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Resonance modes for exterior vibro-acoustic problems, application to a dielectric elastomer loudspeaker

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Abstract

Modal expansion techniques are efficient order reduction tools for bounded problems, but their use for exterior problems is less investigated. In the present article a simple vibrating structure embedded in a heavy exterior acoustic fluid is studied. Using a coupled finite element model, with perfectly matched layers (PMLs) to insure the Sommerfeld boundary condition, we show that the coupled fluid-structure modes can be obtained by solving a linear eigenvalue problem. The full basis comprising the fluid loaded structural modes and numerical PML modes is then used to expand the structural displacement and the radiated acoustic pressure, providing good accuracy. It is shown that the displacement potential appears to be a more natural variable than the acoustic pressure to describe the fluid state. This modal approach is then applied in a real case where a dielectric elastomer loudspeaker is studied, and validated by measurements.

Keywords: Dielectric elastomer, Vibro-acoustic coupling, Perfectly Matched Layers

1 INTRODUCTION

Modal approaches are common model order reduction tools for bounded problems, and are broadly used for all types of dynamical systems. When part of the problem is open, like for exterior acoustic radiation, modal approaches are less investigated. Hein et al. [4] used Perfectly Matched Layers (PMLs) to compute the resonances of open acoustical systems. They were able to retrieve the trapped modes (non-damped modes) that may exist in open systems, but also found so-called leaky modes (modes which are damped because of radiation). However, the obtained modal basis is not used to reconstruct the radiated acoustic field.

Modal superposition methods for the calculation of exterior radiation have been used for example in [8], where resonance modes are used to solve an exterior acoustic problem consisting of an open cavity in an exterior fluid. A FEM/BEM coupled model has been reduced using resonance modes to compute the radiated sound pressure by Peters et al. [11]. In the field of optics and electrodynamics, the use of resonance modes to describe open systems has been studied in a larger extent [7], mainly for laser applications. The modal approach is presented as an efficient method, especially for its easy physical interpretation compared to direct frequency domain approaches.

To the author's knowledge, few publications present acoustic radiation calculations of vibro-acoustic systems using model order reduction techniques based on resonance modes. In order to introduce the proposed method, a simple vibro-acoustic system is considered, consisting in a tensioned membrane embedded in a infinite plane, and immersed in an acoustic fluid. A fully coupled finite element model is set-up, using PMLs to implement the radiation boundary condition. A linear eigenvalue problem is obtained, and solved using a standard eigenvalue solver. The resulting modal basis is used to reconstruct the radiated acoustic field, and compared to semi-analytical results.

To demonstrate the applicability of the method, a real system is then analysed using this modal approach. We consider an dielectric elastomer membrane, which is an active material made of a thin silicone film (200 µm) sandwiched between two compliant electrodes. This material is capable of large deformations (up to 100%) when a high voltage is applied between the electrodes [10]. In the studied configuration this membrane is inflated over a closed cavity, and radiates sound when an audio signal is applied [1, 5].

2 MODAL EXPANSION FOR RADIATION OF FLUID LOADED STRUCTURES

2.1 Description of the studied system

In order to investigate modal methods for exterior radiation of resonant structures, a simple axi-symmetric system is considered, consisting of a flat tensioned membrane embedded in an infinite baffle (see Fig. 1). The speed of sound is denoted c_F, the fluid density ρ_F, the speed of transversal waves in the membrane c_S, the membrane density ρ_S, and the membrane thickness h.

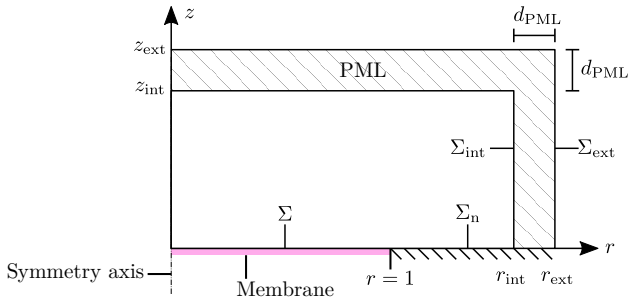

Figure 1. Schematics of the simplified system: a membrane is embedded in an infinite baffle.

The non-dimensional equations governing the dynamics of the system are:

$$\frac{\partial \xi}{\partial t^2} - \frac{\partial \xi}{\partial r^2} + P = P_{\mathrm{force}} \text{ on } \Sigma\,, \qquad \frac{\partial \xi}{\partial r}(r=0) = 0\,, \qquad \xi(r=1) = 0\,, \tag{1}$$

$$\frac{\partial P}{\partial t^2} - \alpha \Delta P = 0 \text{ in } \Omega\,, \qquad \boldsymbol{\nabla} P \cdot \boldsymbol{n} = \beta \frac{\partial \xi}{\partial t^2} \text{ on } \Sigma\,, \qquad \boldsymbol{\nabla} P \cdot \boldsymbol{n} = 0 \text{ on } \Sigma_{\mathrm{n}}\,, \tag{2}$$

where Ω is the whole acoustical domain, ξ the membrane displacement along z, P the acoustic pressure, $\alpha = \rho_F/\rho_S$, and $\beta = c_F^2/c_S^2$. The time is scaled by h/c_S, all lengths by h, and the pressures by $\rho_S c_S^2$. The $P_{\mathrm{force}}(r,t)$ is an external pressure applied to the membrane. In the following the time dependence of all fields is $\mathrm{e}^{\mathrm{i}\omega t}$.

In the rest of the present study, the fluid state is described by the displacement potential $Q = P/\omega^2$ instead of the pressure P. This choice will be justified later. The governing equations using Q instead of P read [9]:

$$-\omega^2 \xi - \frac{\partial \xi}{\partial r^2} + \omega^2 Q = P_{\mathrm{force}} \text{ on } \Sigma\,, \qquad \frac{\partial \xi}{\partial r}(r=0) = 0\,, \qquad \xi(r=1) = 0\,, \tag{3}$$

$$\omega^2 Q + \alpha \Delta Q = 0 \text{ in } \Omega\,, \qquad \boldsymbol{\nabla} Q \cdot \boldsymbol{n} = -\beta \xi \text{ on } \Sigma\,, \qquad \boldsymbol{\nabla} Q \cdot \boldsymbol{n} = 0 \text{ on } \Sigma_{\mathrm{n}}\,. \tag{4}$$

A Sommerfeld radiation boundary condition must be added on Σ_{int}. This condition is implemented using Perfectly Matched Layers (PMLs), by introducing the following complex change of variables:

$$\tilde{r} = \begin{cases} r - \mathrm{i}\int_{r_{\mathrm{int}}}^{r} \sigma_r(s)\mathrm{d}s & \text{for } r > r_{\mathrm{int}} \\ r & \text{otherwise} \end{cases}, \qquad \tilde{z} = \begin{cases} z - \mathrm{i}\int_{z_{\mathrm{int}}}^{z} \sigma_z(s)\mathrm{d}s & \text{for } z > z_{\mathrm{int}} \\ z & \text{otherwise} \end{cases},$$

where $\sigma_r(r) = \sigma_0(r - r_{\mathrm{int}})^2$ and $\sigma_z(z) = \sigma_0(z - z_{\mathrm{int}})^2$ are the attenuation functions, and σ_0 is an attenuation parameter that should be adjusted. The changes of variables imply the following changes of the partial derivatives:

$$\frac{\partial}{\partial \tilde{r}} = \frac{1}{1 - \mathrm{i}\sigma_r(r)} \frac{\partial}{\partial r} = \frac{1}{\gamma_r(r)} \frac{\partial}{\partial r}\,, \qquad \frac{\partial}{\partial \tilde{z}} = \frac{1}{1 - \mathrm{i}\sigma_z(z)} \frac{\partial}{\partial z} = \frac{1}{\gamma_z(z)} \frac{\partial}{\partial z}\,,$$

where the γ functions are defined as:

$$\gamma_r(r) = \begin{cases} 1 - \mathrm{i}\sigma_r(r) & \text{for } r > r_{\mathrm{int}} \\ 1 & \text{otherwise} \end{cases}, \qquad \gamma_z(z) = \begin{cases} 1 - \mathrm{i}\sigma_z(z) & \text{for } z > z_{\mathrm{int}} \\ 1 & \text{otherwise} \end{cases}.$$

The weak form of the structural and acoustic equations is obtained by multiplying Eq. (4) and Eq. (3) by test functions $\hat{Q}$ and $\hat{\xi}$, integrating by parts, and using the change of variables $\tilde{r}$ and $\tilde{z}$:

$$-\omega^2 \int_\Omega Q\hat{Q}\gamma_r\gamma_z\tilde{r}\mathrm{d}r\mathrm{d}z + \alpha \int_\Omega \left(\frac{\gamma_z}{\gamma_r}\frac{\partial Q}{\partial r}\frac{\partial \hat{Q}}{\partial r} + \frac{\gamma_r}{\gamma_z}\frac{\partial Q}{\partial z}\frac{\partial \hat{Q}}{\partial z}\right)\tilde{r}\mathrm{d}r\mathrm{d}z + \alpha\beta \int_\Sigma \xi\hat{Q}\tilde{r}\mathrm{d}r = 0\,, \tag{5}$$

$$-\omega^2 \int_\Sigma \xi\hat{\xi}r\mathrm{d}r + \int_\Sigma \frac{\partial \xi}{\partial r}\frac{\partial \hat{\xi}}{\partial r}r\mathrm{d}r + \omega^2 \int_\Sigma Q\hat{\xi}r\mathrm{d}r = \int_\Sigma P_{\text{force}}\hat{\xi}r\mathrm{d}r\,. \tag{6}$$

2.2 Numerical solving

The weak form obtained in previous section is discretized using finite elements. This is performed in the FreeFem++ software [3], which provides interpolation routines that allow to compute integrals on a common line of functions defined on different meshes. This allows us to build the total mass and stiffness matrices:

$$\left(-\omega^2\begin{bmatrix} M_F & 0 \\ -R^\dagger & M_S\end{bmatrix} + \begin{bmatrix}\alpha K_F & \alpha\beta R \\ 0 & K_S\end{bmatrix}\right)\begin{bmatrix} Q \\ \xi\end{bmatrix} = \begin{bmatrix} 0 \\ F_S\end{bmatrix} \qquad \Leftrightarrow \qquad (-\omega^2 M + K)X = F\,. \tag{7}$$

where M_F and K_F are the mass and stiffness matrices of the fluid, M_S and K_S those of the structure, R the coupling matrix, $R^\dagger$ the hermitian transpose of R, and F_S the force vector on the structure. There is no acoustic source.

The matrix system Eq. (7) can be solved by various methods. The first option is to invert the system for all frequencies of interest. This will be referred to as the **FEM** method:

$$X(\omega) = \begin{bmatrix} Q(\omega) \\ \xi(\omega)\end{bmatrix} = (-\omega^2 M + K)^{-1}F(\omega)\,. \tag{8}$$

Modal methods can also be used, even if the problem involves exterior radiation. Indeed, as the reader may have noticed, the mass and stiffness matrices are frequency independent so a linear eigenvalue problem is obtained. This happens because frequency independent PMLs have been chosen, and because no viscous losses are accounted for. Structural losses are modeled a posteriori using Rayleigh damping. The mass and stiffness matrices are not symmetric, so left and right modeshapes are computed: $(-\omega_n^2 M + K)\psi_n^R = 0$ and $\psi_n^L(-\omega_n^2 M + K) = 0$. If all eigenvalues are of order one, which is the case here, the following bi-orthogonality relations hold: $\psi^L M \psi^R = \mathrm{diag}(\mu_n)$ and $\psi^L K \psi^R = \mathrm{diag}(\kappa_n)$, where ψ^R and ψ^L are the matrices containing the right and left modeshapes respectively, and μ_n and κ_n the modal mass and stiffness of mode n. The total displacement can then be expanded on the right modeshapes as $X = \psi^R c$, where c is the vector of modal amplitudes. The modal amplitudes are easily obtained as:

$$c_n(\omega) = \frac{\psi_n^L F(\omega)}{\mu_n(\omega_n^2 - \omega^2)}\,. \tag{9}$$

This finally yields:

$$X(\omega) = \begin{bmatrix} Q(\omega) \\ \xi(\omega)\end{bmatrix} = \begin{bmatrix} P(\omega)/\omega^2 \\ \xi(\omega)\end{bmatrix} = \sum_{n=1}^{N} c_n(\omega)\psi_n^R\,. \tag{10}$$

The displacement of the membrane and the acoustic pressure calculated using Eq. (10) will be referred to as the **Modal** results. The convergence of this method with the number of modes N taken into account is of primary interest, and is studied in section 2.3.

A last option to compute the radiated field is to use the Rayleigh integral. Indeed, as the membrane is flat and embedded in a infinite plane, the radiated pressure can be computed by:

$$P(\boldsymbol{x}_r, \omega) = -\omega^2\beta \int_\Sigma \xi(\omega)\frac{\mathrm{e}^{-\mathrm{i}k|\boldsymbol{x}_s - \boldsymbol{x}_r|}}{2\pi|\boldsymbol{x}_s - \boldsymbol{x}_r|}\mathrm{d}S(\boldsymbol{x}_s)\,, \tag{11}$$

where the element source is located at $\boldsymbol{x}_s$ and the receiver at $\boldsymbol{x}_r$, and the membrane displacement ξ is computed by Eq. (8). This method for computing the radiated pressure is called **FEM Rayleigh**.

Table 1. Parameters used in all numerical tests

α	β	P_{force}	σ_0	r_{int}	z_{int}	d_{PML}
100	1	$\begin{cases} 1 & \text{if } r < 0.5 \\ 0 & \text{otherwise} \end{cases}$	100	1.5	0.7	0.5

2.3 Results

The membrane is meshed by 50 Lagrangian P2 elements along its radius, and the total mesh is shown in Fig. 2(a). The parameters used in the numerical tests are given in Table 1. These parameters correspond to typical values of the dielectric elastomer loudspeaker studied in section 3. The parameter α is much larger than one, meaning that the first modes of the membrane occur at low frequencies for acoustics (the acoustical wavelength is much larger than the structural wavelength). The parameter β is of order 1, so the fluid loading effect on the membrane is not negligible. The PML absorption parameter σ_0 has been adjusted so that reflections on the PML are as small as possible in the frequency range of interest. The PML efficiency is analysed in Fig. 2 where the **FEM** and **FEM Rayleigh** solutions are compared to each other. They both yield the same results. The **FEM Rayleigh** method implements directly the free-field radiation, so if the **FEM** method gives the same results as the Rayleigh method, it means that no reflections occur on the PML surface Σ_{int}.

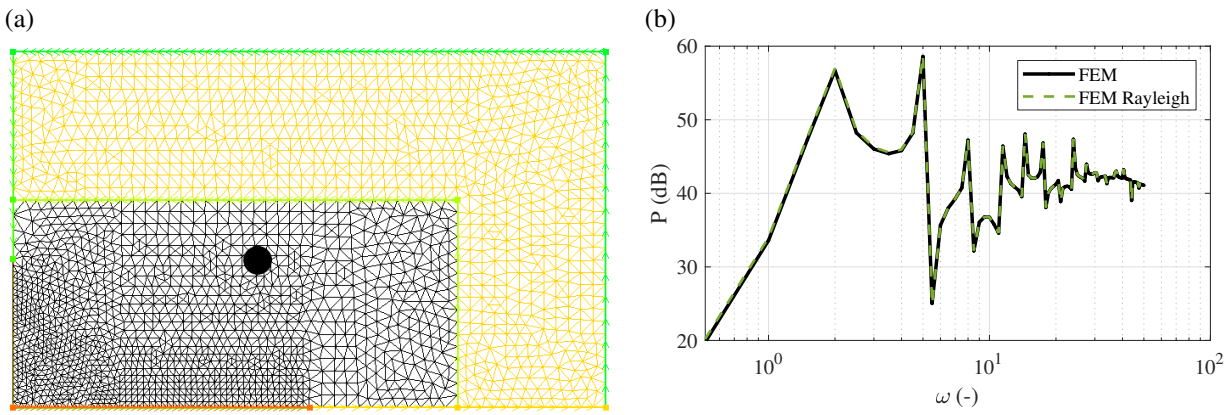

Figure 2. (a) Mesh of the membrane and surrounding fluid. The yellow area is the PML, and the black dot the receiver location. (b) Acoustic pressure at the receiver location.

The first 300 modes (ordered by increasing modulus of the eigenfrequency) are computed, and the eigenfrequencies are plotted in the complex plane in Fig. 3, together with the modal loss factors. This figure clearly shows that there are two categories of modes: the resonant modes of the fluid loaded membrane (which we call membrane modes), and a series of so-called PML modes. The PML modes arise from reflections inside the PMLs. The pressure in these modes is very large inside the PMLs and smaller in the physical part of the domain (see Fig. 4). As the PML is largely damped, PML modes have a high modal loss factor. We thus distinguish PML modes from membrane modes using a simple threshold on the modal loss factor (see Fig. 3(b)). It has also been checked that the membrane modes are independent of the PML parameters, and that the PML modes are not.

The radiated acoustic pressure is calculated using the **FEM** and the **Modal** methods. The convergence of the modal method to the reference **FEM** calculation when the number of modes is increased is studied in Fig. 5. Three subsets of modes are considered: the first 30 membrane modes among the first 300 modes, the first 100 modes including PML modes, and the first 300 modes including PML modes. Figure 5 shows that the

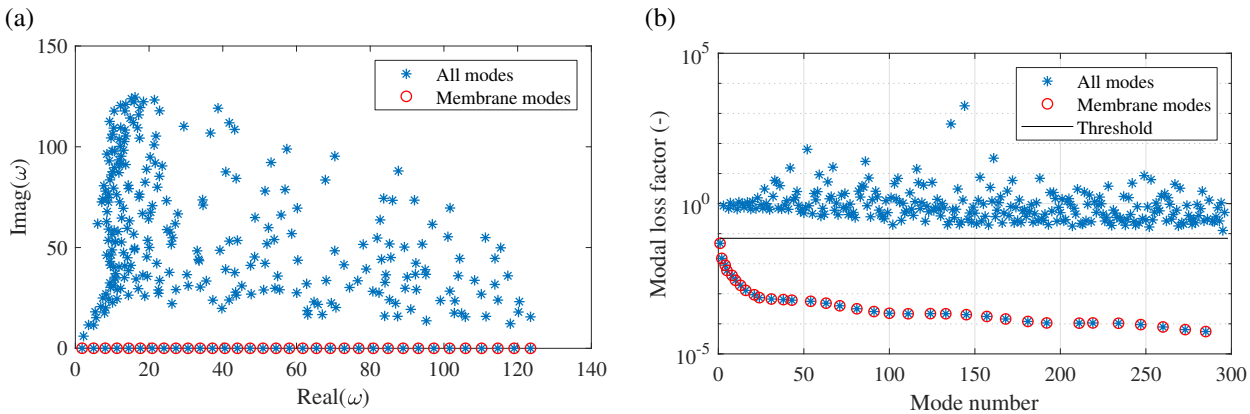

Figure 3. (a) First 300 eigenfrequencies in the complex plane. (b) Modal loss factors of the first 300 modes.

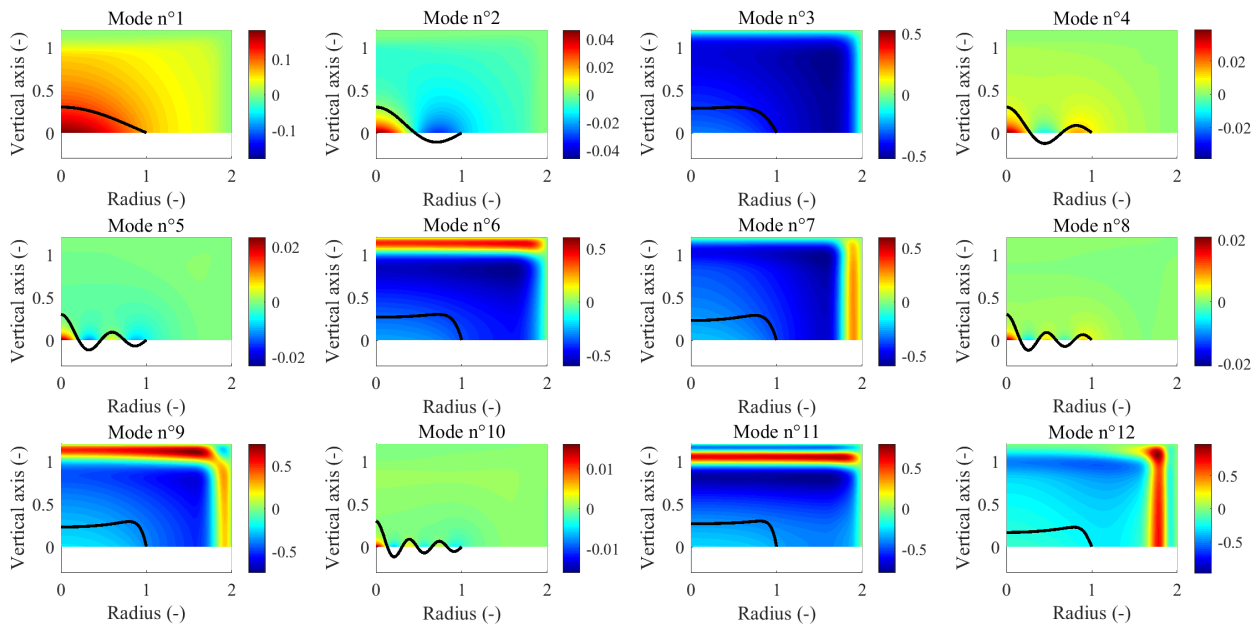

Figure 4. First 12 coupled modes. The black line is the displacement of the membrane, and the colour in the acoustical domain is the real part of the pressure of the mode. Here modes 1, 2, 4, 5, 8 and 10 are membrane modes, the rest are PML modes.

membrane displacement is correctly described by a small number of modes. What is more, if PML modes are removed from the modal expansion (only membrane modes are kept), the membrane displacement is still very well predicted. This suggests that the set of membrane fluid-loaded resonance modes forms a complete basis for the membrane vibrations.

On the other hand, if only membrane modes are used to compute the acoustic radiation, a poor estimation of the radiated pressure is obtained. This means that the basis of fluid loaded membrane modes is not a complete basis to describe exterior acoustics. If the PML modes are included, the **Modal** solution converges to the **FEM** solution. With 300 modes, the **Modal** expansion gives exactly the same solution as the **FEM** calculation. This means that the full basis including PML modes seems to form a complete basis for exterior radiation.

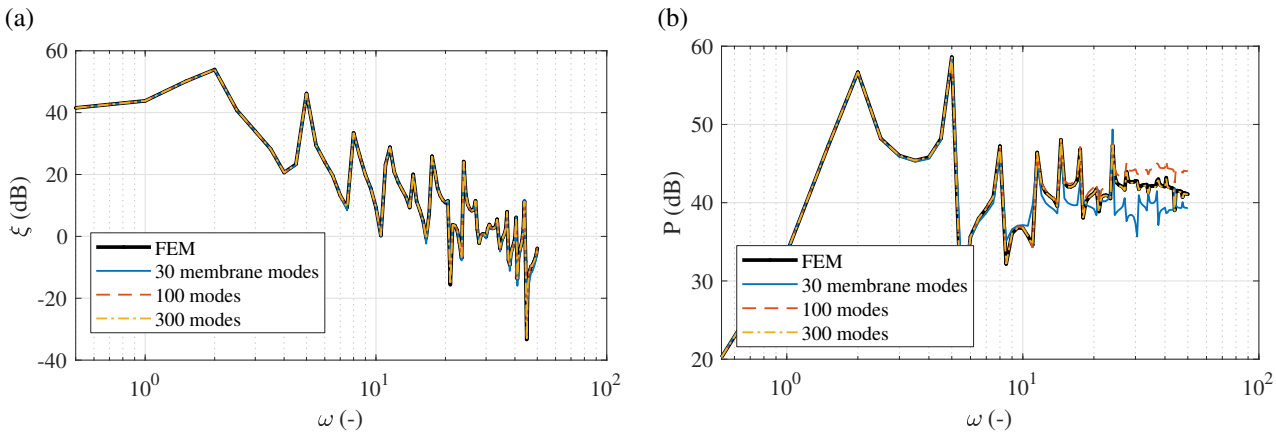

Figure 5. Convergence of the **Modal** solution to the **FEM** solution. (a) Displacement of the membrane at radius $r = 0.5$. (b) Pressure at the receiver location.

Lalanne et al. [7] investigated extensively the use of modal methods to study open resonators, but for optical applications. They also coupled PMLs to finite elements, and concluded that in the general case PML modes should be included in the modal expansion to yield accurate results. The same is observed here: we need to include PML modes to get an accurate prediction of the acoustic response.

If the fluid were described by the pressure P rather than the displacement potential Q, the modal expansion of the pressure would have been of the following form:

$$\begin{bmatrix} P(\omega) \\ \xi(\omega) \end{bmatrix} = \sum_{n=1}^{N} d_n(\omega)\phi_n^R \,, \tag{12}$$

where d_n are the modal amplitudes, and ϕ_n^R the right modeshapes. Comparing Eqs. (10) and (12) shows that if the pressure is used as the main variable, P and ξ have the same frequency dependence, whereas it is not the case in Eq. (10). The convergence of the modal solution to the FEM solution for the acoustic pressure is then much worse, as shown in Fig. 6. Interestingly, if the static contribution of higher order modes is included in Eq. (12) (see [12] for example), the frequency dependence of the modal amplitudes in Eq. (12) is corrected and Eq. (10) is retrieved. To conclude, the displacement potential appears to be a more efficient variable to describe the fluid state for coupled fluid/structure radiation problems.

3 APPLICATION TO THE SOUND RADIATION OF A DIELECTRIC ELASTOMER LOUDSPEAKER

3.1 Description of the studied system

In this section the modal method for exterior sound radiation presented in a simplified case in section 2 is applied to estimate the pressure radiated by a dielectric elastomer loudspeaker. This loudspeaker consists of an inflated silicone membrane coated on both sides with conductive grease, see Fig. 7(a).

When a voltage is applied to the electrodes, the membrane area increases [10]. This effect can be used to radiate sound [5, 1, 6]. We built a prototype using this principle, and measured its sound radiation on axis at a distance of 1 m in an anechoic chamber.

An electro-mechanical model of the membrane dynamics (similar to [13]) is built, and coupled to acoustics inside and outside the cavity. The details of the coupled model are too heavy to be presented here, but will be

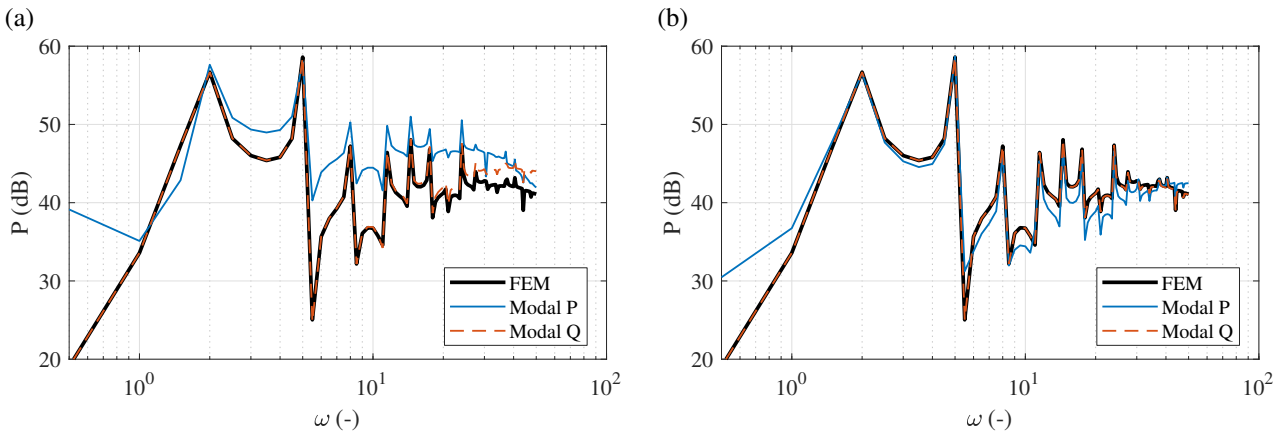

Figure 6. Pressure at the receiver location, calculation with the **FEM** method, and with modal methods. *Modal P* is the modal solution described in Eq. (12), and *Modal Q* is the solution given in Eq. (10). (a) 100 modes included in the modal expansion. (b) 300 modes.

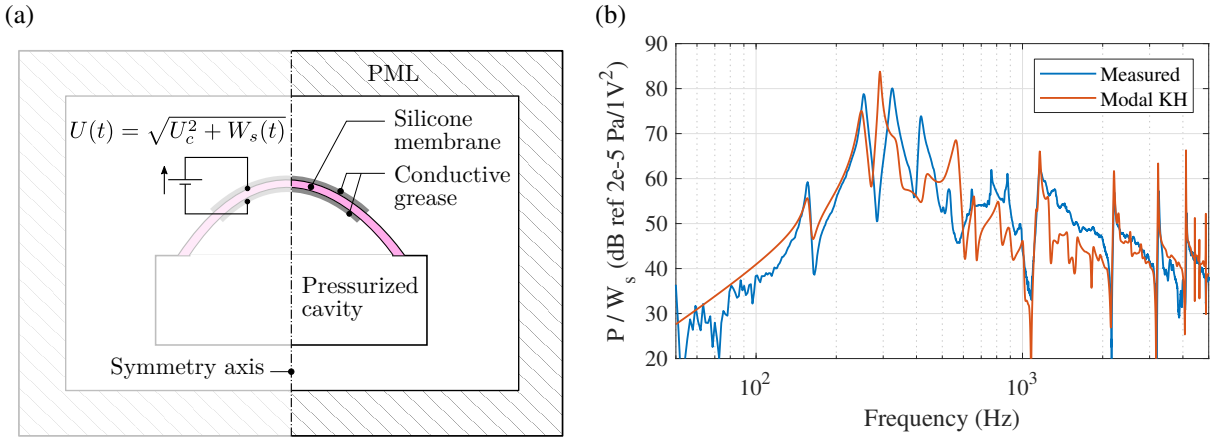

Figure 7. (a) Schematics of the dielectric elastomer loudspeaker. (b) Frequency response function between the radiated pressure on axis at 1m, and the excitation signal W_s.

published soon [2]. The coupled resonance modes of the system are computed, and used as a basis to calculate the acoustical radiation of the loudspeaker, when it is excited by the electro-static force created by the charged electrodes.

The acoustic mesh around the loudspeaker is small, to limit the number of degrees of freedom. The modal method therefore only allows to express the acoustic pressure close to the membrane, which is then propagated to the far field using a Kirshoff-Helmhotlz (KH) integral. Here the first 500 modes (including PML modes) are used in the modal expansion.

3.2 Results

The transfer function between the excitation signal W_s and the radiated acoustic pressure on axis at 1m is plotted in Fig. 7(b). The match of the resonance frequencies is really good, and the peak amplitudes also fit rather well. What is more, the two anti-resonances and resonances around 1100 Hz and 2200 Hz, which

correspond to standing waves in the cavity are reproduced by the coupled model. This is a pure effect of strong vibro-acoustic coupling, demonstrating that a fully coupled model is mandatory to correctly describe the behaviour of this loudspeaker.

4 CONCLUSION

In the present study the use of fluid-loaded resonance modes for the calculation of exterior acoustic radiation has been investigated. It has been shown that an alternative formulation where the displacement potential is used instead of the pressure in the fluid domain allows a much better convergence of the modal expansion. The modal expansion on the resonance modes has then been used to compute the acoustic radiation of a dielectric elastomer loudspeaker, demonstrating its potential application on a real system. This model method is especially well suited for modal force optimizations, for example when the shape of the electrodes of the dielectric elastomer membrane is optimized to improve the spectral balance. Indeed, the system then remains almost identical, so the modes only need to be computed once.

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