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On the cutoff frequency of conical woodwind instruments

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Abstract

Since Arthur Benade, the cutoff frequency of woodwind instruments is a well-known concept that is a result of the acoustic regularity of the tonehole lattice. The distinction between the global cutoff frequency and the local cutoff frequencies was recently introduced. The first can be approximately defined from the input impedance curve, while the second is related to the different cells of the lattice and can be theoretically determined from their individual geometries. From the local cutoff frequencies, it is possible to check the acoustic regularity of an instrument, as was done for the clarinet, even if the geometry is irregular. The present work aims at finding a generalization for conical instruments. The existence of a global cutoff is evident in the impedance curves of oboes, bassoons, and saxophones published by Benade. However, the existence of periodicity is not obvious for a conical geometry because the cells are a priori very different, due to the taper. The present work uses a change of variables to investigate an acoustic periodicity for conical resonators and derives an equation for quantifying the acoustical regularity of a given conical resonator. This method can be applied to the case of a saxophone, for which the local cutoff frequencies are calculated from the geometry of the instrument.

Keywords: Musical acoustics, saxophone, cutoff frequency

1 Introduction

Physical modeling of woodwind instruments typically divides the problem into two parts: the reed as a nonlinear excitation mechanism, and the body as a passive resonator that facilitates the auto-oscillation of the reed in addition to radiating sound. The acoustical response of a resonator is often characterized by its input impedance. If the resonator has a series of open toneholes, the input impedance shows low frequency and high frequency characteristics due to the periodicity of the lattice of toneholes, and the transition occurs at the tonehole lattice cutoff frequency [1].

In general, a global cutoff frequency can be estimated from measurements of the input impedance. In the case of cylindrical woodwind instruments, such as the clarinet, a local cutoff frequency can be defined from the geometry of the instrument by calculating the theoretical Helmholtz resonance of adjacent tonehole cells [2]. This shows that, regardless of cell dimensions, an acoustic regularity can be found such that if each cell has the same local cutoff frequency, the input impedance will display the same global cutoff frequency as any given cell. In the current work, we seek to generalize the concept of acoustic regularity to conical instruments.

The transfer matrices, including a change of variables, that describe wave propagation in a cone with toneholes [3] are presented in Section 2. Acoustic regularity of a conical lattice of toneholes and its application is discussed in Section 3, followed by conclusions and perspectives in Section 4.

2 Simple formula for the cutoff frequency of conical tubes

2.1 Basic equations

Consider a truncated cone of length L, input radius R_1, output radius R_2. The longitudinal coordinate is r. At the abscissa r, the radius is $R = r\tan(\vartheta)$, where ϑ is the half-angle at the apex. The length L is equal to $r_2 - r_1 = (R_2 - R_1)/\tan(\vartheta)$. The area of the cross section is $S = \pi R^2$. In the frequency domain, the Helmholtz equation for the acoustic pressure $P(r)$ is written as:

$$\frac{d^2(rP)}{dr^2} + k^2(rP) = 0 \tag{1}$$

Losses are ignored, $k = \omega/c$ is the wavenumber, where ω is the angular fequency, and c the sound speed. The acoustic velocity V is derived by using the Euler equation, $\partial_r P = -jk\rho c V$, where ρ is the air density.

Between the two abscissae $r_1$and r_2, the following transfer matrix equation can be written as:

$$\begin{pmatrix} Pr \\ Vr - \frac{P}{j\omega\rho} \end{pmatrix}_1 = \begin{pmatrix} \cos kL & j\rho c \sin kL \\ j(\rho c)^{-1}\sin kL & \cos kL \end{pmatrix} \begin{pmatrix} Pr \\ Vr - \frac{P}{j\omega\rho} \end{pmatrix}_2 . \tag{2}$$

When a tonehole is present, we assume that its effect is that of a shunt acoustic mass, $m = \rho h/s$, where s is the cross section area of the hole, and h its chimney height that includes inner and outer length correction. The simplest model for the hole is considered, with the continuity of the planar wave pressure. At the location of the hole, the volume velocity relationship is written as:

$$V_L = V_R + P/(j\omega m S). \tag{3}$$

The subscripts L and R correspond to the left and right sides of the hole, respectively. Therefore the transfer matrix from the left to the right can be written as

$$\begin{pmatrix} Pr \\ Vr - \frac{P}{j\omega\rho} \end{pmatrix}_L = \begin{pmatrix} 1 & 0 \\ Y & 1 \end{pmatrix} \begin{pmatrix} Pr \\ Vr - \frac{P}{j\omega\rho} \end{pmatrix}_R \tag{4}$$

where

$$Y = \frac{1}{j\omega m S}. \tag{5}$$

We use the notations $M_L$and M_h for the matrices defined by Eq. (2) and (4): they are the matrices for the wave propagation over the distance L and through the hole n, respectively. If we build a "*conical*" lattice with the product $(M_L M_h M_L)^N$, the lattice corresponds to the repetition of N cells of tube segments of length $2L$ with a hole at the middle. If the length $2L_n$ and the (*specific*) admittance Y_n of the hole is the same for all cells, the resulting lattice is a periodic medium, despite the fact that the tube is a cone, and the geometry is not periodic. We notice that the model is the same for a cylindrical tube, but here the characteristic quantity of a hole is $1/Y_n = j\omega m_n S_n$, where S_n varies from one cell to the next one. Another difference with a cylindrical tube lies in the term $-P/\mathrm{j}\omega\rho$, which has to be taken into account at the extremities of the lattice. It can be ignored at the open end because the radiation impedance is small but, at the input, it is responsible for a big qualitative difference between the behaviors of cylindrical and conical tubes. This difference is analyzed in many papers.

2.2 Local cutoff frequency

Formally, the analysis of a conical lattice is identical to that of a cylindrical lattice. We do not detail here the explanation given in [2] concerning the distinction between global and local cutoff frequencies.

The local cutoff frequency for a cell of the lattice is calculated by deriving the coefficients (classically denoted A, B, C, D) of the transfer matrix of one cell, $M_L M_h M_L$, and equating $A = D = \pm 1$. This leads to the following equation:

$$j\rho c \cot(kL) = 2j\omega m S. \tag{6}$$

At low frequencies, this leads to the value of the local cutoff f_c

$$f_c \simeq \frac{c}{2\pi L} \frac{1}{\sqrt{2mS/\rho L + 1/3}} = \frac{c}{2\pi} \frac{1}{\sqrt{2mSL/\rho}} \tag{7}$$

The first expression is found by retaining two terms of the expansion of the cotangent function in kL, while the second retains only one term. The first is slightly more accurate than the second. This gives the definition of the local cutoff frequency.

In contrast to the cylindrical case, this local cutoff frequency is not the eigenfrequency of a cell closed at its extremity (see [2, 4]). For a cylindrical geometry, the term $2SL$ defines the volume of the cell, and a simple interpretation is to treat f_c as the Helmholtz resonance of an equivalent cell. This is not valid for a conical resonator because $2SL$ is not the volume of a truncated cone if S is at the midpoint.

3 Analysis of the regularity of a lattice

3.1 Perfect acoustic regularity

In [2] it has been shown that if the (local) cutoff frequencies of the cells are identical, the cutoff frequency seen in the input impedance curve, called the global cutoff frequency, are equal to the local frequencies. The equality of the local cutoff frequencies can be obtained with various dimensions of the holes. Equation (7) provides the condition for acoustical regularity: $2LhS/s$ is independent of the hole. Because this analysis is for conical resonators, it is important to remember that the area S is the cross section of the main tube at the location of the hole. This analysis shows that despite an irregular geometry, an acoustic regularity can be found.

3.2 Analysis of the regularity of an alto saxophone

We measured the geometry of an alto saxophone Senzo, produced by Buffet Crampon. In [2] the aim was to analyse the regularity of a clarinet. It was shown that it is possible to divide the instrument into asymmetric cells with equal local cutoff frequencies. Two methods were used, but an approximate approach was also proposed in order to examine the degree of regularity without requiring the division of the instrument into discrete cells, which is not a trivial task. This later approach is now used for the investigation of a saxophone. Equation (7) is modified as follows:

$$\frac{1}{k_{cn}^2} = 2L_n h_n \frac{S_n}{s_n} + \frac{1}{3} = 2L_n h_n \frac{S_n}{s_n} \tag{8}$$

If the cutoff is identical for two adjacent cells (n and $n+1$), we can write the geometric relationship between adjacent cells as

$$L_n m_n S_n = L_{n+1} m_{n+1} S_{n+1} = \frac{\rho}{2k_c^2}. \tag{9}$$

The distance between hole n and $n+1$ is $d = L_n + L_{n+1}$, so a characteristic cutoff frequency is defined by

$$f_{\mathrm{ch}} = \frac{c}{2\pi}\sqrt{\frac{\rho}{2d}\left(\frac{1}{m_n S_n} + \frac{1}{m_{n+1} S_{n+1}}\right)} \tag{10}$$

If the cutoff frequencies of the adjacent cells are equal, this expression is valid. Therefore, if all pairs of adjacent cells have a similar frequency calculated by Equation (10), the resonator is acoustically regular. Otherwise, if this frequency varies with the considered pair of cells, the variation can be viewed as a measure of irregularity.

Applying this definition of a characteristic frequency to the geometry of a saxophone and comparing with the global cutoff frequency from a measured impedance is continuing as work in progress.

4 Conclusions and perspectives

The global cutoff frequency and a local cutoff frequency defined by the tonehole lattice geometry is treated for cylindrical instruments in Moers et al [2]. Here, we attempt a generalization from the specific case of a cylindrical tonehole lattice that conical case. We derive a local cutoff frequency that can be applied to individual cells. However, this result is not sufficient to analyze a conical tonehole lattice because there is no simple way to divide the lattice into cells. Therefore, we derive a characteristic frequency relating the acoustic masses of two adjacent cells and the length of cone between them. This local value is viewed as a measure of acoustic regularity for a conical tonehole lattice.

In "Fundamentals of Musical Acoustics," Benade provides input impedance measurements to demonstrate the tonehole lattice global cutoff frequency for many instruments including the clarinet, oboe, basson, and even less common instruments such as the tarogato. However, he does not provide similar figures relating to the cutoff frequencies of a saxophone. Although the cutoff frequency is apparent in input impedance measurements (for example the website of Wolfe [5, 6]), the phenomena is not as clear as it is for other instruments. The current work shows that simple eigenfrequency interpretation that works for cylindrical instruments can not be applied to the saxophone, but that a second method of evaluating acoustic regularity shows promise.

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