

Bending wave based characterization of viscoelastic materials

Max Miller⁽¹⁾, Sadeq Malakooti⁽²⁾, Tahereh Taghvaei⁽³⁾, Ning Xiang⁽¹⁾, Hongbing Lu⁽²⁾, Nicholas Leventis⁽³⁾

⁽¹⁾Graduate Program in Architectural Acoustics, Rensselaer Polytechnic Institute, USA, xiangn@rpi.edu

⁽²⁾Department of Mechanical Engineering, University of Texas at Dallas, USA, hongbing.lu@utdallas.edu

⁽³⁾Department of Chemistry, Missouri University of Science and Technology, USA, leventis@mst.edu

Abstract

Obtaining broadband dynamic mechanical properties of viscoelastic materials is challenging. Commercially available characterization equipment is typically limited to about 500 Hz. Ultrasonic testing is a common strategy but requires time-temperature superposition extrapolation to audible frequencies. These techniques are indirect and sensitive to inhomogeneities. Modal methods do not yield broadband data and can fail for materials with high loss factors. A new relatively simple characterization method, relying on bending wave excitation and laser Doppler vibrometer measurement, is proposed. Complex elastic moduli and phase speeds are now attainable over the entire range of building acoustics frequencies. The materials under test are an array of polyurea aerogels with fixed chemical composition but tailorable nanomorphologies. Low cost and ease of synthesis add appeal to materials already noted for their sound transmission loss performance [4].

Keywords: Bending, Flexural, Characterization, Damping, Transmission

1 INTRODUCTION

Modeling and simulation of noise and vibration control problems requires accurate material properties to be effective. Time spent on considering complex propagation paths inherent to most problems is wasted if the propagation variables are not realistic. The current work proposes a method to determine the viscoelastic properties of damping materials. Viscoelastic properties are in general frequency dependent, a feature that complicates their determination. Several characterization techniques exist, our aim is to address some of the deficiencies plaguing other methods. The ultimate goal is to predict the sound transmission loss performance of composite structures.

The American Society for Testing and Materials (ASTM) maintains a standard [2] for the testing of dynamic elastic properties. The procedures in general entail impulsively exciting a specimen of known geometry and measuring different fundamental modes. Determining the elastic properties in this manner elevates the importance of boundary conditions [1] and only directly yields properties at limited modal frequencies. Another popular technique for determining elastic moduli is Dynamic Mechanical Analysis [5]. This method subjects a small, generally bar shaped, specimen to a sinusoidal force. The force and strain responses of the sample are simultaneously recorded. The extent to which the strain signal is out of phase with the driving signal, coupled with the magnitudes of each, allows viscoelastic property determination. Downsides here include an upper frequency bound of about 1 kHz, and sensitivity to clamping conditions of the setup.

Ultrasonic methods circumvent the aforementioned issues but at the cost of being indirect, requiring extrapolation to audible frequencies. The proceeding sections will outline a more direct, broadband, minimally invasive material characterization technique.

2 METHOD

This method relies on the propagation of bending waves. Bending waves excited in a controlled manner are measured using a laser Doppler vibrometer [7]. The frequency dependent real part of the complex phase velocity and the magnitude decay of the transverse velocity are determined in a fairly direct manner. The viscoelastic material properties are then afforded by algebraic manipulation.

2.1 Bending wave equation

The wave equation governing bending waves is reproduced here for completeness, comprehensive derivations are readily available [3]. The differentials operate on the transverse velocity, v_y , of the wave but the equation is valid for several other field variables as well.

$$-\left(\frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial z^4}\right)v_y(x, z, t) = \frac{m''}{B'} \frac{\partial^2 v_y(x, z, t)}{\partial t^2}. \quad (1)$$

The mass per unit area of the plate is represented by m'' , with the bending stiffness per unit length, B' . The work at hand is concerned with two dimensional and complex bending wave propagation. The x-z plane is parallel to the plate's surface with the thickness along the y-axis. The equation is satisfied by a standard spatiotemporal harmonic solution. The phase velocity takes the form [3]:

$$c_B = \sqrt[4]{\frac{B'(1+j\eta)}{m''}} \sqrt{\omega}, \quad (2)$$

with

$$B' = \frac{I'E}{(1-\mu^2)}. \quad (3)$$

The moment of inertia, I' , is defined per unit width,

$$I' = \frac{h^3}{12}, \quad (4)$$

with h representing the plate's thickness. Propagation in both the x and z directions requires consideration of the Poisson ratio, μ . Presently the Poisson ratio is assumed based on static measurements. The damping capacity of the material is captured by the loss factor, η . As the two dimensional complex phase velocity contains the desired elastic properties, experimental measurement allows determination via inversion. Phase velocity and loss factor calculations are possible using a transfer function approach. The transverse velocity of bending waves excited in a plate sample is measurable using a laser Doppler vibrometer. The transfer function between two points, of known radial separation relative to the source, yields the phase delay as a function of frequency. The real part of the complex phase velocity directly follows. The transfer function also contains magnitude decay information. The magnitude decay, ΔL (in Nepers), allows calculation of the bending wave derived loss factor [3],

$$\eta = \frac{\lambda_B \Delta L}{\pi \Delta x}, \quad (5)$$

where the wavelength of the bending wave is λ_B , and the spacing between the transfer function points is Δx .

2.2 Experimental setup

Small phase change discernibility is crucial for the success of this method. Figure 1 illustrates a single axis slide as a precise means of positioning a laser Doppler vibrometer. The sample geometry is an acoustically thin plate of sufficient size to allow isolating direct sound from interfering reflections. To this end, the piezo transducer is mounted with thin transfer tape near the center of the panel. The measurement points may require

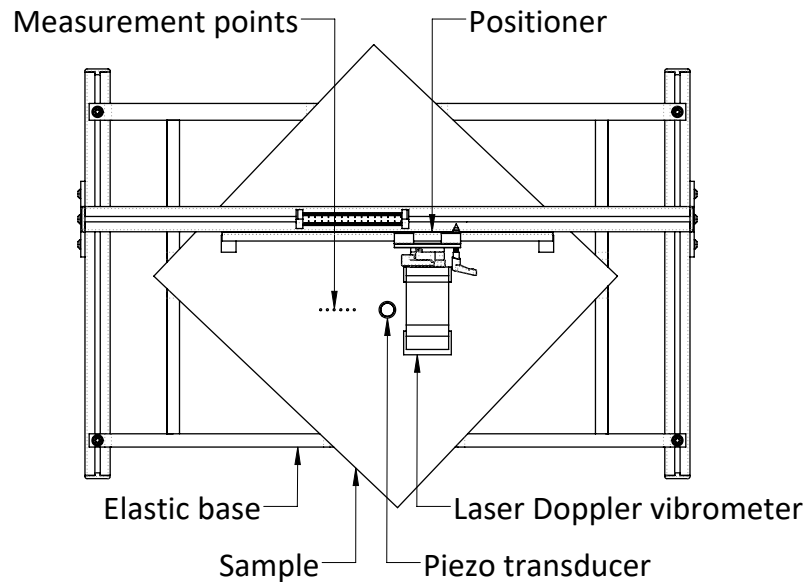


Figure 1. Viscoelastic material characterization is enabled using a setup capable of measuring bending wave propagation. The laser Doppler vibrometer records the transverse velocity of propagating waves.

adhering retro-reflective targets to the sample for signal to noise ratio improvement. The sample is simply supported on a noise isolating elastic base.

The piezoelectric diaphragm style transducer serves as a continuous excitation source. Impulse responses are experimentally measured as the laser Doppler vibrometer is positioned to predetermined locations. Pairs of impulse responses from any two points allow formulation of the transfer functions required for further analysis.

3 RESULTS

The samples under test for this work are polyurea aerogels of shared chemical composition but differing densities and tailored nanostructures [6]. Plotted in Fig. 2, the real part of the complex two-dimensional bending wave phase velocity is calculated over a broad frequency range. The curve's validity is promising as bending waves are dispersive, with the phase velocity proportional to the square root of the angular frequency.

Assumption of a constant Poisson ratio and determination of the frequency dependent loss factor allows for calculation of the complex elastic modulus of the material. The complex elastic moduli of three sample densities are shown in Fig. 3. The dynamic moduli are in line with the reported static values of closely related materials [6].

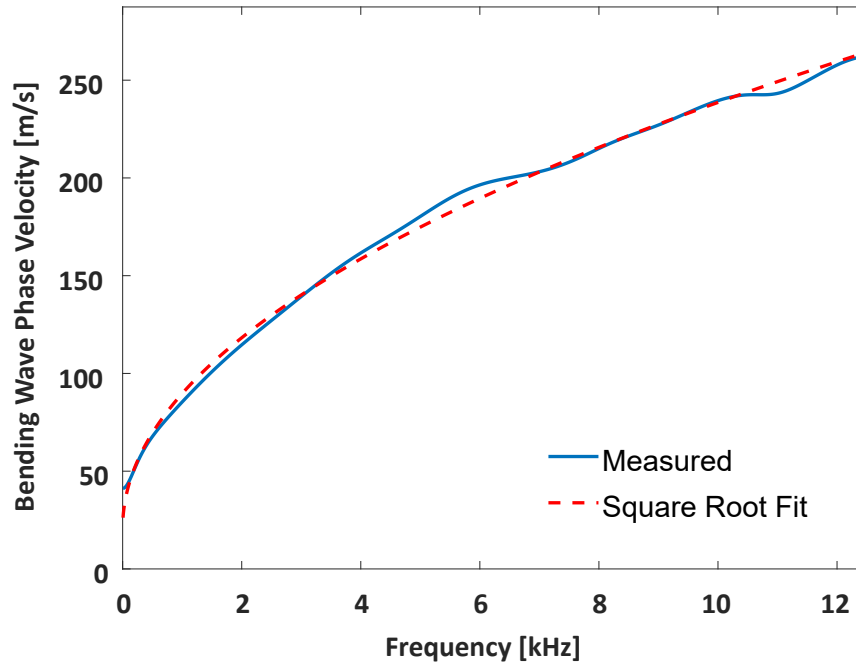


Figure 2. Bending wave phase velocity for a 6.1 mm thick panel of 0.27 g/cc polyurea aerogel. Notice the square root dependence on frequency, as expected per the bending wave model.

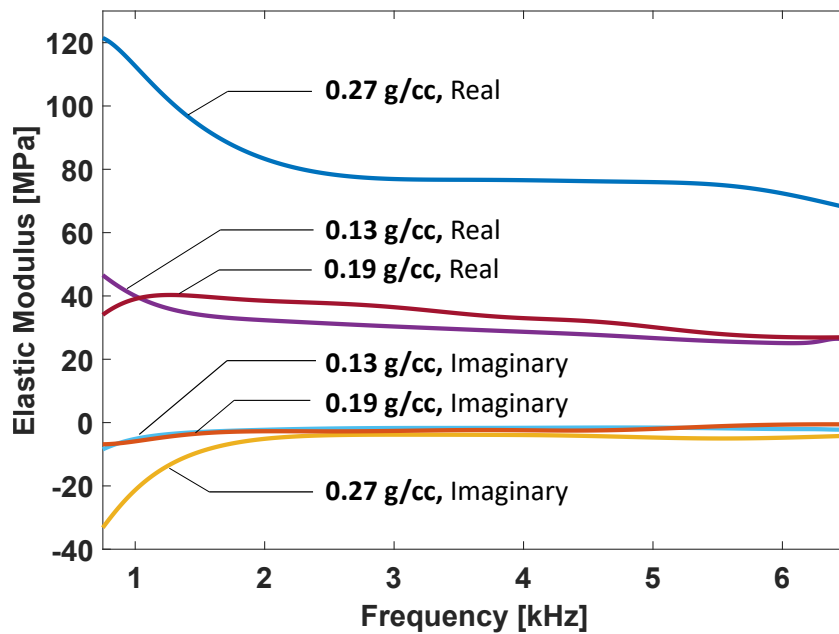


Figure 3. Complex elastic moduli derived from bending wave phase velocity and magnitude decay measurements.

4 CONCLUSIONS

Bending wave phase velocity and amplitude decay measurements enable determination of the complex elastic moduli of viscoelastic polyurea aerogels. Windowing the direct sound from interfering reflections helps free the method from dependence on system resonances. A low mass source transducer and non-contact measurement, using a laser Doppler vibrometer, alleviate the negative influence of mass loading effects. The frequency range of the method is dictated by the window length required to isolate the direct sound, the source's properties, as well as the sample's damping capacity and geometry.

The relatively large sample size required for this method may be reduced using reflection removal strategies. Adapting this method to insitu measurements of large scale surfaces may render this consideration inconsequential and represent another potential benefit of this technique.

ACKNOWLEDGEMENTS

We are grateful of the support from Nashi New Materials, NSF 1636306/1661246/172043, and the Louis A. Beecherl Jr. Chair.

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