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Analysis of a vibrating structure as an airborne sound source by means of matrix inversion

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ABSTRACT

The decomposition of a complex airborne sound source into a number of simple sources has found wide application in transfer path analysis. One of the most advanced ways to solve this task is to use the matrix inversion method. This method works well for describing concentrated sources like monopoles. In practice, however, it is also desirable to analyze distributed sources such as a vibrating structure (e.g., the housing of an engine). A large number of grid points, representing emitting partial surfaces, and an even larger number of measurement points complicate the task. The large matrix of transfer functions between source points and measurement points that needs to be inverted is often ill-conditioned. Mathematical approaches such as regularization techniques are required for the matrix inversion. Understanding the ways to apply these approaches and their physical meaning is one of the most important steps in finding suitable solutions to the mathematical inversion problem without losing physical information. In this paper, new ways to improve the matrix inversion method are analyzed for the example of a vibrating plate.

Keywords: Matrix inversion; Airborne TPA; Source estimation

1. INTRODUCTION AND MOTIVATION

Characterization of a complex sound source corresponds to the prediction of main contributors to the sound emission. If sound emission is caused by vibration of a flat structure (e.g., a housing of any aggregate), the main contributors are spread over the structure. This can be seen for instance in the sound intensity map of an industrial gearbox in Figure 1. The measurement took place in an anechoic chamber, therefore the hot spots in the sound intensity map can be interpreted as locations of sound sources of the gearbox. Their positions strongly vary with frequencies. The hot spots are determined by the properties of the gearbox structure (mass and stiffness distribution) and the characteristics of the excitation (here gear mesh excitation inside the gearbox).

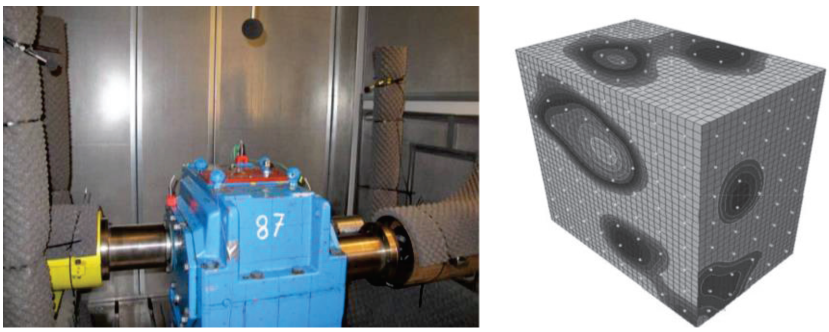

Figure 1 – Left: Gearbox on a test rig, Right: Sound intensity map for a hull of the gearbox [1]

To determine such hot spots different techniques can be considered depending on the final goal. Beamforming or intensity measurements can be applied to predict positions of the sources. However,

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these methods do not provide a characteristic of the source itself independently of its surroundings. To be able to predict the behavior of the source in any other environment, the volume velocity of the source has to be derived. For this goal, one can utilize a novel particle velocity-based technique as was proposed in [2]. In the current work, however, we use a more traditional matrix inversion method (MIM) that requires only customary sound field measurements.

MIM aims to decompose a complex sound source into a number of simple sound sources (components), such as monopoles. In order to “catch” such unstable distributed components as we observe for a vibrating structure, one needs to use a large number of measurement points. However, that will lead to a high matrix condition number and possibly large error. In this paper, we describe the way of coping with this problem and propose a strategy for treating the matrix according to the frequency range and the used measurement setup.

2. MATRIX INVERSION METHOD

2.1 DESCRIPTION OF THE METHOD

The main steps of measurements and calculation of the matrix inversion method are presented in Figure 2 through the example of an engine characterization. The engine is represented by monopoles (yellow dots). Volume velocity ($Q_{\mathrm{op}\,k}$) of each of the monopoles is sought. We will call such monopoles Q-sources. Transfer functions between each Q-source location and each microphone position are measured in the near field (NF) by means of a volume velocity source, while the engine is not active. These transfer functions are named (by analogy to the structure-borne case) acoustic impedances ($H_{\mathrm{AI}\,l,k}$). At the second step, sound pressure under operational conditions ($p_{\mathrm{op}\,l}$) is measured at the same measurement points. The model of sound emission of a complex source can then be described as follows:

$$\begin{bmatrix} H_{\mathrm{AI}\,1,1}(f) & \cdots & H_{\mathrm{AI}\,1,K}(f) \\ \vdots & \ddots & \vdots \\ H_{\mathrm{AI}\,L,1}(f) & \cdots & H_{\mathrm{AI}\,L,K}(f) \end{bmatrix} \cdot \begin{pmatrix} Q_{op\,1}(f) \\ \vdots \\ Q_{op\,K}(f) \end{pmatrix} = \begin{pmatrix} p_{op\,1}(\mathrm{f}) \\ \vdots \\ p_{op\,L}(f) \end{pmatrix} \tag{1}$$

The complex source is modeled by K monopoles. To find the desired vector Q_{op}, the acoustic impedance matrix has to be inverted (using the pseudo inverse). The resulting acoustic admittance matrix multiplied by a vector of operational sound pressure will provide the desired result.

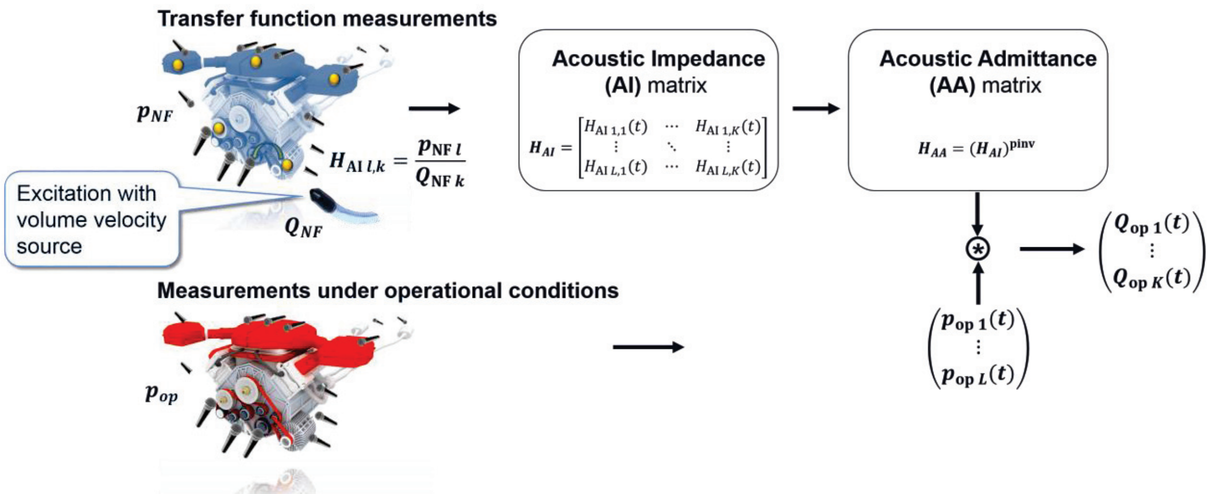

Figure 2 – Description of the application of the matrix inversion method for the airborne case

The main critical points of the application of MIM are the measurements with a sufficient number of points and the inversion of the acoustic impedance matrix. They are in some contradiction to each other: larger numbers of measurement points with highly correlated transfer functions will usually lead to an ill-conditioned matrix (increase of matrix condition number), while exclusion of some points for decreasing the condition number can lead to a loss of the resolution of Q-sources. In the paper we show how to find a compromise between these two parameters.

2.2 ILL-CONDITIONED MATRIX TREATMENT

Regarding this specific application of a vibrating structure, one can refer to observations made by Nelson and Yoon in [3]. The paper presents their study on the dependency of the matrix condition number on the relationship between the parameters of a measurement setup and a frequency range.

Figure 3 shows the investigated model with 100 Q-sources and 100 microphones. The variation of the condition number of the matrix of transfer functions between all the Q-sources and microphones is observed for the variation of the frequency range and a further parameter defined as distance of the Q-source plane to the sensor plane (r_{ms}) in relation to the distance between the sources (r_{ss}). The distance between the microphones r_{mm} and between the Q-sources r_{ss} is equal, eccentricity $e = 0$, that means that the microphones are placed above the Q-sources without any shift in the horizontal plane. The strong variation of the condition number with the varying parameters can be seen. That means that for a specific pair of parameters, for example frequency and r_{ms}, one can find an optimal r_{ss} in terms of the conditioning of the problem. In the current paper we show a similar observation for a vibrating structure and use it as a basis for the processing of an acoustic impedance.

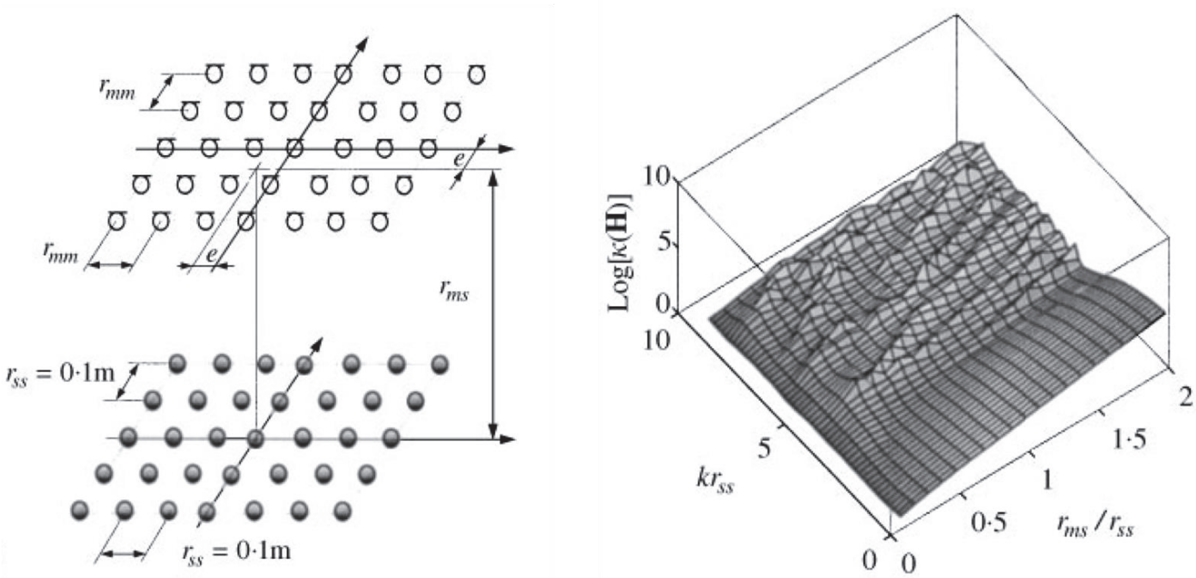

Figure 3 – Left: A geometrical arrangement of sources and microphones, Right: Variation of the condition number $k(H)$; kr_{ss} - non-dimensional frequency, where r_{ss} is the distance between sources, r_{ms} - distance between the source plane and the sensor plane [3]

3. IDEA

Trying to describe a real vibrating structure in a wide frequency range, one will probably make measurements with a large regular grid of measurement points at some fixed short distance r_{ms} to the structure for the full frequency range. For higher frequencies, even the full grid can be relevant, because the transfer functions will vary at different points due to short wavelengths, and the acoustic impedance matrix can be well-conditioned. For lower frequencies (longer wavelengths) the elements of the matrix will be highly correlated and the matrix will become ill-conditioned. For the determination of points (matrix elements) that are relevant for each specific frequency with the fixed distance r_{ms}, we decided to observe the condition number changing over the varied r_{ss}, similar to Nelson in [3]. Having this dependency, we can estimate an optimum r_{ss} for this frequency, and used r_{ms}. This will help to adjust a model of the complex source and a corresponding matrix for each frequency.

The goal of this study is to find a model of a vibrating plate consisting of point sources using the matrix inversion method for the airborne case, and to discuss challenges due to requested spatial resolution with respect to the requested frequency range. In the first step, this analysis is done based only on numerical results. Future work will deal with the manufactured assembly and experimental tests.

4. DESCRIPTION OF THE EMPLOYED MODEL

For this study, an application example is requested with several sound sources, preferably with changing source positions and source number depending on the considered frequency range to be able to investigate the method of sound source localization under different conditions. Of course, there should also be a link to a problem of the practical field, but with known boundary conditions and available data for the purpose of numerical investigations.

A vibrating plate clamped on two sides can represent the introduced acoustical behavior of a wall of a gearbox. Based on numerical pre-studies the model shown in Figure 4 was derived. An asymmetric aluminum plate is fixed on two steel beams by screws in such a way that the plate itself mostly dominates the airborne noise in a frequency range of 150-3000 Hz. The prerequisite is that the excitation takes place on the plate itself. The excitation (F) is by an impact normal to the plate surface. The position is marked in Figure 4.

The asymmetry is chosen to get non-symmetric source patterns, which exhibit different levels of source strength.

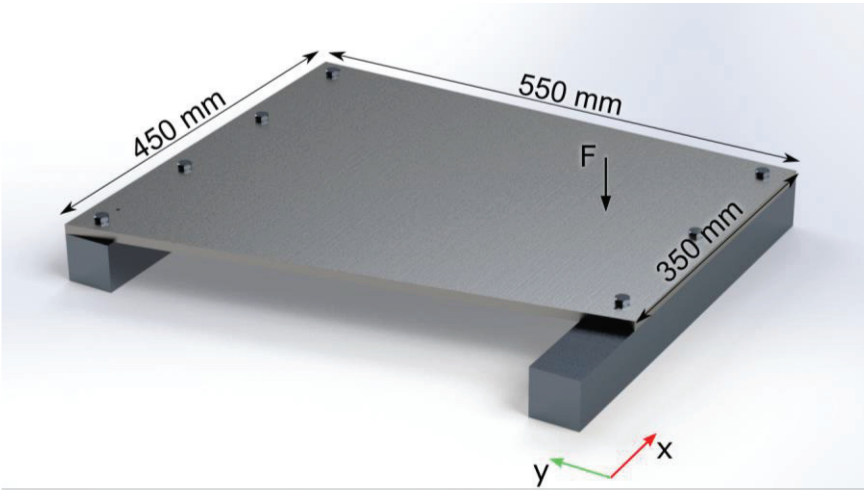

Figure 4 – Application example

For the acoustical evaluation of the plate-construction, MSC Software is used. The meshing process for the structure parts, the definition of bolt connections and application of the input force for frequency response calculations are carried out with *MSC Apex*. Thanks to the *MSC Nastran* solver the resulting velocities at the nodes of the finite elements are calculated and transferred to *MSC Actran* for enabling the airborne calculation.

For the simulation of transfer functions and operational measurements the arrangement shown in Figure 5 was chosen.

The point volume velocity sources were placed on the top of the plate. The microphones were placed above the Q-sources at a distance $r_{ms} = 5$ cm. The sources were randomly distributed over the plate with minimum distance between the points $\min(r_{ss}) = \min(r_{mm}) = 3$ cm. The simulations were performed for free field conditions, without any geometrical constraints.

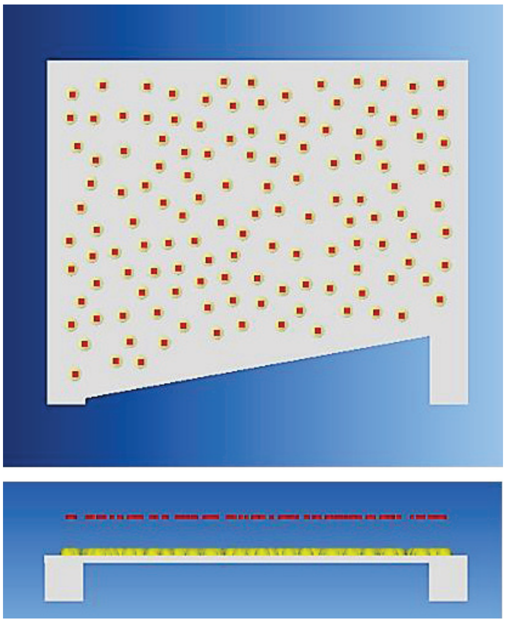

Figure 5 – Setup for simulated measurements: yellow dots – point sources, red dots – microphones
Top: Top view, Bottom: Side view

In Figure 6 and Figure 7 patterns of the structure-borne normal velocity and of the sound pressure are shown at low and mid frequencies. As expected, both quantities show a very similar pattern. The areas of the plate involved in the movement at most can be considered as the main contributors to the sound radiation. The desired representation of the vibrating plate by several Q-sources leads to a very similar pattern.

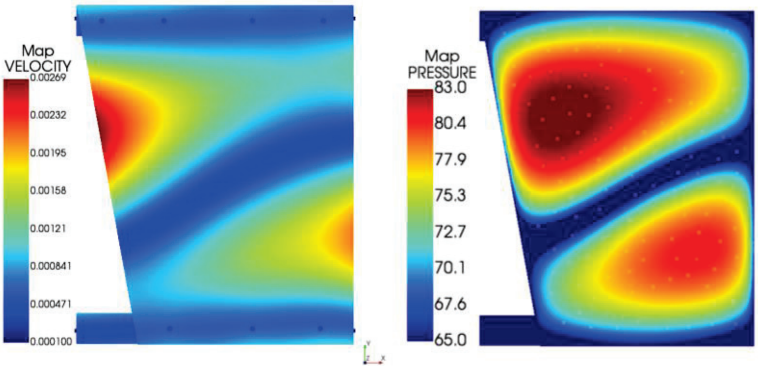

Figure 6 – Left: Velocity map, Right: Sound pressure map on the plate at 260 Hz

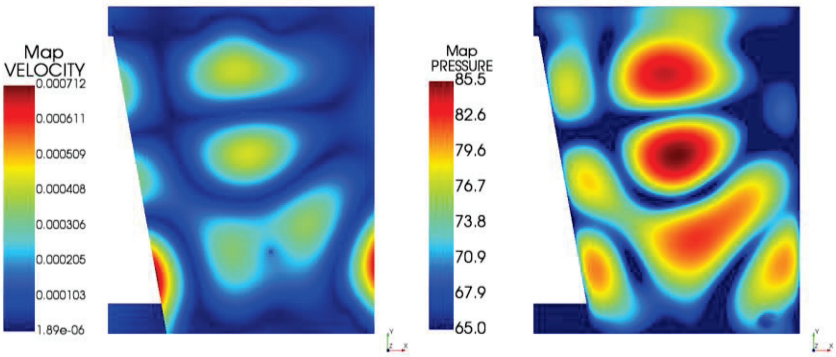

Figure 7 – Left: Velocity map, Right: Sound pressure map on the plate at 1650 Hz

5. ANALYSIS OF THE TRANSFER FUNCTION MATRICES AND DERIVATION OF THE POINT SOURCES

The condition number of the transfer function matrices related to frequency range and distance between the Q-sources (r_{ss}) is shown in the center of Figure 8. The distance r_{ss} was changed by means of the selection of the measurement points from the full grid. That means that the size of the matrix was reduced with the rise of r_{ss}. In such a case the change of the distance corresponds to a change of the density of the points. Indeed, the selection of the points for each r_{ss} is not unique, however for other arrangements the tendency of the condition number remains the same.

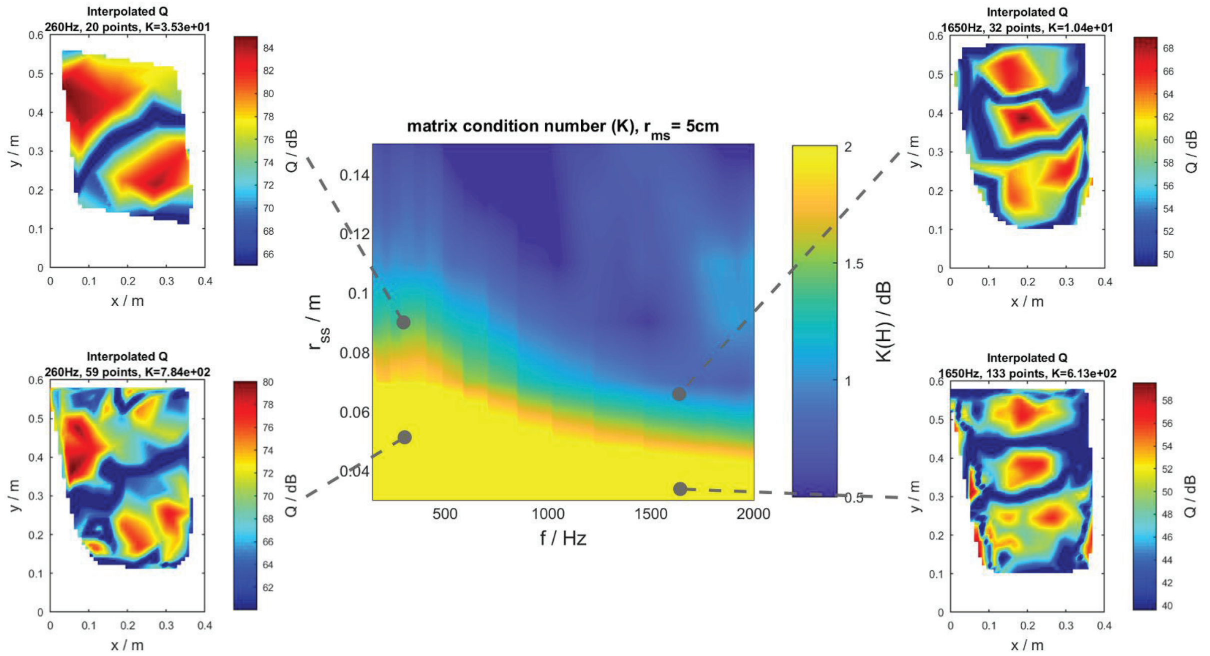

Figure 8 – Mid: Matrix condition number depending on frequency and distance between the assumed sources, At the margin: Examples of interpolated volume velocity patterns for different measurement setups at particular frequencies

As expected, the preferred distance r_{ss} in terms of a better condition number grows with increasing wavelength, which means decrease of the frequency. That suits the results in Figure 3. According to the shown analysis we can treat the ill-conditioned matrix by means of the selection of only those matrix elements that correspond to the estimated optimum r_{ss}. In such a way, the model of the vibrating plate is adjusted for different frequencies. The corresponding transfer function matrix is then used to define the volume velocity of the components placed at the selected points.

In Figure 8 the results of such calculations are presented for 260 and 1650 Hz for both “bad” and “good” cases. For each frequency and each case, the acoustic impedance matrix was modified corresponding to the shown r_{ss}. The matrix was inverted to find the volume velocity of the Q-sources. To show the distribution of the volume velocity over the plate, the results were interpolated. The patterns’ edges are defined by the selected Q-sources.

The resulting interpolated volume velocity obtained from the modified models with lower matrix condition number suit the velocity maps shown in Figure 6 and Figure 7. As can be seen for the higher frequency, even the complete 133x133 acoustic impedance matrix shows a good condition number for the calculation of the volume velocity. However, the reduction of the model helped to obtain an even better match to the velocity map. For the lower frequency the benefit of the matrix decrease is more significant.

In the described example only the single Q-sources at distance r_{ss} to each other were selected. Indeed, in the vibrating plate the sound-emitting parts correspond rather to a surface than to a point. That is why we have considered a way of including the information from the neighboring surface to the Q-source.

This is realized by the discretization of the Q-sources according to their positions, as follows. The surface of the plate in the current example was divided into 32 slots (Figure 9). One slot is then considered as one source component. The transfer functions corresponding to the Q-sources placed in one slot were averaged. The center of each slot is taken then as a position of the joint Q-source.

It is worthwhile to mention that the sound pressure at any measurement point can be affected by several initial Q-sources. The basic assumption here is that for the targeted frequency there is only a slight difference between the transfer functions of neighboring sources to all initially defined measurement points. Additionally, the amplitude of assumed sources should not differ strongly from each other. Otherwise, the information of different source strengths would be lost. As an indicator to avoid such a negative effect on spatial resolution, the measured sound pressure is analyzed. When the mentioned requirements are fulfilled, it is consistent not to try to dissolve these neighboring sources whereby the problem size can be minimized.

Thanks to this procedure, the matrix size for the inversion can be reduced. For example, instead of an initial configuration of 133 assumed sources and 133 measuring points, the problem size can be adjusted to only 32 sources remaining for gaining the pressures at 133 measuring points. In Figure 9 the grid with the resulting Q-source levels is shown. In Figure 10 the interpolated result is shown on the left side.

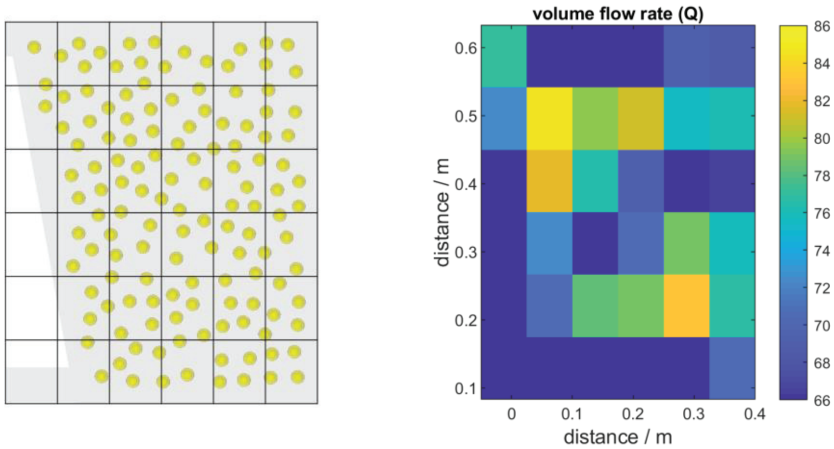

Figure 9 – Left: Illustration of the separation of the model into 32 spots and grouping of the initial Q-source, Right: Resulting volume velocity for the modified model (for 260 Hz)

This result is compared with the result based on the reduction of the model by selecting single Q-sources as shown in Figure 8 (for 260 Hz). The comparison shows that both procedures deliver a similar result with respect to the position of sources on the plate.

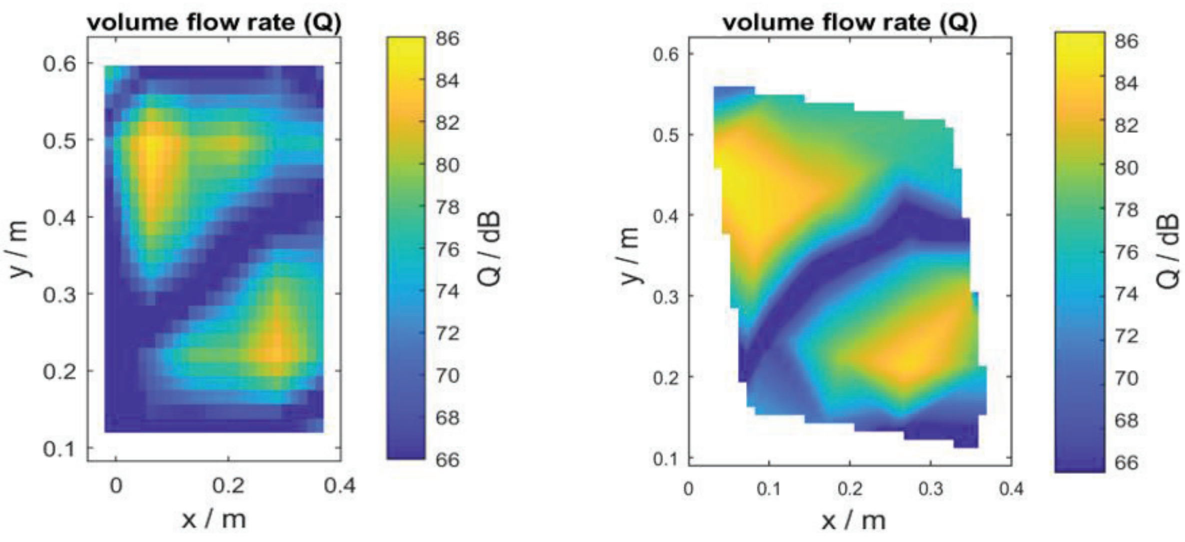

Figure 10 – Interpolated volume velocity of the plate for 260 Hz
Left: Model of 32 joint Q-sources, Right: 20 single Q-sources

6. SUMMARY AND OUTLOOK

A sound field produced by a distributed source was observed through the example of a vibrating plate. The plate was characterized using the matrix inversion method in a frequency range of 150-2000 Hz. A strategy based on the analysis of the matrix condition number for the adjustment of the source model was proposed. It was shown, that for a vibrating structure an optimum distance between the Q-sources can be estimated. Using the derived distance, the model was changed in two different ways: reduction of the initial model by means of the selection of single Q-sources, and grouping of several Q-sources into clusters. The results of both investigations match the simulated velocity of the plate.

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