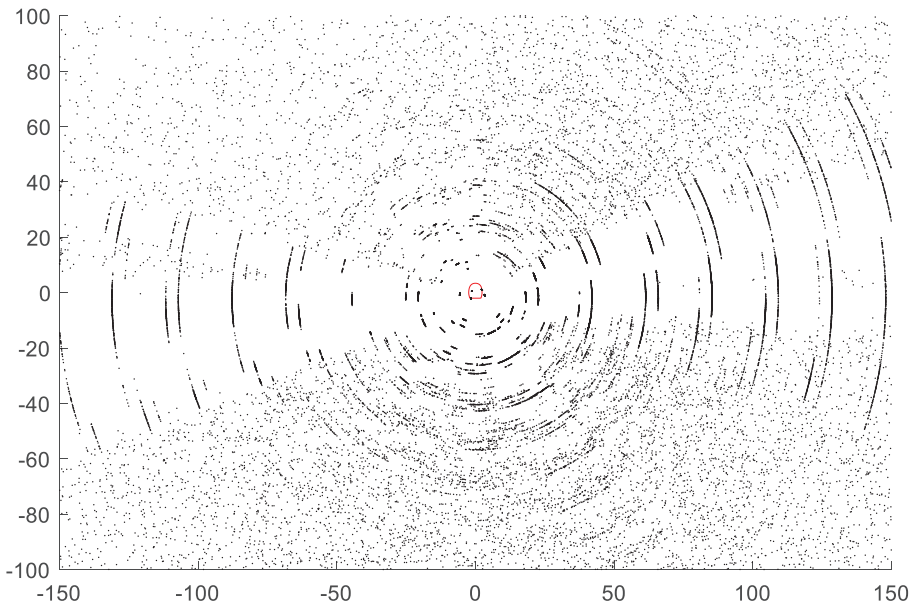

Figure 6 Plot of reconstructed image sources. The tunnel is shown in red in the centre of the plot.

As Figure 4 illustrated, the rays continue in these closed-loops for a certain number of reflections, before finally being reflected upwards or downwards sufficiently to escape the closed-loops. They are unstable paths. Subsequently, the ray path becomes unpredictable and the billiard shows chaotic properties. Like the Bunimovich stadium, Thurgoland tunnel shows ergodic motion.

Figure 6 shows a plot of estimated image sources. This was constructed from the arrival time and angle of the rays at the receiver. This shows two regular rings of image sources that create the periodic

orbits and flutter echo. The smaller spacing between the double rings is too small to create perceived temporal flutter. It is the larger spacings between the rings of image sources that is responsible.

2.4 Wave-based modelling

Diffraction plays an important role in room acoustics when geometries become similar in size to wavelength. This is investigated using a 2D BEM model of the cross-section.[10] The frequency-domain pressures are laborious calculated for the entire spectrum and an inverse Fourier Transform used to recover the impulse response. Two filters were applied before the inverse Fourier Transform: (1) a low pass filter to reduce time aliasing (Chebychev, 2nd order, 1dB ripple, -3dB point = 3200 Hz) and (2) a high pass filter (Butterworth, 1st order, -3dB point= 50Hz). To reduce computation time: (1) the sampling frequency was 8000 Hz, and (2) the surface admittance was set to give the measured reverberation times, capped at a maximum value of ten seconds. The length of the Fourier Transform was sufficient to simulate a decay of 60 dB.

Figures 2 and 3 showed the BEM impulse response envelope and its autocorrelation function. The results are similar to that obtained by the ray tracing. This indicates that the temporal flutter is not strongly influenced by diffraction effects.

3. Booking Hall, National Theatre Station, Oslo

The booking hall at the Parkveien Exit of the National Theatre Station in Oslo was designed by Architect Arne Eggen; it is shown in Figure 7. He remarked in an interview to the first author, *"I knew that there would be a flutter echo, but not that it would be that strong and attract interest, even after 15 years."* The warble is so strong that a plaque was added to the middle of the floor to celebrate this accidental 'Akustisk Skulptur.'

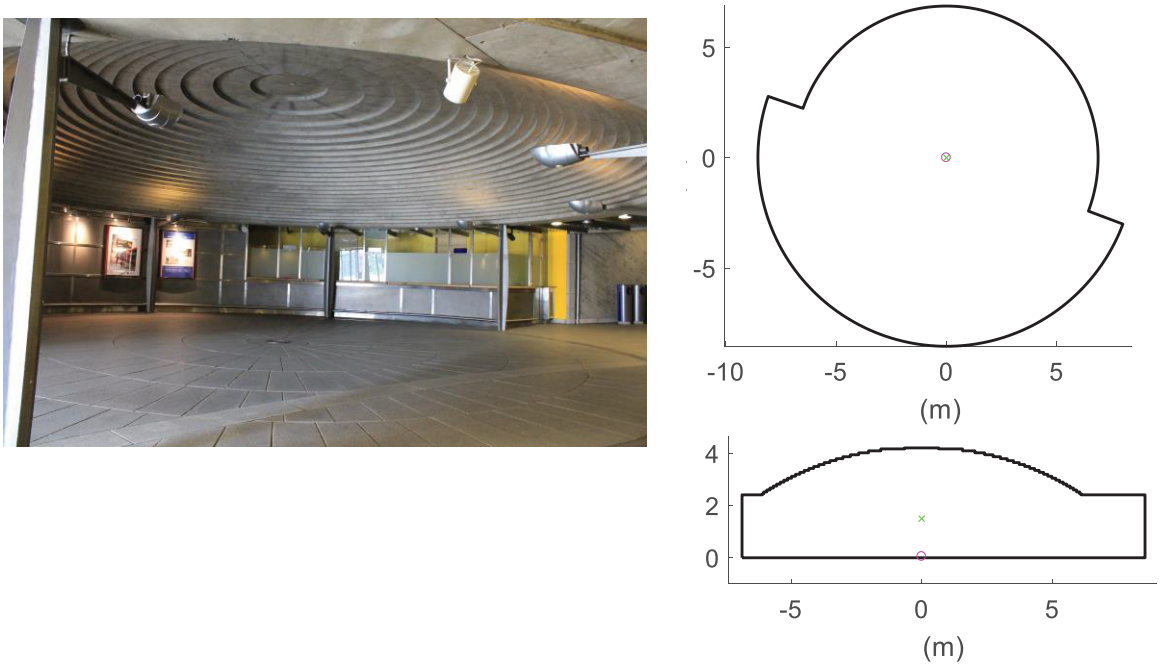

Figure 7 The Oslo booking hall. Top right plan (exits and entrances not included), bottom right section.

3.1 Measurements

The dimensions of the space were provided by the architect. The acoustic measurements were unfortunately relatively rudimentary; at the time, these were no intention of doing a detailed acoustic analysis. Four balloon bursts were recorded on an Edirol R-09 using its built-in microphones. The source and receiver positions were roughly the same for every balloon burst and are marked on the plan and section. Reverberation times are shown in Table 3. With similar source and receiver position being used for every measurement, the standard deviations are small.

On first inspection of the drawings and photograph, there are several possible causes of the temporal flutter. What is responsible: the corrugated ceiling? the domed roof? Or the two semi-cylinder radii visible in the plan view?

Table 3 – Average reverberation time and 95% confidence intervals in the Oslo Booking Hall.

Frequency (Hz)	125	250	500	1000	2000	4000	8000
T_{25} (s)	23 ± 2	19 ± 1	9 ± 1	2.2 ± 0.4	1.2 ± 0.1	0.99 ± 0.03	0.66 ± 0.03

3.2 Corrugated ceiling

Staircase structures are known to create chirps through repetition pitch.[2] Examining the measured impulse responses, there are broad peaks either around 1,700 and/or 2,600 Hz (varies between measurements). It seems likely these arise due to amplification by the room rather than inherent resonances of the balloon source. Could these be caused by the corrugations? A simple model was constructed that calculated the expected first-order arrival times for sound reflecting from each corner of the corrugations. This was done for both the source and the image source created by the floor. The reciprocal of the difference in the arrival times was used to estimate likely repetition pitch. This indicated repetition pitch effects around 1,500 Hz and 2,200 Hz. Consequently, the corrugated ceiling is probably the cause of the peaks in the measured spectra. This also means the corrugations, because the delays between reflections are too short.

In section, the Booking Hall has similarities to the Thurgoland tunnel (consider the booking hall section rotated by ninety degrees and mirror-imaged in the floor). There are concave surfaces and source and receiver positions not at the focal point. However, the Booking Hall has a highly corrugated ceiling. 2D ray-tracing were carried out for the Booking Hall with a smooth concave ceiling and with the corrugations properly modelled. While temporal flutter is created for the smooth ceiling, there was poor correspondence with the measurements. The presence of the corrugations in the model completely removed the temporal flutter creating a smooth decaying impulse response. The implication of this is that the temporal flutter in the booking hall is driven by the concave surfaces seen in the plan rather than the dome.

3.3 2D Ray-tracing of plan

The billiard modelled is shown in Figure 7 (top right). The exit that can be seen on the right of the photograph was modelled as a completely absorbing surface. Figure 8 shows the envelope and autocorrelation of the envelope for a ray-tracing of the floor plan. This 2D model captures the temporal flutter correctly, although the rate of modulation is not predicted correctly. For example, the first side lobe peak in the measurement is at 49ms, whereas in the 2D simulation this is at 40ms. In the 2D simulation, sound is forced to remain in a single plane and thus the repetition paths are the shortest they can be. In the real space, focussed reflections from the side walls can return by longer routes (e.g. by also reflecting off the floor), and so increasing the possible time between repeated reflections.

Figure 9 shows the reflection angle verses element number. For a source in the middle, zero dimensional orbits are created with the sound repeatedly following radial paths. (The small spread around zero degrees is because the curved surface was modelled as a large number of facets and the ray-tracing receiver is finite in extent.) The two different radii (Figure 7 right) leads to two repeat times, and hence the autocorrelation of the envelope shows double peaks for the first side lobe.

4. CONCLUSIONS

Two sonic curiosities have been investigated that have strong temporal flutter in the impulse response. Both feature constructions that lead to long reverberation times and convex surfaces that create focusing.

It is the horseshoe shape of the Thurgoland tunnel that creates a complex temporal warble. The phenomenon is not a single flutter but a complex mixture of invariant deterministic orbits and closed orbits. These orbits retain greater energy than the surrounding diffuse energy due to the focusing effects of the smooth curved boundaries. The complex paths mean that the flutter sounds different to a simple repeating pattern that might be heard at the centre of a large sphere.

The Oslo Booking Hall floor plan consists of two semi-cylinders of different radii. This feature is primarily responsible for the complex temporal flutter via zero dimensional orbits with sound repeatedly following radial paths. The corrugated domed roof may play a role in modifying the temporal flutter, but to understand this requires 3D modelling. Unfortunately, ray-tracing models are

not well suited to model the corrugation that produces scattering, especially one that does not follow a lambert distribution and produce mainly radial scattering.

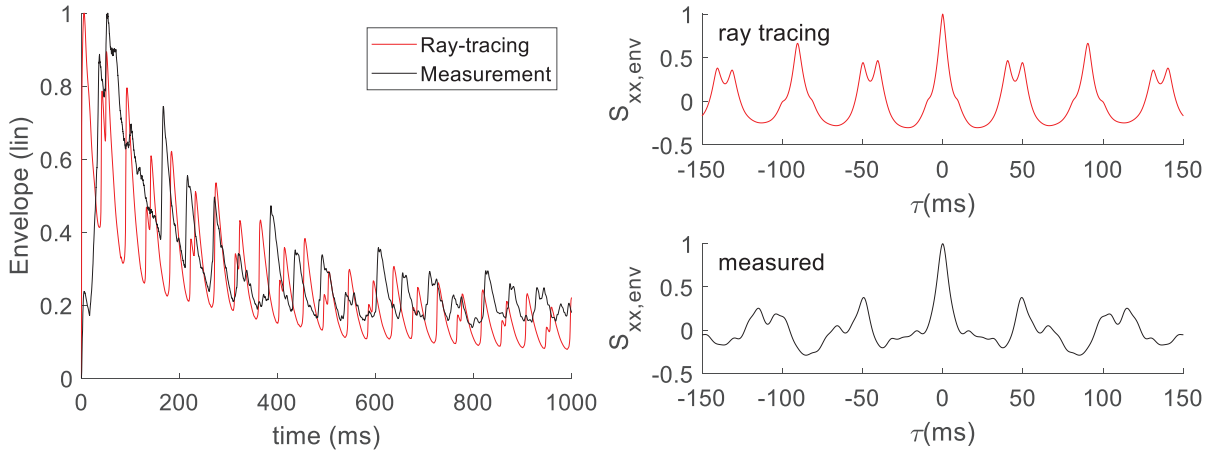

Figure 8, Left: Broadband smoothed envelope of measured and modelled impulse responses. Each envelope is normalised to its maximum value. Right: Autocorrelation of the envelope

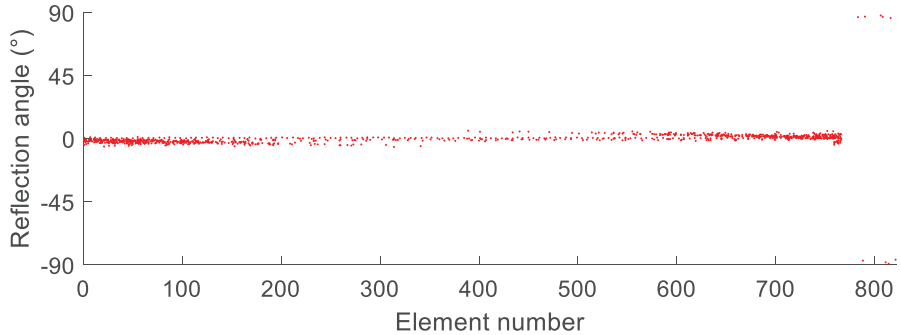

Figure 10. Reflection angle to surface normal vs element number. The elements are numbered in a clockwise direction starting from the element at -72º. 1000 rays traced up to order 500

ACKNOWLEDGEMENTS

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