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Design of damping layer by topology optimization and Non-Negative Intensity

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Abstract

Non-Negative Intensity (NNI) is a quantity which could avoid the near-field cancellation effects in sound intensity and provide direct visualization of surface contributions to sound power. Hence, the NNI and its variants are implemented to be the objective function of topology optimization for damping layer design. Regarding vibro-acoustic systems, the structural vibrations are analyzed by the finite element method (FEM), and fast multipole boundary element method (FMBEM) is used for the acoustic analysis. A two-way coupling is established between the structural and the acoustic domains. By using the FMBEM and the implicitly restarted Arnoldi method (IRAM), the eigenvalue analysis for the symmetrized acoustic impedance matrix is performed with efficiency. Then, the NNI can be easily computed based on the eigen-solutions and the FEM-FMBEM analysis. Further, these eigen-solutions can be recycled in the optimization iterations since they are independent of the solutions of the coupled system. This reduces the computational efforts. For calculating the gradients of the objective function, the adjoint variable method is applied. With the evaluated gradients, the optimization problem is solved by the method of moving asymptotes (MMA) and the optimized distribution of damping layer is obtained.

Keywords: Topology optimization, Structural-acoustic, Boundary element method, Non-negative intensity

1 INTRODUCTION

Reducing the sound emission of machines, systems and structures is important. One efficient tool is to suppress the sound radiation from the most contributing component. This require a straightforward identification of the contribution to sound radiation from individual components of a structure. Sound intensity has been widely used to analyze the contribution to radiation in far-field, and the integration of sound intensity over closed surface, i.e., sound power has also been widely accounted for the objective function in a number of papers [1, 2]. However, sound intensity has both negative and positive values, and thus cannot provide direct visualization of surface contributions to sound radiation. Hence, sound power is also unable to evaluate the sound contribution from partially components rather than whole system. To correctly identify the most contributing area, the non-negative intensity (NNI), was proposed by Marburg et al. [3] to compute the surface contributions to radiated sound power. In this work, we implemented the integration/sum of NNI on surface of interest as the objective function. This allows for minimizing the sound radiation from predefined surface rather than the total surfaces.

2 VIBRO-ACOUSTIC ANALYSIS

For exterior structural–acoustic problem, the finite element method (FEM) is applied for structures and the boundary element method (BEM) is used for fluid. By using the equilibrium and continuity conditions over the

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interface, the coupling between FEM and BEM can be described by [4]

$$\begin{bmatrix} \mathbf{K}-\omega^2\mathbf{M} & -\mathbf{C}_{sf} \\ -\mathrm{i}\omega\mathbf{G}\boldsymbol{\Theta}^{-1}\mathbf{C}_{fs} & \mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_s \\ 0 \end{bmatrix}, \tag{1}$$

where $\mathbf{K}$ and $\mathbf{M}$ are stiffness and mass matrices of the structure, respectively. For convenience, we use $\mathbf{K}_d$ for representing $\mathbf{K}-\omega^2\mathbf{M}$ in the following. $\mathbf{H}$ and $\mathbf{G}$ are the coefficient matrices for the BEM. $\mathbf{C}_{sf}$ and $\mathbf{C}_{fs}$ are the coupling matrices, $\mathbf{C}_{sf}=\mathbf{C}_{fs}^{\mathrm{T}}$, $\boldsymbol{\Theta}$ is the symmetric boundary mass matrix. The vectors $\mathbf{f}_s$ is the structural load.

3 NON-NEGATIVE INTENSITY

Following references [3], the non-negative intensity $\eta(\mathbf{x})$ can be defined by

$$\eta(\mathbf{x}) = \frac{1}{2}\beta^*(\mathbf{x})\beta(\mathbf{x}), \tag{2}$$

where $\beta(\mathbf{x})$ is only a quantity without physical significance, $()^*$ denotes the conjugate of complex values. Obviously, β is non-negative and thus is called by non-negative intensity (NNI). After discretizing, the NNI at collocation points can be calculated by [3]

$$\boldsymbol{\beta} = \mathbf{B}\boldsymbol{\Theta}\mathbf{v}_f = \boldsymbol{\Phi}\sqrt{\boldsymbol{\Lambda}}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Theta}\mathbf{v}_f, \tag{3}$$

where $\mathbf{v}_f = -\mathrm{i}\omega\boldsymbol{\Theta}^{-1}\mathbf{C}_{fs}\mathbf{u}$ is the particle velocity vector at collocation points. $\boldsymbol{\Phi}$ and $\boldsymbol{\Lambda}$ are the matrix storing eigenvector Φ and diagonal matrix storing eigenvalue λ by solving the following generalized eigenvalue problem

$$\mathbf{Z}_R\Phi = \lambda\boldsymbol{\Theta}\Phi, \tag{4}$$

where $\mathbf{Z}_R$ is the resistive impedance matrix defined by [3]

$$\mathbf{Z}_R = -\Re\left(\mathbf{G}^{\mathrm{T}}\mathbf{H}^{-\mathrm{T}}\boldsymbol{\Theta}\right), \tag{5}$$

where the negative sign is introduced because the direction of the given normal for emitted sound power is opposite to the direction of outward normal defined in the BEM analysis. Based on the NNI, the sound power can be computed as

$$W = \frac{1}{2}\boldsymbol{\beta}^{\mathrm{H}}\boldsymbol{\Theta}\boldsymbol{\beta}, \tag{6}$$

where $()^{\mathrm{H}}$ denotes the transpose conjugate of a complex matrix.

4 Optimization problem

To decrease radiation contribution from chosen surface Γ_c, we define the objective function as

$$W_{\mathrm{NNI}} = \frac{1}{2}\boldsymbol{\beta}^{\mathrm{H}}\boldsymbol{\Theta}_c\boldsymbol{\beta}, \tag{7}$$

where $()^{\mathrm{H}}$ denotes the conjugate transpose, and $\boldsymbol{\Theta}_c$ is the corresponding boundary mass matrix for surface Γ_c. When $\boldsymbol{\Theta}_c = \boldsymbol{\Theta}$, $W_{\mathrm{NNI}} = W$. For vibration and noise control, the damping layer design has been proved to be efficient, and is also adopted in this work. The damping effects are included by introducing an imaginary component to the Young's modulus E. The optimization problem can be formulated as

$$\begin{cases} \min\limits_{\boldsymbol{\mu}} & \Pi = \dfrac{1}{2}\boldsymbol{\beta}^{\mathrm{H}}\boldsymbol{\Theta}_c\boldsymbol{\beta} \\ \text{s.t.} & \sum\limits_{e=1}^{N_e}\mu_e v_e - f_v\sum\limits_{e=1}^{N_e} v_e \leqslant 0 \\ & 0 \leqslant \mu_e \leqslant 1 \qquad (e = 1,\ldots,N_e) \end{cases} \tag{8}$$

where $\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_{N_e}]^{\mathrm{T}}$ is the artificial elemental density of damping layer, and only can be 0 or 1 originally. When $\mu = 1$, the damping layer element exists, otherwise it vanishes. N_e denotes the number of chosen design elements. The symbol v_e denotes the volume of e-th element, and f_v denotes the corresponding volume fraction constraint. Because the design variables can only be two discrete values (0 or 1), the optimization problem is hard to solve mathematically. However, this difficulty can be overcome by converting discrete design variables into continuous values using a continuous interpolation function, which is the basic idea of density-based approaches [5]. In the present work, we apply a gradient based algorithm, namely the method of moving asymptotes (MMA) [6], to solve the optimization problem (8). Hence, the derivative of objective function, i.e., the sensitivity information is necessary. As discussed before, we utilize the adjoint method for the sensitivity analysis based on our previous work [2].

5 NUMERICAL TESTS

In numerical tests, we validate the proposed optimization approach by optimizing the damping layer on a four-edge simply supported plate. By setting $\boldsymbol{\Theta}_c = \boldsymbol{\Theta}$, we confirm the optimizations with objective function being sound power and integration of NNI give highly matched results. This is reasonable. Then the center area and corners of the plate are chosen to be investigated area, and optimized damping layer distributions are quite different.

6 CONCLUSIONS

In this work, the non-negative intensity (NNI) and its variant are selected to be the objective function in topology optimization. By applying this technique, an optimized damping layer distribution can be found which reduces the sound radiation from predefined components of structure. The FEM and BEM are applied for structural and acoustic domains, respectively, and a fully coupled FEM-BEM analysis system can be constructed. Thus, the feedback interaction between the structural and the surrounding acoustic domain can be taken into account in the optimization. Due to the large number of design variables corresponding to the damping layer distribution, a gradient based optimization algorithm, namely the method of moving asymptotes, has been used. To compute the objective function's gradients efficiently, an adjoint variable method (AVM) has been developed for the fully coupled acoustic–structure system. Numerical examples have been provided to validate the optimization technique.

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