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DOA Estimation of Underwater Targets via Improved Monopulse Method with Sonar Array

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Abstract

In this paper, a DOA estimation method using improved monopulse to resolve two targets in a single beam is proposed. We first establish an echo model of two targets with sonar array. Then we derive the maximum likelihood angle estimation based on a numerical optimization solution (Levenberg-Marquardt method). After that an improved monopulse method is proposed to estimate the DOA of the targets according to the maximum likelihood estimation principle. Finally, the simulation results prove that, the improved monopulse method performs very well in many aspects, including small estimation error and less computational complexity.

Keywords: DOA estimation; two targets; sonar array; Monopulse

1 INTRODUCTION

In the field of sonar detection, the most commonly used method for DOA estimation of underwater targets is the conventional beamforming (CBF) algorithm. It has the advantages in small computation, easy implementation, high robustness and wide application range. However, due to the limitation of Rayleigh limit, the method cannot resolve multiple targets located within one beam. Based on array signal processing, the spatial spectrum analysis can effectively achieve the multi-target super-resolution, and the MUSIC algorithm[1] and the ESPRIT algorithm[2] are the most commonly used methods. Based on these two methods, scholars have proposed many improved methods, such as root-MUSC[3], LS-ESPRIT[4], etc.. These algorithms have high accuracy and good robustness, but require that the signal sources are narrow-band and uncorrelated. When the conditions are not met, the performance of these algorithms will drop rapidly and sometimes even fail. In the 1980s, a new class of DOA estimation algorithms called subspace fitting algorithms, including the maximum likelihood (ML) algorithm[5] and the weighted subspace fitting (WSF) algorithm[6], was proposed. The subspace fitting algorithm has excellent estimation performance. But it has large computational complexity and is inconvenient for real-time engineering processing.

In order to reduce the amount of calculation, this paper studies DOA estimation of two targets in a single beam with sonar array. We first establish an echo model of two targets received by the array, and develop the maximum likelihood angle estimation based on Levenberg-Marquardt(LM) method. Since it has a slow convergence rate, then we propose a dual-monopulse method to estimate the DOA of the targets according to the maximum likelihood estimation principle. The method is expected to gain advantages in DOA estimation accuracy and moderate requirement on the system complexity as it is based on the dual-monopulse system. The core idea the proposed method is that, two monopulse systems are formed and each of them points at one of the targets, and iterations are then carried out to reduce the DOA estimation errors. In the Section 5, the performance of this method is demonstrated via Monte Carlo simulations.

2 TWO-TARGET ECHO MODEL OF SONAR ARRAY

Consider the uniform linear array consisting of N elements, the single snapshot received by the sonar array in the case of two targets is

$$\boldsymbol{x} = A_1\boldsymbol{s}(u_1) + A_2\boldsymbol{s}(u_2) + \boldsymbol{n} \tag{1}$$

where

$$\boldsymbol{s}(u_i) = [e^{-j\frac{(N-1)d\pi}{\lambda}u_i}, e^{-j\frac{(N-3)d\pi}{\lambda}u_i}, \dots, e^{j\frac{(N-3)d\pi}{\lambda}u_i}, e^{j\frac{(N-1)d\pi}{\lambda}u_i}]^{\mathrm{T}}$$

is the array steering vector,$u_i = \sin\theta_i$, $i = 1,2$, θ_1 and θ_2 are the angles of the two targets with respect to the normal of the array. λ is the wavelength, d is the distance between adjacent array elements. $A_1, A_2 \in \mathbb{C}^{1\times 1}$ are the complex amplitudes of the two target echoes, respectively. $\boldsymbol{n} \in \mathbb{C}^{N\times 1}$ represents the thermal noise received by array, which is independently and identically distributed white Gaussian noise with zero mean and a variance of σ^2 .

3 MAXIMUM LIKELIHOOD DOA ESTIMATION

According to the distribution of the noise in (1), one can conclude in the following Gaussian density function of the array measurement with respect to A_1, A_2, u_1, u_2,

$$p(\boldsymbol{x}; u, \boldsymbol{a}) = [1/(\pi\sigma^2)]^N \exp\{-[1/\sigma^2](\boldsymbol{x} - \boldsymbol{S}\boldsymbol{a})^{\mathrm{H}}(\boldsymbol{x} - \boldsymbol{S}\boldsymbol{a})\} \tag{2}$$

where $\boldsymbol{a} = [\ A_1 \quad A_2\ ], \boldsymbol{S} = [\ \boldsymbol{s}(u_1) \quad \boldsymbol{s}(u_2)\ ]$. The maximum likelihood estimates can then be obtained by maximizing the above formula, which equals to minimizing $(\boldsymbol{x} - \boldsymbol{S}\boldsymbol{a})^{\mathrm{H}}(\boldsymbol{x} - \boldsymbol{S}\boldsymbol{a})$. For notational convenience, we normalize $\boldsymbol{x}$ as $\frac{1}{\sqrt{N}}\boldsymbol{x} = \frac{1}{\sqrt{N}}(A_1\boldsymbol{s}(u_1) + A_2\boldsymbol{s}(u_2) + \boldsymbol{n}) \overset{\Delta}{=} A_1\boldsymbol{s}_1 + A_2\boldsymbol{s}_2 + \boldsymbol{n}'$, in which $[\ \boldsymbol{s}_1 \quad \boldsymbol{s}_2\ ] = \left[\ \frac{1}{\sqrt{N}}\boldsymbol{s}(u_1) \quad \frac{1}{\sqrt{N}}\boldsymbol{s}(u_2)\ \right]$ $\overset{\Delta}{=} \bar{\boldsymbol{S}}$ and $\boldsymbol{n}' = \frac{1}{\sqrt{N}}\boldsymbol{n}$. Denote $Q = (\boldsymbol{x} - \bar{\boldsymbol{S}}\boldsymbol{a})^{\mathrm{H}}(\boldsymbol{x} - \bar{\boldsymbol{S}}\boldsymbol{a})$, then A_1, A_2, u_1, u_2 can be estimated by setting the partial differentiations of Q with respect to them to 0, i.e.,

$$\partial Q/\partial A_1 = -\boldsymbol{s}_1^{\mathrm{H}}\boldsymbol{x} + A_1 + A_2\boldsymbol{s}_1^{\mathrm{H}}\boldsymbol{s}_2 = 0 \tag{3}$$

$$\partial Q/\partial A_2 = -\boldsymbol{s}_2^{\mathrm{H}}\boldsymbol{x} + A_1\boldsymbol{s}_2^{\mathrm{H}}\boldsymbol{s}_1 + A_2 = 0 \tag{4}$$

$$\partial Q/\partial u_1 = 2\mathrm{Re}[-A_1^{\mathrm{H}}\boldsymbol{s}_1'^{\mathrm{H}}\boldsymbol{x} + A_1^{\mathrm{H}}A_2\boldsymbol{s}_1'^{\mathrm{H}}\boldsymbol{s}_2] = 0 \tag{5}$$

$$\partial Q/\partial u_2 = 2\mathrm{Re}[-A_2^{\mathrm{H}}\boldsymbol{s}_2'^{\mathrm{H}}\boldsymbol{x} + A_2^{\mathrm{H}}A_1\boldsymbol{s}_2'^{\mathrm{H}}\boldsymbol{s}_1] = 0 \tag{6}$$

which conclude in the following estimates according to (3) and (4),

$$\hat{A}_1 = \frac{\boldsymbol{s}_1^{\mathrm{H}}\boldsymbol{x} - \boldsymbol{s}_2^{\mathrm{H}}\boldsymbol{x}\boldsymbol{s}_1^{\mathrm{H}}\boldsymbol{s}_2}{1 - ||\boldsymbol{s}_1^{\mathrm{H}}\boldsymbol{s}_2||^2} \tag{7}$$

$$\hat{A}_2 = \frac{\boldsymbol{s}_2^{\mathrm{H}}\boldsymbol{x} - \boldsymbol{s}_1^{\mathrm{H}}\boldsymbol{x}\boldsymbol{s}_2^{\mathrm{H}}\boldsymbol{s}_1}{1 - ||\boldsymbol{s}_2^{\mathrm{H}}\boldsymbol{s}_1||^2} \tag{8}$$

However, it is difficult to obtain the expressions of $\hat{u}_1, \hat{u}_2$ straightforwardly based on these equations[7]. Let $\boldsymbol{\omega} = (u_1, u_2)$, and consider $Q = \|\boldsymbol{x} - \bar{\boldsymbol{S}}\boldsymbol{a}\|^2$. $minQ(\omega)$ becomes a least square problem. So we make use of a numerical optimization solution to obtain the value of $\hat{u}_1, \hat{u}_2$.

The Levenberg-Marquardt method[8] is used to solve this problem. For the least square problem $minQ(\boldsymbol{\omega}) = \frac{1}{2}\|\boldsymbol{F}(\boldsymbol{\omega})\|^2$, the LM method defines that: $\boldsymbol{g}(\boldsymbol{\omega}) = \nabla Q(\boldsymbol{\omega}) = \boldsymbol{J}(\boldsymbol{\omega})^T\boldsymbol{F}(\boldsymbol{\omega})$. Where $\boldsymbol{J}(\boldsymbol{\omega}) = \boldsymbol{F}'(\boldsymbol{\omega}) = [\nabla\boldsymbol{F}_1(\boldsymbol{\omega}), \dots, \nabla\boldsymbol{F}_m(\boldsymbol{\omega})]^T$. And the search direction $\boldsymbol{d}_k$ is defined as $\boldsymbol{d}_k = -(\boldsymbol{J}_k{}^T\boldsymbol{J}_k + \mu_k\boldsymbol{I})^{-1}\boldsymbol{J}_k{}^T\boldsymbol{F}_k$, where $\boldsymbol{J}_k = \boldsymbol{J}(\boldsymbol{\omega}_k)$, $\boldsymbol{F}_k = \boldsymbol{F}(\boldsymbol{\omega}_k)$,and $\mu_k > 0$. According to the iteration rule $\boldsymbol{\omega}_{k+1} = \boldsymbol{\omega}_k + \alpha_k\boldsymbol{d}_k$, then we derive the iteration sequence $\{\boldsymbol{\omega}_k\}$. The last value is the minimum point of optimization problem. The step factor α_k is determined by the Armijo search method[9].

We propose LM-based maximum likelihood angle estimation(LM-ML)[10].The process of the LM-ML is showed in the following:

Step one: choose the initialization value of $\boldsymbol{\omega}$ as $\boldsymbol{\omega}_0$, set the permissible error $0 \leq \varepsilon \leq 1$, the maximum number of iterations K and $\mu_0 > 0$, let $k = 0$;

Step two: calculate $\boldsymbol{J}_k$ and get $\boldsymbol{g}(\boldsymbol{\omega}_k) = \boldsymbol{J}_k{}^T \boldsymbol{F}_k$, when $\|\boldsymbol{g}(\boldsymbol{\omega}_k)\| < \varepsilon$ or $k \geq K$, stop and output the $\boldsymbol{\omega}_k$ as the optimal solution;

Step three: calculate $\boldsymbol{d}_k$, and get $\boldsymbol{d}_k = -(\boldsymbol{J}_k{}^T \boldsymbol{J}_k + \mu_k \boldsymbol{I})^{-1} \boldsymbol{J}_k{}^T \boldsymbol{F}_k$;

Step four: obtain the step factor α_k by the Armijo search method;

Step five: set $\boldsymbol{\omega}_{k+1} = \boldsymbol{\omega}_k + \alpha_k \boldsymbol{d}_k$, $k = k+1$. Update μ_k and substitute $\boldsymbol{\omega}_{k+1}$ into Eq.(7) and (8) to update A_1,A_2, then go to step two.

After several iterations, we obtain the minimum point $\boldsymbol{\omega}_k = (u_{1k}, u_{2k})$, which is the DOA estimation of the LM-ML.

4 DUAL-MONOPULSE BASED DOA ESTIMATION METHOD

Since the LM-ML has a slow convergence rate and a long computation time, we propose a dual-monopulse method for angle estimation following the guidelines of the above derivations. In the method, two monopulse systems are constructed and the angles of the two unresolved targets are estimated via iterations[11]. It can be concluded from Eq. (7) and (8) that,

$$\hat{A}_1 = \frac{\boldsymbol{s}_1^{\mathrm{H}} \boldsymbol{x} - \boldsymbol{s}_2^{\mathrm{H}} \boldsymbol{x} \boldsymbol{s}_1^{\mathrm{H}} \boldsymbol{s}_2}{1 - ||\boldsymbol{s}_1^{\mathrm{H}} \boldsymbol{s}_2||^2} = \frac{(\boldsymbol{P}_{\mathrm{s}_2 \perp} \boldsymbol{s}_1)^{\mathrm{H}} \boldsymbol{x}}{\|\boldsymbol{P}_{\mathrm{s}_2 \perp} \boldsymbol{s}_1\|^2} = \boldsymbol{w}_{\Sigma_1} \boldsymbol{x} \tag{9}$$

$$\hat{A}_2 = \frac{\boldsymbol{s}_2^{\mathrm{H}} \boldsymbol{x} - \boldsymbol{s}_1^{\mathrm{H}} \boldsymbol{x} \boldsymbol{s}_2^{\mathrm{H}} \boldsymbol{s}_1}{1 - ||\boldsymbol{s}_2^{\mathrm{H}} \boldsymbol{s}_1||^2} = \frac{(\boldsymbol{P}_{\mathrm{s}_1 \perp} \boldsymbol{s}_2)^{\mathrm{H}} \boldsymbol{x}}{\|\boldsymbol{P}_{\mathrm{s}_1 \perp} \boldsymbol{s}_2\|^2} = \boldsymbol{w}_{\Sigma_2} \boldsymbol{x} \tag{10}$$

where $\boldsymbol{P}_{\mathrm{s}_1 \perp} = \boldsymbol{I} - \boldsymbol{s}_1 \boldsymbol{s}_1{}^{\mathrm{H}}$ and $\boldsymbol{P}_{\mathrm{s}_2 \perp} = \boldsymbol{I} - \boldsymbol{s}_2 \boldsymbol{s}_2{}^{\mathrm{H}}$ are the orthogonal projection matrices with respect to $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$, respectively, which satisfy

$$\|\boldsymbol{P}_{\mathrm{s}_1 \perp} \boldsymbol{s}_2\|^2 = \|\boldsymbol{P}_{\mathrm{s}_2 \perp} \boldsymbol{s}_1\|^2 = 1 - |\boldsymbol{s}_2{}^{\mathrm{H}} \boldsymbol{s}_1|^2 \tag{11}$$

Moreover,

$$\boldsymbol{w}_{\Sigma_1} = \frac{(\boldsymbol{P}_{\mathrm{s}_2 \perp} \boldsymbol{s}_1)^{\mathrm{H}}}{\|\boldsymbol{P}_{\mathrm{s}_2 \perp} \boldsymbol{s}_1\|^2}, \quad \boldsymbol{w}_{\Sigma_2} = \frac{(\boldsymbol{P}_{\mathrm{s}_1 \perp} \boldsymbol{s}_2)^{\mathrm{H}}}{\|\boldsymbol{P}_{\mathrm{s}_1 \perp} \boldsymbol{s}_2\|^2} \tag{12}$$

It can be easily obtained that $\boldsymbol{w}_{\Sigma_1}{}^{\mathrm{H}} \boldsymbol{s}_1 = 1, \boldsymbol{w}_{\Sigma_1}{}^{\mathrm{H}} \boldsymbol{s}_2 = 0$ and $\boldsymbol{w}_{\Sigma_2}{}^{\mathrm{H}} \boldsymbol{s}_1 = 0, \boldsymbol{w}_{\Sigma_2}{}^{\mathrm{H}} \boldsymbol{s}_2 = 1$.

Thus $\hat{A}_1$,$\hat{A}_2$ can be deemed as the sum beams of the monopulse system I and II, respectively, which are denoted as Σ_1, Σ_2 in the rest of the paper. $\boldsymbol{w}_{\Sigma_1}, \boldsymbol{w}_{\Sigma_2}$ are the weight vectors of Σ_1, Σ_2 . The formulation of the sum beams helps to enhance the target signal of interest and suppress the signal of the other target at the same time.

According Eq.(5) and (6), one can obtain that,

$$\Delta_1 = \boldsymbol{s}_1'^{\mathrm{H}} \boldsymbol{x} - A_2 \boldsymbol{s}_1'^{\mathrm{H}} \boldsymbol{s}_2 = \boldsymbol{s}_1'^{\mathrm{H}} (\boldsymbol{x} - A_2 \boldsymbol{s}_2) \tag{13}$$

$$\Delta_2 = \boldsymbol{s}_2'^{\mathrm{H}} \boldsymbol{x} - A_1 \boldsymbol{s}_2'^{\mathrm{H}} \boldsymbol{s}_1 = \boldsymbol{s}_2'^{\mathrm{H}} (\boldsymbol{x} - A_1 \boldsymbol{s}_1) \tag{14}$$

Substituting Eq.(9) and (10) into Eq.(13) and (14) respectively yields

$$\Delta_1 = \boldsymbol{s}_1'^{\mathrm{H}} \left(\boldsymbol{x} - \frac{(\boldsymbol{P}_{\mathrm{s}_1\perp}\boldsymbol{s}_2)^{\mathrm{H}}\boldsymbol{x}}{\|\boldsymbol{P}_{\mathrm{s}_1\perp}\boldsymbol{s}_2\|^2}\boldsymbol{s}_2 \right) = \boldsymbol{w}_{\Delta_1}\boldsymbol{x} \tag{15}$$

$$\Delta_2 = \boldsymbol{s}_2'^{\mathrm{H}} \left(\boldsymbol{x} - \frac{(\boldsymbol{P}_{\mathrm{s}_2\perp}\boldsymbol{s}_1)^{\mathrm{H}}\boldsymbol{x}}{\|\boldsymbol{P}_{\mathrm{s}_2\perp}\boldsymbol{s}_1\|^2}\boldsymbol{s}_1 \right) = \boldsymbol{w}_{\Delta_2}\boldsymbol{x} \tag{16}$$

where

$$\boldsymbol{w}_{\Delta_1} = \boldsymbol{s}_1'^{\mathrm{H}} \left(\boldsymbol{I} - \frac{\boldsymbol{s}_2(\boldsymbol{P}_{\mathrm{s}_1\perp}\boldsymbol{s}_2)^{\mathrm{H}}}{\|\boldsymbol{P}_{\mathrm{s}_1\perp}\boldsymbol{s}_2\|^2} \right) = \boldsymbol{s}_1'^{\mathrm{H}} - \frac{\boldsymbol{s}'_1{}^{\mathrm{H}}\boldsymbol{s}_2}{\|\boldsymbol{P}_{\mathrm{s}_1\perp}\boldsymbol{s}_2\|^2}(\boldsymbol{P}_{\mathrm{s}_1\perp}\boldsymbol{s}_2)^{\mathrm{H}} \tag{17}$$

$$\boldsymbol{w}_{\Delta_2} = \boldsymbol{s}_2'^{\mathrm{H}} \left(\boldsymbol{I} - \frac{\boldsymbol{s}_1(\boldsymbol{P}_{\mathrm{s}_2\perp}\boldsymbol{s}_1)^{\mathrm{H}}}{\|\boldsymbol{P}_{\mathrm{s}_2\perp}\boldsymbol{s}_1\|^2} \right) = \boldsymbol{s}_2'^{\mathrm{H}} - \frac{\boldsymbol{s}'_2{}^{\mathrm{H}}\boldsymbol{s}_1}{\|\boldsymbol{P}_{\mathrm{s}_2\perp}\boldsymbol{s}_1\|^2}(\boldsymbol{P}_{\mathrm{s}_2\perp}\boldsymbol{s}_1)^{\mathrm{H}} \tag{18}$$

It can be easily concluded that $\boldsymbol{w}_{\Delta_1}{}^{\mathrm{H}}\boldsymbol{s}_1 = 0$, $\boldsymbol{w}_{\Delta_1}{}^{\mathrm{H}}\boldsymbol{s}_2 = 0$, $\boldsymbol{w}_{\Delta_2}{}^{\mathrm{H}}\boldsymbol{s}_2 = 0$, $\boldsymbol{w}_{\Delta_2}{}^{\mathrm{H}}\boldsymbol{s}_1 = 0$

Thus Δ_1, Δ_2 can be deemed as the difference beams of monopulse system I and II respectively, and $\boldsymbol{w}_{\Delta_1}, \boldsymbol{w}_{\Delta_2}$ are the weight vectors of Δ_1, Δ_2 . The difference beams of the two systems help to suppress the jamming signal. The iterative formulas of monopulse system I and II are

$$u_{1t} = u_{10} + k^{-1}\left(\frac{\Delta_1}{\Sigma_1}\right), \quad u_{2t} = u_{20} + k^{-1}\left(\frac{\Delta_2}{\Sigma_2}\right) \tag{19}$$

where k is the slope of the monopulse response curve (MRC) and its value is derived according to literature [12] when the angle estimation error is very small. The formula for setting the value of MRC is

$$k = [\boldsymbol{s}''(u)^{\mathrm{H}}\boldsymbol{s}(u) + \boldsymbol{s}(u)^{\mathrm{H}}\boldsymbol{s}''(u)]/\boldsymbol{s}^{\mathrm{H}}(u)\boldsymbol{s}(u) = 8\sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}}\left[\pi n(\frac{d}{\lambda})\right]^2/N \tag{20}$$

According to the above derivations and Eq.(19), the dual-monopulse system for angle estimation can be constructed as that shown in Fig.1.

According to Eq.(19), this system estimates the directions of the two unresolved targets based on the current beam pointing orientations. Then the beam orientation is updated according to the refreshed DOA estimates. The iteration is terminated when a predefined convergence criterion is satisfied. In this paper, the convergence criterion is set when the difference between the new DOA estimates and the beam pointing orientations is less than a certain iterative threshold in a single cycle. In Fig.1, u_{10}, u_{20} are the sine values of the current beam pointing angles and u_{1t}, u_{2t} are the sine values of the DOA estimates of the two targets, γ is the iterative threshold.

5 SIMULATION

Consider a vertical uniform linear array whose inter-element distance equals half-wavelength. The number of array elements is N=100, then the beamwidth is $\theta_{\mathrm{3dB}} = 0.886\frac{\lambda}{Nd} \approx 1.016^{\circ}$. The iterative threshold $\gamma = 10^{-8}$. SNR is defined as the ratio of the direct signal power to the noise variance of each array element, i.e., $\mathrm{SNR_i} = \frac{|A_i|^2}{\sigma^2}$. The root mean square error (RMSE) defined as $\mathrm{RMSE_i} = \sqrt{E[(\hat{\theta}_i - \theta_i)^2]}$ is introduced for DOA estimation

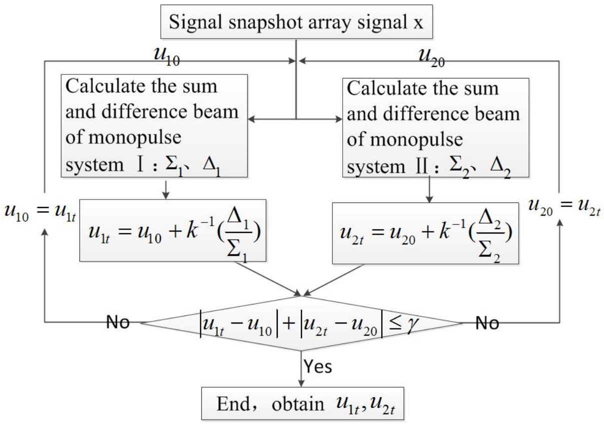

Figure 1. The process of the dual-monopulse based DOA estimation method

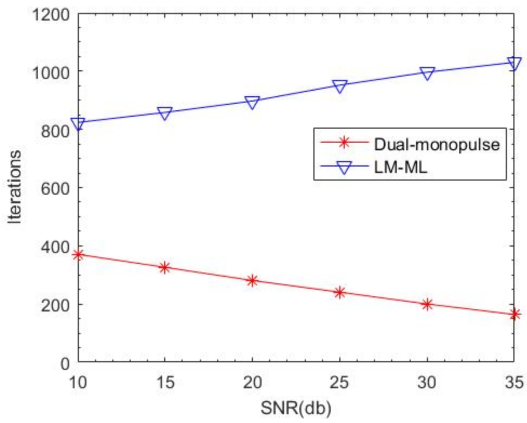

Figure 2. Iterations of dual-monopulse method and LM-ML for different SNR

precision evaluation. In order to enhance the generality of the conclusions, the target azimuth, the included angle and the RMSE are normalized with respect to the beam width, the unitary azimuth of the target is $\bar{\theta}_i = \theta_i/\theta_{3\mathrm{dB}}$, the unitary inter-target angle is $\overline{\Delta\theta} = |\theta_1 - \theta_2|/\theta_{3\mathrm{dB}}$, the unitary root mean square error (URMSE) is URMSE $=$RMSE$/\theta_{3\mathrm{dB}}$.

The influences of the SNR and the azimuth on the performance of the proposed method are studied in the following simulations. When the influence of a certain factor on the performance is studied, the other parameters are fixed on typical values. In each situation, 1000 Monte Carlo simulations are carried out to obtain the statistical performances.

5.1 Relationship between the Number of Iterations and the SNR

We compare the number of iterations of the dual-monopulse method with that of LM-ML algorithm. In this experiment, we assume the radian of the two target azimuths to be $\theta_1 = -0.007, \theta_2 = 0.008$, respectively, and the corresponding unitary azimuths are $\bar{\theta}_1 = -0.41, \bar{\theta}_2 = 0.47$. Fig.2 shows the relationship curve between the iterations of two methods and the SNR with $\mathrm{SNR} = \mathrm{SNR}_1 = \mathrm{SNR}_2$.

Fig.2 shows that the number of iterations of the dual-monopulse method is less than that of the LM-ML method and it decreases with increasing SNR.

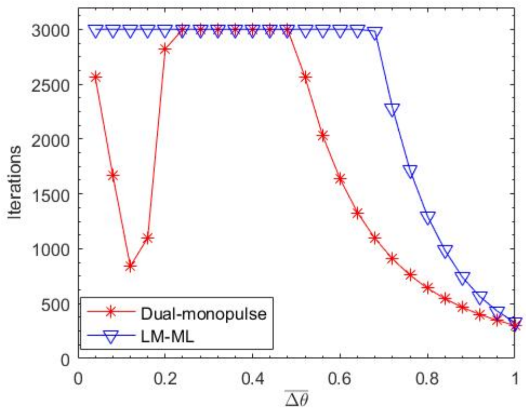

Figure 3. Iterations of dual-monopulse method and LM-ML for different unitary inter-target angle

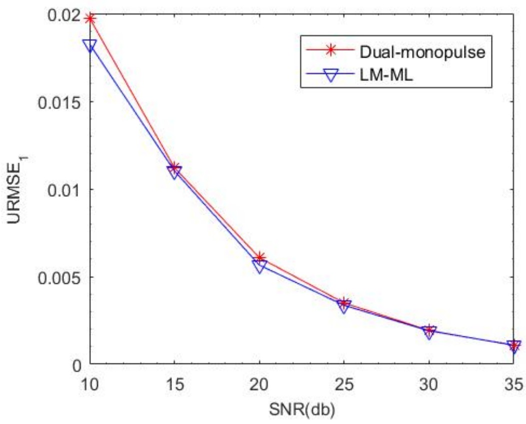

Figure 4. URMSE1 derived from two methods for different SNR

5.2 Relationship between the Number of Iterations and the Inter-Target Angle

In this experiment, we assume the SNR of the two signals as $SNR_1 = SNR_2 = 30dB$, Fig.3 shows the relationship curve between the iterations of two methods and the unitary inter-target angle $\overline{\Delta\theta}$.

Fig.3 shows that the number of iterations of the dual-monopulse method decreases significantly when the inter-target angle increases. And when the inter-target angle is smaller than 0.7 times the beamwidth, the LM-ML method cannot reach the convergence condition after 3000 iterations. From the above results one can conclude that, compared with the LM-ML method, the dual-monopulse method converges fast and has low computational complexity. So it can estimate DOA of the targets more quickly and efficiently.

5.3 Relationship between the DOA Estimation Accuracy and the SNR

We compare the DOA estimation accuracy of the dual-monopulse method with that of the LM-ML method. In this experiment, we assume the radian of the two target azimuths to be $\theta_1 = -0.007, \theta_2 = 0.008$, respectively. Fig.4 shows the relationship curve between the URMSE1 of the first target of two methods and the SNR with $SNR = SNR_1 = SNR_2$.

Fig.4 shows that the RMSE decreases and the angle estimation accuracy improves with increasing SNR. The

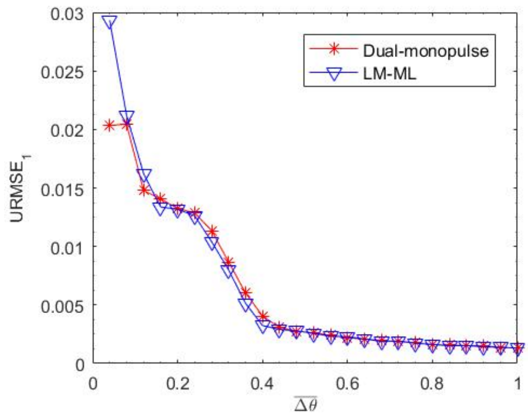

Figure 5. URMSE1 derived from two methods for different unitary inter-target angle

angle estimation error of the dual-monopulse method is close to that of the LM-ML method.

5.4 Relationship between the DOA Estimation Accuracy and Inter-Target Angle

In this experiment, we assume the SNR of the two signals as $SNR_1 = SNR_2 = 30$dB, Fig.5 shows the relationship curve between the URMSE1 of two methods and the unitary inter-target angle $\overline{\Delta\theta}$.

Fig.5 shows that the URMSE increases when the inter-target angle decreases. The phenomenon accords with the experience of most sonar users that, when the inter-targer angle decreases, the two targets get closer and become more difficult to be resolved, and the precision of DOA estimation decreases accordingly. Moreover, the angle estimation error of the dual-monopulse method is close to that of the LM-ML method, which approximates CRLB[10].

6 CONCLUSIONS

This paper studied the problem of resolving two unresolved targets with sonar array, and a dual-monopulse based DOA estimation method is proposed. The simulation results indicate that, this method works effectively, and it gains advantages in small estimation error. Moreover, compared with the maximum likelihood angle estimation algorithm, the proposed method is faster and less computationally intensive.

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