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An analytical model for broadband sound transmission loss of a Finite Single Leaf Wall using a two degree of freedom resonant Metamaterial

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Abstract
Acoustic metamaterials (AM) have emerged as an academic discipline within the last decade. Metamaterials can exhibit high transmission loss at low frequencies despite having low mass per unit area. This paper investigates the possibility of using AMs for increasing the sound insulation of finite single leaf walls (SLW), focusing on the coincidence effect problem. Formulas are derived using a variational technique for the forced sound transmission of finite SLW with a coupled array of two degree of freedom resonators. An analytical model is presented and the effects of the band gap in sound transmission and radiation are analyzed and compared to the single degree of freedom case. Moreover, numerical simulations verify the two degree of freedom model. Finally, some conclusions are drawn regarding the effectiveness of the proposed model, possible applications, and future work.
Keywords: Sound Insulation, Transmission, Metamaterial

1 INTRODUCTION

The importance of sound insulation has increased in cities with the ever-growing population. Buildings are in closer proximity to each other. Also growing number of vehicles has given rise to noise pollution in cities, which in-turn has necessitated sound insulation in buildings. In offices, it is essential to keep the noise level at the minimum to enhance employee efficiency [1, 2, 3]. In schools, audio comfort is one of the primary conditions necessary for an effective learning environment.[4].

Acoustic metamaterials have emerged as an academic discipline within the last decade. The definition may be broadly interpreted as systems or materials that display (as a whole) extraordinary properties not found in natural materials with respect to sound and vibration characteristics, such as negative apparent mass and/or bulk modulus. Metamaterials can show high transmission loss (TL) at low frequencies despite having low mass per unit area [8]-[11]. They owe this behavior to internal subwavelength periodic structures. One of the most important characteristics of the AM is the so-called band gaps (BG), a frequency region where wave propagation is not possible. This property shows great promise to be a good tool to be used in sound insulation, absorption, and even radiation. Sound insulation of walls in buildings or vehicles is a broadband problem, and for a single homogeneous structure, the sound insulation is mainly given by the mass per unit area of the wall, which leaves not much room for improvement [6, 7].

This paper investigates the possibility of using AMs for increasing the sound insulation of single leaf walls, focusing on the coincidence effect problem. The approach utilized in this paper is the same as Brunskog's when investigating the forced sound transmission of single leaf walls using a variational technique [14] and a continuation of the authors work in [21]. In the present research formulas are derived for the forced sound transmission of a finite single leaf wall with a coupled array of two degree of freedom resonators. An analytical model is presented for this case, and the effects of the band gap in sound transmission and radiation are analyzed. The developed model is restricted to the low frequency range where the wavelengths of the wall is much longer than the periodic distance of the resonators. Numerical simulations are carried out to check the analytical model results and also parameter optimization is done. Moreover, a comparison between the SDOF

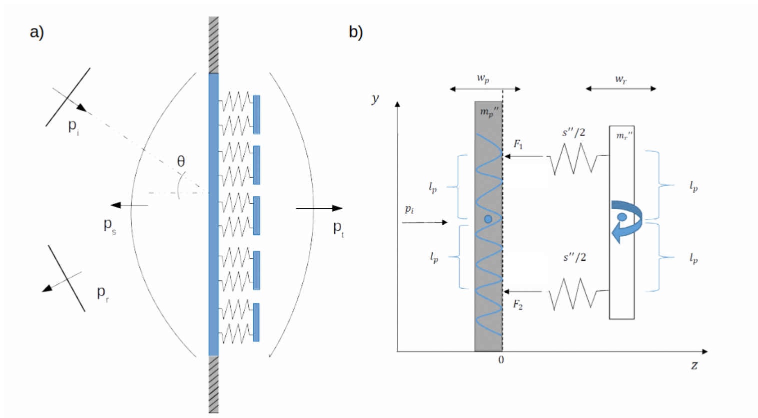

Figure 1. (a) A finite wall of dimensions $a \times b$ coupled with a series of mass-spring resonators located inside a rigid baffle in the $x-y$ plane, at $z = 0$. (b) Simplified diagram of a small section

and the 2DOF is presented and conclusions drawn.

2 THEORY

This section will present the theory utilized in this paper. The variational formulation of the problem used throughout this study is based on Brunskog's work [14] and is extended for the case described in the following section.

2.1 Problem description

Consider a finite thin plate with mass per unit area m_p'' lying in the $x-y$ plane coupled with periodically attached resonators as seen in figure 1a. The plate is located inside a rigid baffle at $z = 0$. For $z < 0$ the acoustic field consists of an incident plane wave, a reflected plane wave, and one scattered field due to the motion of the finite wall. On the positive side of z only the transmitted wave is present (p_t). The resonators have mass m_r'', stiffness s'', and mass moment of inertia j_r'', all per unit area. Structural damping of the spring is considered by assigning the inherent losses to the spring element. For harmonic motion this can be represented by a complex stiffness $\underline{s}'' = s''(1+i\eta_s)$ where η_s is the damping loss factor and s'' is the real part of the complex spring constant. The transverse displacement of the resonators is w_r and the plate w_p. The distance to a reference point is l_p. It is of interest to analyze the transmission through this structure and the influence of the resonators.

2.2 Model Development

The following expression can be developed to describe the model

$$\nabla^4 w_p - w_p \left(k_b^4 + \frac{m_r'' \omega^2 s''(1+i\eta_s)}{B'[m_r''(\omega_0^2-\omega^2)+i\eta_s s'']} - k_x^2 \frac{l_p^2 J_r'' \omega^2 s''(1+i\eta_s)}{B'[J_r''(\omega_\theta^2-\omega^2)+l_p^2 i\eta_s s'']} \right) = 0. \quad (1)$$

This modified Helmholtz equation is the same as the one developed for the SDOF resonators case [21] with an extra term now including the rotation of the mass of the resonators. For simplicity this approximate expression was developed considering rotation only around the x axis. The natural resonance is $\omega_0 = \sqrt{s''/m''}$ and the angular resonance is $\omega_\theta = \sqrt{l_p''^2 s''/j_r''}$. From Eq 1, it can be seen that the modified wavenumber is

$$k_{mod} \approx \sqrt[4]{k_b^4 + \frac{m_r'' \omega^2 s''(1+i\eta_s)}{B'[m_r''(\omega_0^2 - \omega^2) + i\eta_s s'']} - k_x^2 \frac{l_p^2 J_r'' \omega^2 s''(1+i\eta_s)}{B'[J_r''(\omega_\theta^2 - \omega^2) + l_p^2 i\eta_s s'']}}, \tag{2}$$

2.3 The wall impedance

Re-writing and developing Eq. 1 it is possible to get an expression for the wall impedance. It can be interpreted as the impedance operator of the plate plus a term controlled by the resonators. Assuming that the traveling wave is in the form of $e^{-i(k_x X + k_y Y)}$, k_x and k_y being wavenumbers, the wall impedance is reduced to

$$z \approx \frac{B'}{i\omega}\left(k_x^2 + k_y^2\right)^2 + i\omega m_p'' - \left(\frac{m_r'' \omega s''(1+i\eta_s)}{m_r'' i(\omega_0^2 - \omega^2) - \eta_s s''} - k_x^2 \frac{l_p^2 J_r'' \omega s''(1+i\eta_s)}{J_r'' i(\omega_\theta^2 - \omega^2) - l_p^2 \eta_s s''}\right), \tag{3}$$

In order to include the losses in the plate, the loss factor η_p is included as an imaginary part of the Young's modulus $E \longrightarrow E(1+i\eta_p)$ and by that also to the bending stiffness $B' \longrightarrow B'(1+i\eta_p)$.

2.4 Radiation impedance

The formula of the radiation impedance of a finite plate utilized in this paper was derived and explained in a previous study [14], so here it is only presented for clarity

$$z_f = \frac{ik}{2\pi S}\int_0^a \int_0^b 4\cos(k\mu_x \kappa)\cos(k\mu_y \varrho)\frac{e^{-ik\sqrt{\kappa^2+\varrho^2}}}{\sqrt{\kappa^2+\varrho^2}} \times (a-\kappa)(b-\varrho)d\kappa d\varrho \tag{4}$$

where a and b are the dimensions of the plate in the x and y direction, S is the area of the plate ($S = ab$), k is the wavenumber on air, $\mu_x = \sin(\theta)\cos(\varphi)$, and $\mu_y = \sin(\theta)\sin(\varphi)$, θ is the evaluation angle and φ is the azimuth angle.

2.5 Effective mass

It is of interest to develop an expression of the effective mass (also referred to as apparent mass) of the proposed model. It is straightforward to do so from the wall impedance

$$m_{eff}'' \approx \left(m_p'' + \frac{i m_r'' s''(1+i\eta_s)}{i m_r''(\omega_0^2 - \omega^2) - \eta_s s''} - k_x^2 \frac{l_p^2 s''(1+i\eta_s) i J_r''}{J_r'' i(\omega_\theta^2 - \omega^2) - l_p^2 s \eta_s}\right) \tag{5}$$

An approximate expression can be found if losses in the spring are neglected

$$m_{eff}'' \approx \left(m_p'' + \frac{s''}{\omega_0^2 - \omega^2} - k_x^2 \frac{l_p^2 s''}{\omega_\theta^2 - \omega^2}\right) \tag{6}$$

In this form it is much easier to understand the behavior of the effective mass. For low frequencies the mass of the plate and resonators are added. This means that, in this frequency region, the proposed model is effectively working as a wall with mass equal to the sum of the plate and resonators. If this frequency region is also below the angular resonance, the mass moment of inertia is subtracted making the effective mass smaller. This expression show a complex interaction between the resonances, even in this simplified model.

2.6 Diffuse field transmission

The same approach utilized in Brunskog's paper [14] will be used in this study. For the sake of simplicity, the derivation of the equations will not be repeated here. The diffuse field transmission using Paris formula reads

$$\tau = \frac{1}{\pi}\int_0^{2\pi}\int_0^{\pi/2} \frac{4\rho^2 c^2 \Re\{z_f\}}{|z+2\rho c z_f|^2} \sin\theta d\theta d\varphi \tag{7}$$

where ρ is the density of air, c is the speed of sound in air, θ is the incidence angle, φ is the azimuth angle, z_f is the radiation impedance and z is the wall impedance, equation 4 and 3 respectively in this paper. It is assumed that the resonators do not contribute to the sound radiation. In the following sections transmission loss (TL) will be used to analyze the proposed model. It is defined as

$$R = 10\log\frac{1}{\tau} \tag{8}$$

3 RESULTS AND ANALYSIS

A test case will be presented in this section to analyze the analytical model and the sound insulation behavior of the proposed structure. A brick wall with dimension $2\times3\times0.05m$ is used. Material properties are shown in Table 1. The stiffness of the springs are selected specifically to tune the resonant frequency of the resonators above the coincidence frequency of the plate. As known from previous research [21], this is the optimized design in terms of sound insulation. The mass ratio M is one. The mass moment of inertia and the distance to the reference point are selected to tune the angular resonance below the coincidence frequency.

Table 1. Material properties

Property	Value
Young's modulus	$17\times10^9[Pa]$
Density	$2000[Kg./m^3]$
Poisson's ratio	0.2

This case is considering losses in the springs and the plate. Losses are $\eta = 0.03$ for both. Results are presented in Figure 2.

The effect of the angular resonance is barely visible as it has been damped. As expected the general behavior of the structure remains the same as the SDOF model. For frequencies above the band gap the TL of the metamaterial is lower than the one of just the plate. So there seems to be a trade-off between the two approaches.

4 NUMERICAL SIMULATION

In order to validate the results of the analytical model a numerical simulation was carried out using Comsol software. Solving the diffuse case numerically would take a long time and make the simulation harder to set up, so a single incidence angle of $\theta = \pi/6$ was chosen (more incident angles were tested but not shown for the sake of brevity). The case analyzed in this test is a brick wall as used in section 3 but with dimensions $15\times3\times0.04m$ and losses $\eta = 0.003$ in order to be able see the angular resonance. The angular resonance is tuned below the natural resonant frequency of the system. Results are shown in figure 3. The numerical result seem to validate the analytical one. The mismatch seen in higher frequencies may be because of meshing issues. This is going to be investigated further in the future.

5 OPTIMIZATION

The objective of optimization is to find a combination of mass ratio, spring loss factor,spring stiffness, mass moment of inertia and the distance to the reference point that would provide the best possible sound insulation.

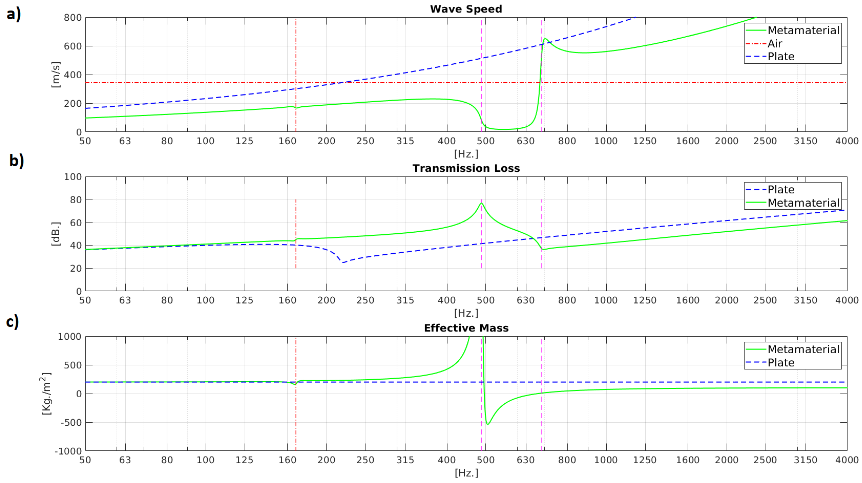

Figure 2. Results for the case considering structural losses of the plate and springs a) wave speed, b) transmission loss, and c) effective mass. Vertical dashed line indicates BG limits of SDOF case and vertical dashed-dot line the angular resonant frequency

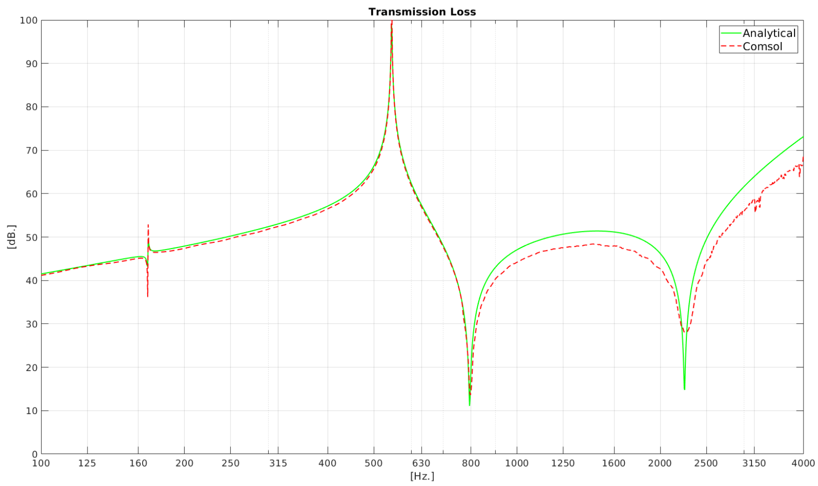

Figure 3. Analytical and Comsol simulation for incidence angle $\theta = \pi/6$

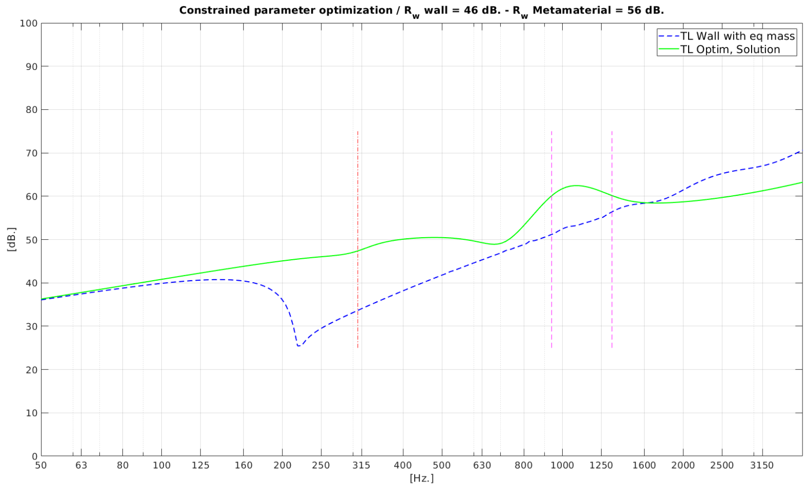

Figure 4. Constrained parameter optimization TL results. Vertical dashed line indicates BG limits. Red vertical dashed-dot line indicates angular resonant frequency

However, defining what is better in terms of sound insulation is not a trivial task. ISO standard 717 [19] defines single-number quantities for airborne sound insulation of building elements such as walls. This provides a solid base to compare different solutions, at least if considering building structures.

5.1 Cost function

The weighted sound reduction index R_w [19] is the selected value to judge which solution is better. The cost function used in this paper is

$$\varphi = \left(\frac{1}{R_w}\right)^2 \tag{9}$$

5.2 Constrained Optimization test case

The developed cost function is used to optimize the case presented in section 3. In order to keep the solutions confined upper and lower bounds were used. The constraints and optimized values are shown in table 2.

Table 2. Constraints used for optimization and optimized values

Property	Lower Bound	Upper Bound	Optimized Value
M	0.001	1	1
η_s	0.001	0.5	0.4255
s''	1×10^4	1×10^{10}	3.477×10^9
J_r''	10×10^{-4}	1	0.5976
l_p	10×10^{-3}	40×10^{-3}	0.0254

A genetic algorithm was used for the optimization (Matlab's "ga" function). In this way most permutations are tested, almost ensuring a good result at the cost of efficiency. At this time, efficiency is not a concern so this is an acceptable drawback. Figure 4 presents the result of the optimization.

The weighted sound reduction index of the optimized metamaterial solution is $R_w = 56dB$ meanwhile the value for the wall with equivalent mass is $R_w = 46dB$. Furthermore, the minimum TL value of the optimized structure is bigger than the minimum value of just the wall. The optimization process seems to have maximized the frequency range where the transmission loss of the metamaterial is larger than the wall and tuned the BG above the coincidence frequency as in the SDOF case. The angular resonance was tuned below the band gap. The coincidence effect and angular resonance are smoothed by the losses in the springs and overall design. This result is almost identical to the one reached for the SDOF [21]. In other words, no advantage can be seen in the usage of this model. The feasibility of the optimized values shown in Table 2 are not explored in this study. On the other hand, for frequencies above the band gap the TL of the wall is higher than the metamaterial.

6 DISCUSSION

The developed analytical model for sound transmission loss of a finite SLW with a coupled array of two degree of freedom resonators was tested and analyzed. The behavior of the proposed metamaterial structure is on par with the SDOF model presented in [21]. Around the frequency of the angular resonance there is a fast decrease in TL followed by an increase, explained by the effective mass. For frequencies above the BG, the mass of the resonators and the plate are moving out of phase, while below they are in phase. If the frequency region below the BG is also below the angular resonance, the mass moment of inertia is subtracted making the effective mass smaller. This model show a complex interaction between the resonances, even in this simplified model. The developed model is restricted to the frequency range where the wavelengths of the vibrations traveling through the wall are much longer than the periodic distance between the resonators. This provides a limitation when designing these types of structures.

The numerical simulation presented in section 5 is an initial step towards the validation of the analytical result. Further analysis has to be done, including a comparison with experimental data.

Furthermore, it is of interest to discuss the feasibility of the proposed structure as a tool to increase sound insulation at constant mass per unit area, paying special attention to the coincidence effect problem. The angular resonance does not seem to provide an advantage in terms of sound insulation. The optimization process finished with an equivalent result to the one obtained for the SDOF case in [21]. It is worth noting that the approximation presented in this study is intended to illustrate the behavior and influence of the angular resonance in sound insulation of this type of structure in an easier manner. The real case would be much more complex and harder to analyze. In future work it would be interesting to validate the analytical model with experimental data. In order to create and analyze more complex designs it is pertinent to explore the possibilities brought by numerical methods techniques.

7 CONCLUSIONS

The analytical model developed in this paper is useful to better understand metamaterials composed of two degree of freedom resonators and how the different parameters affect their behavior. The proposed structure is an effective sound insulator but present not perceivable advantage over the SDOF case. The possibility of using springs with high losses and tuning the band gap above the coincidence frequency still emerges from the optimization process as a possible solution to this phenomenon, rendering the effect of the angular resonance almost null.

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