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Effective Radiation Area (Sd) for Axisymmetric Loudspeakers Radiating in an Infinite Baffle Using a Near Field Analysis

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ABSTRACT

In this paper, the effective radiation area (Sd) is obtained (theoretically and experimentally) for different loudspeakers. For this purpose, is considered a near field approach. This method is theoretically based in previous works that use the geometrical properties of different axisymmetric pistons and calculate the effective radiation area by positioning the pistons in an infinite baffle. In the case of loudspeakers, the calculations are redefined with the new geometrical conditions, in order to increase the precision of the results. Finally, these results are compared with the listed effective radiation area of the loudspeakers data sheets, and other techniques, in order to set the accuracy of the measurement.

Keywords: Axisymmetric Piston, Effective radiation area.

1. INTRODUCTION

Exact calculation and experimental determination of the Effective Radiation Area (Sd) has always been a very important topic in order to establish loudspeaker parameters. For that purpose, different approaches have been found. In previous work, theoretical analysis has been developed using axisymmetric pistons with different shapes, and then solving the Rayleigh-Sommerfeld equation in order to obtain a simplified expression for this value.

Based on a previous work from Moreno (1) it has been verified that a relation between pressure and acceleration can lead to the value of the diameter used to obtain the Effective Radiation Area (Sd). With the purpose of verifying this relation, two different mathematical methods have been tested. The first method consider a mathematical simplification of the equations based on the cylindrical symmetry of the pistons (2). The second one correspond to an adaptation of a previous publication from Alba (3) that used multiple concentric rings as an approximation for circular pistons. Both methods were adapted for a near field analysis for the calculation of the Sd.

However, in both methods were detected that for conical and convex pistons the value of the error can exceed the 10% of the projected area of them. And for that reason, even when this method probe to work for concave pistons with a small error, it must be tested in real speakers.

2. EFFECTIVE RADIATION AREA(Sd)

In accordance with Beranek (4) (5), the Sd parameter helps to determine the sound pressure on the axis and total acoustic power output. A precise value of Sd is required to calculate the Thiele-Small parameters with a certain degree of reliability.

As Klipple (6) mentioned, in order to determine the Sd, it is desired to find the radius of the flat piston that generates the same sound pressure at a distant point (far field) than a generic axisymmetric piston. This piston radiates independently from the occupied volume in a uniform way, always considering the components in the direction of the symmetry axis.

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Moreno (1) also says, that some “ruler method” exist, that consist, in taking the diameter in the middle of the surround or edge area.

2.1 Near-Field Theoretical Analysis

For the theoretical analysis, here is presented an extract from (2) and (7). The general model for the sound pressure produced by the vibration of a piston mounted in an infinite baffle is called the Rayleigh-Sommerfeld Integral. If the piston shape is arbitrary and the normal velocity component is u_p; we will have a time-dependent arbitrary function.

$$p(x,y,z;t) = \rho_0 \int_S \frac{\dot{u}_p\left(x',y';t-\frac{R}{c}\right)}{2\pi R} dS \tag{1}$$

Where R is the distance to the point where the sound pressure L will be determined.

This calculation is an integral, usually very complex and is often solved with numerical calculation methods.

It is important to analyze the axisymmetric piston generically, and for this purpose, two graphics have been prepared to see the piston from different perspectives. Then, from this analysis, the integral of Rayleigh-Sommerfeld can be evaluated.

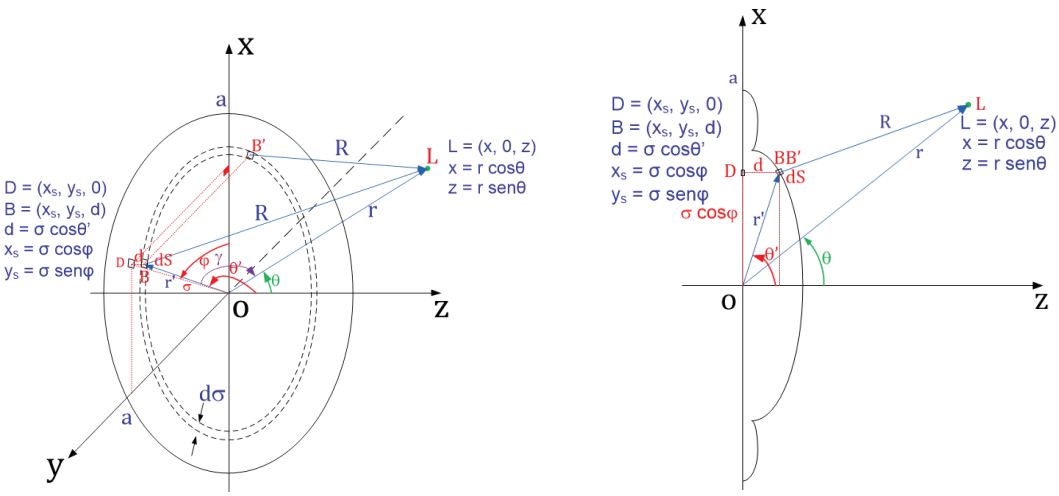

Figure 1: Circular axisymmetric piston. Ring with a width dσ at a height d on the XY plane

From the figures, we can define the characteristics of the Rayleigh-Sommerfeld integral for the plane axisymmetric piston. That expression can be seen below in equation (2)

$$p(x,y,z;t) = \rho_0 \int_S \frac{\dot{u}_p\left(x',y';t-\frac{R}{c}\right)}{2\pi R} dS \tag{2}$$

Based on these graphics it can be proposed the following analysis.

2.2 Calculation methods

2.2.1 Integral simplification

Using the expressions from Hasegawa (8) (9) an applying them for the near field the equation (2) can be transform into:

$$\frac{e^{-jk\sqrt{r'^2+r^2-2rr'\cos\gamma}}}{\sqrt{r'^2+r^2-2rr'\cos\gamma}} = j\sum_{m=0}^{\infty}(2m+1)\frac{K_{m+1/2}(kr')}{\sqrt{r'}}\frac{I_{m+1/2}(kr)}{\sqrt{r}}P_m(\cos\gamma) \tag{3}$$

In order to use different geometries we need to include terms d and σ with the expressions that relate these with r' and $\cos\theta'$. After these modifications we have an expression that can be used to obtain the relation between pressure and acceleration. This expression, which have been called Beta (β) is shown here:

$$\beta_{\text{flat piston}} = \frac{p}{\rho_0 \cdot acc} \tag{4}$$

Now, for the near field it can be obtain the Beta expression for a uniform speed:

$$\beta_{\text{p.axisymmetric NF}} = -jk\sum_{m=0}^{\infty}(2m+1)j_m(kz)\int_0^a \sigma h_m^{(1)}\left(k\sqrt{d^2+\sigma^2}\right)P_m\left(\frac{d}{\sqrt{d^2+\sigma^2}}\right)d\sigma \tag{5}$$

2.2.2 Numerical approximation

Using the work from Alba (3), and the same geometry, it will be assumed that the piston radiation can be decomposed as the addition of the pressure generated by two differential rings applying the superposition principle, but considering both velocity in opposite direction. Even when this principle is applied originally for the far field analysis, it can be used also in the transition from near to far field with the Blackstock (10) equations.

It is considered that the pressure expression for a piston in the z axis:

$$= P_0[e^{j(\omega t-kr)} - e^{j(\omega t-k\sqrt{r^2+a^2})}] \tag{6}$$

Then, it can be define:

- The rings have a width: $\Delta\sigma = \frac{a}{\#rings}$
- The formation rule for the pistons consider the radius $\sigma_i = \Delta\sigma \times i$ where i goes from 1 to #rings
- The z position of the ring is:

$$z_{prom} = \frac{z(\sigma_i\ ) + z(\sigma_{i-1})}{2} \tag{7}$$

Finally, for the near field it can be define the Beta expression for a uniform speed:

$$\beta_{p.axi.NF} = \frac{j}{k}\sum_{i=1}^{\#anillos}\left\{cos\left(k\sqrt{\left(r-z_{prom}\right)^2+\sigma_i{}^2}\right)-cos\left(k\sqrt{\left(r-z_{prom}\right)^2+\sigma_{i-1}{}^2}\right)-jsen\left(k\sqrt{\left(r-z_{prom}\right)^2+\sigma_i{}^2}\right)+jsen\left(k\sqrt{\left(r-z_{prom}\right)^2+\sigma_{i-1}{}^2}\right)\right\} \quad (8)$$

These expressions can be used to verify that the radius that lead to the effective radiation area can be obtain from the relation between the pressure and the acceleration. It should be also consider that the expression for Beta is related to a surface and not for a single point. For the experimental part it will be necessary to measure these values and obtain the relation accurately enough to compare the results with the speakers data sheets or another technique.

3. EXPERIMENTAL DETERMINATION

For the purpose of the paper it have been developed two set ups in order to compare the effective radiation area or the radio that lead to its calculation.

For the validation process, the following speakers were measure. The Tables 1, 2 and 3 shows the Thiele-Small (T-S) parameters of the speakers:

Table 1: T-S Speakers parameters 12" Speaker

	Free Air	Empty Box 1 Vol = 86.45l	Empty Box 2 Vol=156l
Sd(m^2)	0.0475	0.0475	0.0475
Re(Ohm)	5.9	5.9	5.9
fs(Hz)	50.26	62.04	56.87
Bl (Txm)	15.89	16.23	16.23
Qms	2.90	3.47	3.30
Qes	0.46	0.57	0.53
Qts	0.39	0.49	0.46
Mms (g)	61.55	64.70	65.00
Rms (Kg/s)	6.73	7.26	7.04
Cms (m/N)	0.000163	0.000102	0.000121
Vas (l)	51.90	32.00	37.90
No (%)	1.41	1.29	1.28

Table 2: T-S Speakers parameters 6.5" Speaker

	Free Air	Empty Box 1 Vol=10.5l	Empty Box 2 Vol=44l
Sd(m^2)	0.0135	0.0135	0.0135
Re(Ohm)	7.5	7.5	7.5
fs(Hz)	94.26	136.67	101.19
Bl (Txm)	5.28	5.15	5.49
Qms	3.80	6.13	3.88
Qes	1.00	1.55	1.05
Qts	0.79	1.24	0.82
Mms (g)	6.33	6.43	6.80
Rms (Kg/s)	1.00	0.91	1.11
Cms (m/N)	0.000455	0.000211	0.000388
Vas (l)	11.37	5.27	9.20
No (%)	0.92	0.84	0.88

Table 3: T-S Speakers parameters 8" Speaker

	Free Air	Empty Box 1 Vol=10.5l	Empty Box 2 Vol=44l
Sd(m^2)	0.0219	0.0219	0.0219
Re(Ohm)	6.2	6.2	6.2
fs(Hz)	59.30	123.92	78.01
Bl (Txm)	8.88	8.62	8.84
Qms	1.87	3.90	2.35
Qes	0.43	0.91	0.55
Qts	0.35	0.74	0.44
Mms (g)	14.63	14.13	14.50
Rms (Kg/s)	2.96	2.83	3.02
Cms (m/N)	0.000494	0.000117	0.000299
Vas (l)	33.77	7.97	19.60
No (%)	1.60	1.62	1.65

3.1 Near-Field method

The first set up is based in the theoretical approach that is explained in the previous part of the paper. So, is necessary to measure the pressure and the acceleration of the speaker, and for that reason the next configuration has been deployed:

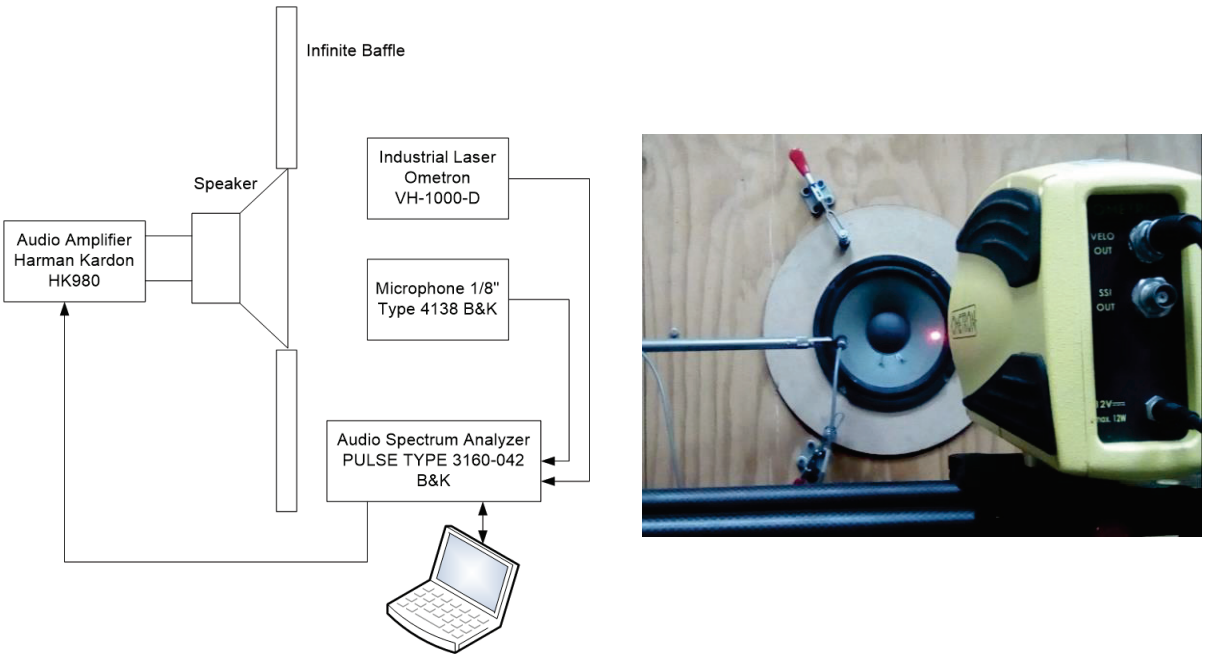

Figure 2: Block Diagram and image for measuring pressure and acceleration

The value of Beta is measured for 3 different speakers using this setup, for different distances from the surface in the z axis and for different frequencies. It has been taken into account the resonance frequency as a reference. Finally, the value of Beta has been measured along the radius of the speaker in order to calculate its mean in the projected surface of the speaker at the edge.

3.2 Differential Volume method

As a way to verify the value of the Effective Radiation Area it has been performed the measure of the speaker parameters in different volumes, according to Klippel (6). In this case, however, it has been measured the compliance of the speakers in free air, and also in the two volumes in order to eliminate the error of measuring the additional speaker volume when it is installed in a box.

By using the measuring method proposed by Moreno (11) and the considerations of Jonsson (12) the Effective Radiation Area (Sd) is obtained.

4. RESULTS

First, it will be shown the graphics from measuring Beta along the z-axis and the measuring of Beta along the radius, for the 12” and 6.5” speakers as an example.

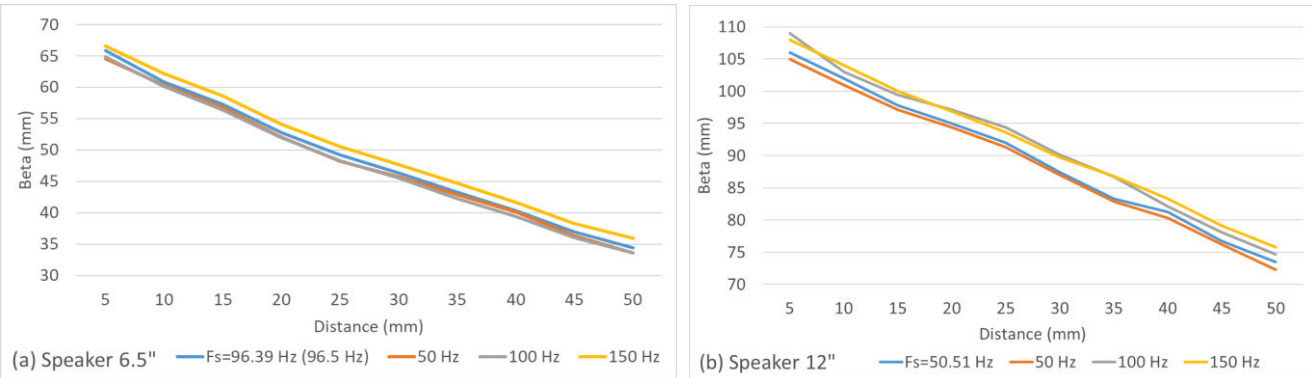

Figure 4: Beta vs. Distance in z-axis from the Speaker (a) Case of 6.5” Speaker, (b) Case of 12” Speaker.

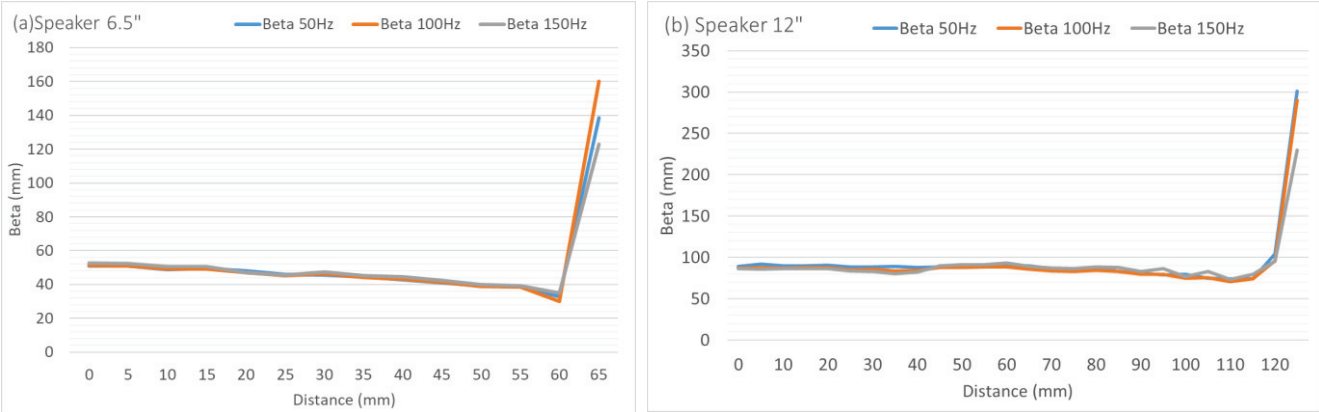

Figure 4: Beta vs. Distance along the radius, from the center of the Speaker (a) Case of 6.5" Speaker, (b) Case of 12" Speaker

From this values, we can obtain the radius that lead to the Effective Radiation Area (Sd) and compare the results with the calculated by using the volume method.

Two approaches have been used. The first one is related with the previous work from Moreno (1), measuring the value of Beta in the surface of the speaker, but with the addition on this paper, of the measuring of Beta along the axis, as it is showed in Figure 4 (a) and, (b).

The second approach, consider the measuring of Beta along the radius and obtaining from these values (See Figure 4) the surface average, in order to obtain a value that represent the effect all over the surface from the center until the edge.

The results can be seen in the next table:

Table 4 - Beta comparison with different approaches.(All values in mm.)

	Aproximation	50Hz	error (%)	100Hz	error (%)	150Hz	error (%)	Average from Compliance Boxes
6.5" Speaker (fs=94.26Hz)	Exponential (z-axis aprox)	69.73	-10.61	69.77	-10.68	71.50	-13.42	63.03
	Polinomial (z-axis aprox)	69.78	-10.70	69.84	-10.80	71.51	-13.45	
	Surface average (along radius aprox)	60.94	3.33	63.92	-1.40	59.57	5.49	
8" Speaker (fs=59.3Hz)	Exponential (z-axis aprox)	83.96	-1.41	82.72	0.08	87.50	-5.69	82.79
	Polinomial (z-axis aprox)	85.49	-3.27	84.70	-2.30	89.27	-7.82	
	Surface average (along radius aprox)	81.84	1.14	78.94	4.65	84.29	-1.81	
12" Speaker (fz=50.26Hz)	Exponential (z-axis aprox)	110.27	11.51	113.50	8.92	112.79	9.49	124.61
	Polinomial (z-axis aprox)	108.58	12.87	112.96	9.35	112.22	9.94	
	Surface average (along radius aprox)	120.22	3.52	116.68	6.37	114.40	8.19	

5. CONCLUSIONS

As can be seen, the inclusion of the Beta average over the surface increase the precision of the technique. Also, it is clear that the value is more precise when the frequency of the measuring is close to the resonance of the speaker. Even when the z-axis approximation on the 6.5” and the 12” speakers have an error around 10%, with the surface average in the worst case is near 3.5% around the resonance frequency. It is important to mention that in the case of the determination of the Effective Radiation Area (Sd) with the volume technique, in the need of obtaining accuracy, the measuring must be performed many times because the value of the compliance is very sensitive to the amplifier characteristics. Finally, this paper succeed in proposing an improvement to previous techniques for the determination of the Effective Radiation Area in a Near Field approach.

REFERENCES

1. Moreno JN, Jonsson S, Moscoso R. Measurement of the Effective Radiating Surface Area of a Loudspeaker using a Laser Velocity Transducer. In 96th Convention Audio Engineering Society preprint; 1994; Amsterdan.
2. Velarde A, Moreno J. Effective radiation area (Sd) for an axisymmetric piston radiating in an infinite baffle. In 22nd International Congress on Acoustics ICA 2016 PROCEEDINGS; 2016; Buenos Aires, Argentina: Asociación de Acústicos Argentinos.
3. Alba J, Arenas JP, del Rey R. Determinación de la presión sonora radiada por pistones circulares y anillos noplanos usando un método numérico simplificado. Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería. 2014; 30(4): p. 221-228.
4. Beranek LL. Acoustics: Amer Inst of Physics; Rev Sub edition; 1986.
5. Beranek LL, Mellow TJ. Acoustics: Sound Fields and Transducers Oxford: Elsevier Inc.; 2012.
6. Klippel W, Schlechter J. Dynamical Measurement of the Effective Radiuation Area SD. In 128th Convention Audio Engeneering Society; 2010; London.
7. Velarde A, Moreno J. Análisis de Campo Cercano para el Cálculo del Area Efectiva de Radiación (Sd) en un Pistón Axisimétrico Radiando en un Plano Infinito. In ; 2018; Cádiz, España.
8. Hasegawa T, Inoue N, Matsuzawa K. A new theory for the radiation from a concave piston source. Journal Acoustical Society of America. 1987; 82(2): p. 706-708.
9. Hasegawa T, Matsuzawa K, Inoue N. A new expansion for the velocity potential osf a circular concave piston. Journal Acoustical Society of America. 1986; 79(4): p. 927-931.
10. Blackstock DT. Fundamentals of Physical Acoustics Austin, Texas: John Wiley & Sons, Inc.; 2000.
11. Moreno J, Jonsson S, Bog H. Measuring of Closed Box Loudspeaker Suyetem Parameters using a Laser Velocity Transducer and an Audio Analyzer. In 92nd Convention AES; 1992; Vienna.
12. Jonsson S. [Application Note: Accurate Determination of Loudspeaker Parameters using Audio Analyzer Type 2012 and Laser Velocity Transducer Type 3544].; 1996 [cited 2019 April. Available from: https://www.bksv.com/en/.