PROCEEDINGS of the
23rd International Congress on Acoustics
9 to 13 September 2019 in Aachen, Germany

Measurement of the four pole matrix of a sample in a transmission tube

Rene BOONEN[(1)]

[(1)]Nabla Technical Consulting, Germany, Boonen.Rene@gmail.com

Abstract

A transmission tube has been developed to measure the four pole matrix of a sample of absorbing material. The electrical analogy of a sample in a transmission tube can be considered as a T-equivalent circuit of three impedances, two in series and one in parallel. As consequence, three measurements with three different closing impedances needs to be carried out. From these measurements, a set of equations will result from which the three impedances of the T-equivalent circuit can be determined in real and imaginary parts in terms of frequency. Once these impedances are known, any four pole matrix, such as the transfer or the scattering matrix, of the sample can be set up. The method has been validated by measurements of several sample combinations in the impedance tube. A good agreement has been established between transmission tube and impedance tube measurements in a range of 40Hz-4kHz.

Keywords: Kundt, Transmission, Impedance, Transfer, Matrix

1 INTRODUCTION

The Kundt's tube is used to measure the acoustical absorption, reflection or impedance of a sample of absorbing material or a muffler. The sample or muffler needs to be backed by a known impedance, such as a hard wall, an open end or an anechoic end. However, if one wants to calculate layers of absorbing materials or muffler systems, four pole data such as the transfer matrix are needed. Therefore, a transmission tube has been developed which measures the four pole data with high accuracy. The sample is represented as a T-equivalent circuit [1] of three impedances, two in series and one in parallel. Three impedance measurements with three different closing impedances will be carried out according to the ISO 10534-2 standard [2]. From these measurements, a set of equations will result from which the three impedances of the T-equivalent circuit can be determined in real and imaginary parts in terms of frequency. Once these impedances are known, any four pole matrix, such as the transfer or the scattering matrix, of the sample can be set up. The method has been validated by measurements of several sample combinations in the impedance tube.

2 SETUP OF THE TRANSMISSION TUBE

Figure 1 presents a scheme of a normal Kundt tube using the two microphone transfer function method according to the ISO 10534-2 standard.

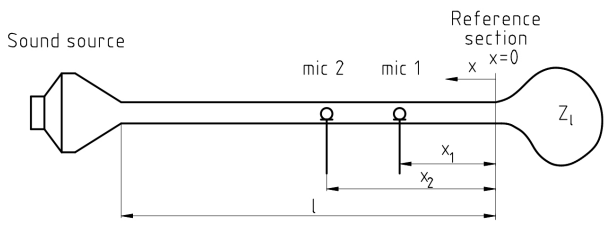

Figure 1. Scheme of an impedance tube using the two microphone transfer function method according to the ISO 10534-2 standard.

The wave pattern in the wave guide is governed by the one-dimensional Helmholtz wave equation, which describes the pressure distribution along the wave guide [3]. At each position x, the pressure p in terms of the angle frequency ω in the wave guide is expressed as:

$$p(x,\omega) = \phi_g \frac{Z_0 Z_g}{Z_0 + Z_g} \frac{e^{-jkl}}{1 - \Gamma_l \Gamma_g e^{-j2kl}} (e^{jkx} + \Gamma_l e^{-jkx}) \tag{1}$$

wherein ϕ_g the flow and Z_g the internal impedance of the acoustic source, Z_0 the characteristic tube impedance, k the wave number, l the length of the tube, Γ_l and Γ_g the reflection coefficient on the load and the source impedance respectively.

By taking the transfer function T_{12} between the acoustic pressures at two positions x_1 and x_2, all the quantities mentioned above disappear except the reflection coefficient at the load. Consequently, this reflection coefficient Γ_l can be determined. The load impedance Z_l is then determined from the reflection coefficient Γ_l.

$$T_{12} = \frac{p(x_1,\omega)}{p(x_2,\omega)} = \frac{e^{jkx_1} + \Gamma_l e^{-jkx_1}}{e^{jkx_2} + \Gamma_l e^{-jkx_2}} \tag{2}$$

$$\Gamma_l = -\frac{e^{jkx_1} - T_{12} e^{jkx_2}}{e^{-jkx_1} - T_{12} e^{-jkx_2}} \quad \Rightarrow \quad Z_l = Z_0 \frac{1+\Gamma_l}{1-\Gamma_l} \tag{3}$$

The load impedance Z_l is the impedance of everything appearing behind the reference section. By taking the transfer function T_{12}, the source reflection coefficient is eliminated. This implicates that the structures in the source tube, i.e. everything between the microphone at position x_2 and the sound source is eliminated from the expression of the load impedance. These properties form the basis of the newly developed transmission tube.

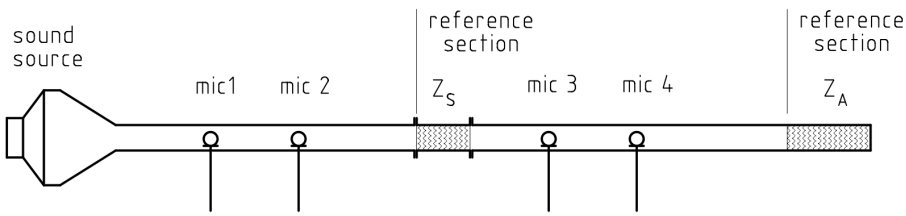

Figure 2. Scheme of the transmission tube with two microphone heads, sample Z_S and closing impedance Z_A.

A scheme of the transmission tube is presented in figure 2. In this tube, a second microphone arrangement has been introduced between the sample Z_S and the closing impedance Z_A. The measurement head with microphones 1 and 2 measures the impedance behind the first reference section, consisting of the sample impedance, the impedance of the second measurement head and the closing impedance. The measurement head with microphones 3 and 4 measures the impedance behind the second reference section, consisting of only the closing impedance Z_A.

To determine the three impedances Z_1, Z_2 and Z_3, three impedance measurements at each measurement head have to be carried out with three different closing impedances using the ISO 10534-2 two microphone transfer function method. From these measured impedances, a set of three algebraic equations will be set up from which Z_1, Z_2 and Z_3 will be solved in amplitude and phase. Once these three impedances are known, any four pole matrix such as the impedance matrix, the admittance matrix, the transfer matrix and the scattering matrix can be obtained.

The transfer matrix $\mathbf{H} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ can be determined from the three impedances Z_1, Z_2, Z_3 and the characteristic impedance Z_0:

$$\mathbf{H} = \begin{bmatrix} 1 + \frac{Z1}{Z3} & Z0\,Z1 + Z0\,Z2 + \frac{Z0\,Z1\,Z2}{Z3} \\ \frac{1}{Z3\,Z0} & 1 + \frac{Z2}{Z3} \end{bmatrix} \tag{4}$$

The transmission loss TL can then be calculated from transfer matrix:

$$TL = 20\log(.5\,(A + \frac{B}{Z0} + C\,Z0 + D)) \tag{5}$$

3 EXPERIMENTAL RESULTS USING THE TRANSMISSION TUBE

Figure 3 presents the newly developed transmission tube.

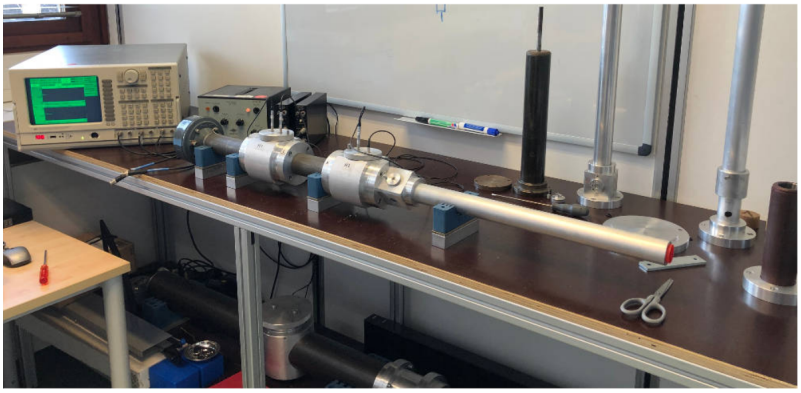

Figure 3. Transmission tube with internal diameter of 45 mm with two measurement heads and closing impedance. Total length is 1.4 m

At the left end situates the loudspeaker, then the first measurement head with two microphones, then the sample holder, then the second measurement head with the closing impedance tube at the right side. The closing impedance tube has a valve system to create the three closing impedances without disassembling the setup. The transmission tube has a frequency range from 40 Hz until 4 kHz.

The test sample is presented in figure 4. It is a glass fiber sample of 20 mm thickness which fit slightly tight in the sample holder with a surplus of 0.2 mm.

The figures 5, 6 and 7 presents the measurement results of the three impedances Z_1, Z_2 and Z_3 of the T-equivalent circuit between 40 Hz and 4 kHz. The two series impedances Z_1 and Z_2 are more or less resistive of nature. Their magnitude is mainly horizontal. The phase remains around zero which indicates the resistive nature. At 800 Hz, a sample resonance occur and above this resonance, the magnitude is slightly increasing and the phase tends towards $+90^{\circ}$, indicating that the sample behaves more as an acoustic inertia. The impedance Z_3 behaves as an acoustic volume. The magnitude decreases with a slope of -20 dB/decade and with a phase of -90°.

From these three impedances, the transfer matrix $\mathbf{H}$ is calculated using expression (4). Then, the transmission loss of this sample is determined using expression (5) and is presented in figure 8. Once the transfer matrix has been obtained for a sample, they can be collected in a library of sample transfer matrices. These transfer matrices can be used to predict the acoustic reflection and absorption of different assemblies of absorption layers without prior measurements with the impedance tube.

Figure 4. Sample of glass fiber material with diameter 45mm and 20mm thickness.

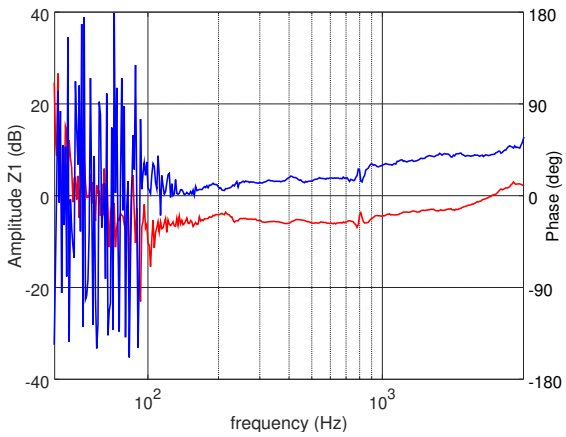

Figure 5. Measured impedance Z_1 from the T-equivalent circuit. (magnitude in red line with reference 0 dB is Z_0, phase in blue line).

To illustrate this possibility, as a first case, the transfer matrix $\mathbf{T}$ of the sample against a hard wall will be obtained by multiplying the measured transfer matrix by the transfer matrix of a hard wall:

$$\mathbf{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_H} & 1 \end{bmatrix} \tag{6}$$

wherein Z_H is the hard wall impedance, which is set to 1000 times the characteristic impedance Z_0.
The impedance Z_T of the sample against the hard wall is then determined from the the elements $\mathbf{T}(1,1)$ and $\mathbf{T}(2,1)$ of the transfer matrix $\mathbf{T}$:

$$Z_T = \frac{\mathbf{T}(1,1)}{\mathbf{T}(2,1)} \tag{7}$$

The reflection and absorption coefficient Γ and α are then determined as:

$$\Gamma = \frac{\frac{Z_T}{Z_0} - 1}{\frac{Z_T}{Z_0} + 1} \quad \text{and} \quad \alpha = 1 - \Gamma\Gamma^c; \tag{8}$$

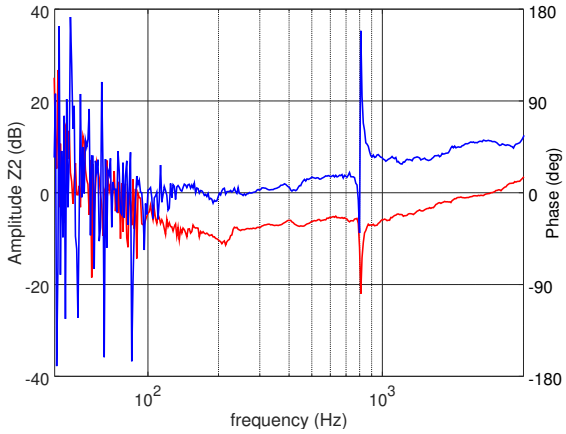

Figure 6. Measured impedance Z_2 from the T-equivalent circuit. (magnitude in red line with reference 0 dB is Z_0, phase in blue line).

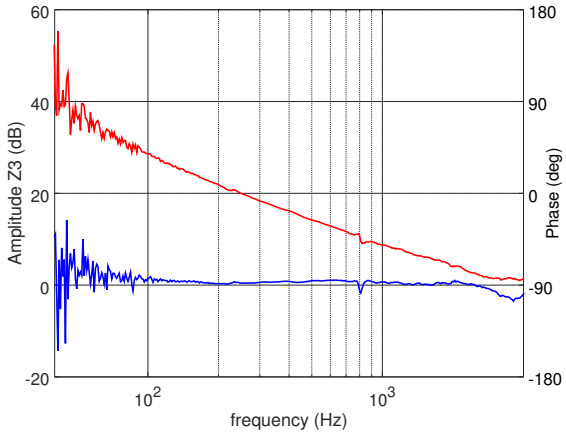

Figure 7. Measured impedance Z_3 from the T-equivalent circuit. (magnitude in red line with reference 0 dB is Z_0, phase in blue line).

To evaluate the result, a benchmark measurement has been carried out in a classical impedance tube. In the sample holder, the sample has been put directly against the hard wall. Then, the reflection and absorption coefficient are measured and plotted against the predicted coefficients by using the transfer matrix. Figure 9 presents the result. In red, the real part of the reflection coefficient obtained from the transfer matrix is plotted against the classical tube measurement in green. In dark blue, the imaginary part of the reflection coefficient obtained from the transfer matrix is plotted against the classical tube measurement in light blue. Both plots show a remarkable agreement.

Figure 10 presents the absorption coefficient. In red, the absorption coefficient obtained from the transfer matrix is plotted against the classical tube measurement in green. Also these plots show a remarkable agreement.

As a second case, an assembly consisting of the sample and an air gap of 100 mm closed by a hard wall will be considered. The total transfer matrix **T** of the assembly will be obtained by multiplying the measured sample

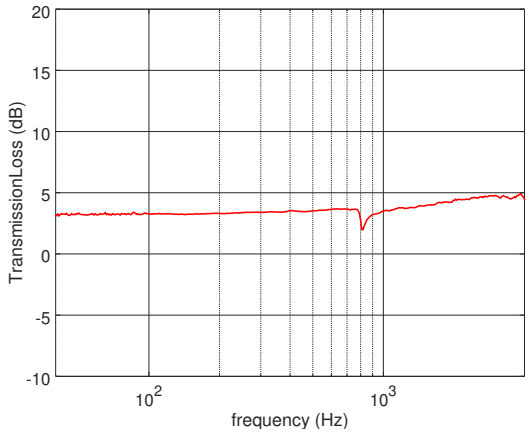

Figure 8. Transmission loss of the sample presented in figure 4 between 40 Hz and 4 kHz.

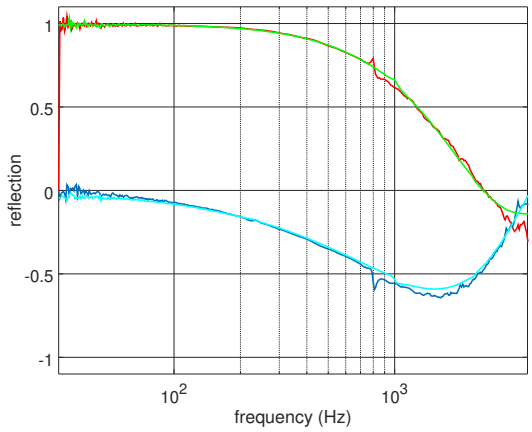

Figure 9. Comparison between the measured reflection coefficient of the sample in the impedance tube and the predicted reflection coefficient from the transfer matrix of the sample measured in the transmission tube in real/imaginary parts.

transfer matrix by the transfer matrix of a wave guide of 100 mm air gap en then by the transfer matrix of a hard wall:

$$\mathbf{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \cos kL & jZ_0 \sin kL \\ \frac{j}{Z_0} \sin kL & \cos kL \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_H} & 1 \end{bmatrix} \tag{9}$$

Then, the reflection coefficient and the absorption coefficient will be determined again by using the expressions (7) and (8). The same assembly will be realized in the sample holder of the impedance tube. Between the sample and the hard wall situates an air gap of 100 mm. Figure 11 presents the resulting reflection coefficient. In red, the real part of the reflection coefficient obtained from the transfer matrix is plotted against the classical tube measurement in green. In dark blue, the imaginary part of the reflection coefficient obtained from the transfer matrix is plotted against the classical tube measurement in light blue. Again, both plots show a remarkable agreement.

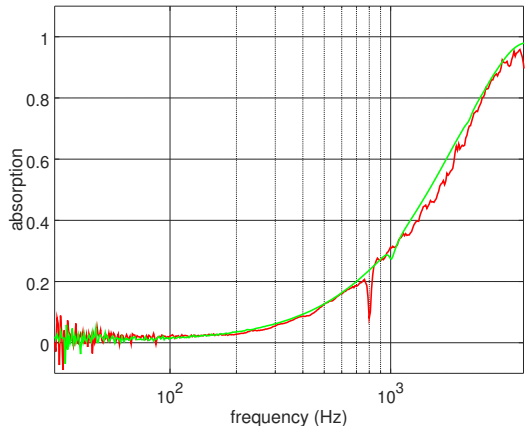

Figure 10. Comparison between the measured absorption coefficient of the sample in the impedance tube and the predicted absorption coefficient from the transfer matrix of the sample measured in the transmission tube.

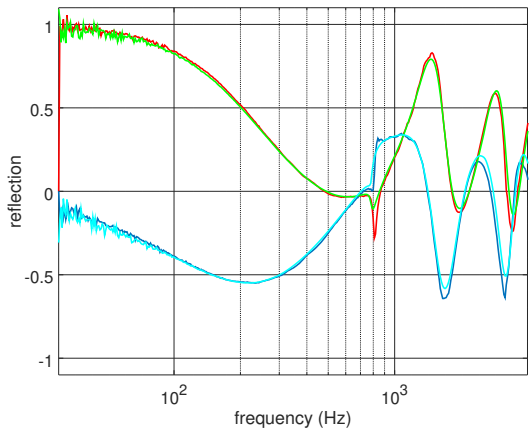

Figure 11. Comparison between the measured reflection coefficient of the sample in the impedance tube and the predicted reflection coefficient from the transfer matrix of the sample measured in the transmission tube in real/imaginary parts.

Figure 12 presents the absorption coefficient. In red, the absorption coefficient obtained from the transfer matrix is plotted against the classical tube measurement in green. Also these plots show a remarkable agreement.

These benchmarks prove that is possible to reconstruct reflection and absorption coefficients in several assemblies by manipulating transfer matrices of samples measured in this newly developed transmission tube.

4 CONCLUSIONS

A new method for determination four pole parameters by impedance measurements has been presented. The sample is represented by a T-equivalent circuit of three impedances. These three impedances will be identified from three impedance measurements with three different closing impedances. The measurements are conducted

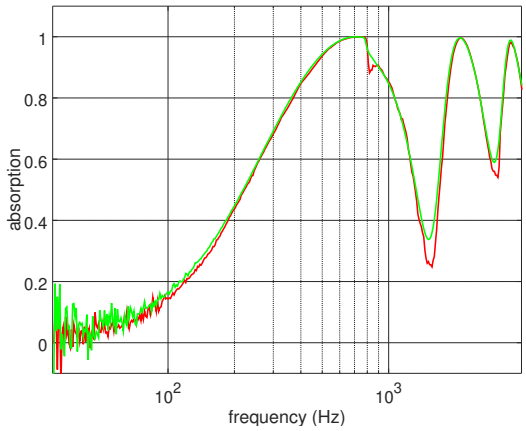

Figure 12. Comparison between the measured absorption coefficient of the sample in the impedance tube and the predicted absorption coefficient from the transfer matrix of the sample measured in the transmission tube.

in such a way that the source impedance does not appear in the measurements. No prior calibrations of closing impedances are required. It has become possible to collect material transfer matrices in a library, from which reflection and absorption coefficients for several assemblies can be predicted.

ACKNOWLEDGEMENTS

The author whishes to thank Mark Brennan and Jan Vandenbroeck from Huntsman Polyurethanes for evaluating the transmission tube.

REFERENCES

[1] Terman, F.E., Electronic and Radio Engineering, McGraw-Hill Kogakusha Ltd., 1955.

[2] ISO, International Standard ISO 10534-2: Determination of sound absorption coefficient and impedance in impedance tubes – Part 2: Transfer-function method, 1998.

[3] R. Boonen; P. Sas; W. Desmet; W. Lauriks; G.Vermeir; Calibration of the two microphone transfer function method by determining the hard wall impedance at shifted reference sections. Proceedings of the International Conference on Noise and Vibration Engineering, Leuven, Belgium, 2008, Vol. 3; pp. 1413 - 1424