

Investigation of elastic surface and edge modes in finite-size three-dimensional phononic crystals

Tian-Xue MA¹; Quan-Shui FAN²; Chuanzeng ZHANG¹; Yue-Sheng WANG³

¹ Department of Civil Engineering, University of Siegen, Siegen, D-57076, Germany

² Institute of Engineering Mechanics, Beijing Jiaotong University, Beijing 100044, PR China

³ School of Mechanical Engineering, Tianjin University, Tianjin 300350, PR China

ABSTRACT

Three-dimensional (3D) phononic crystals give rise to the so-called 3D phononic bandgaps, which are frequency ranges where the propagation of elastic or acoustic waves is prohibited along all spatial directions. Phononic crystals show various potential applications, such as vibration suppression, noise isolation, waveguiding and filtering, etc. By appropriately terminating the 3D phononic crystals, the elastic waves can be guided at the surface and/or the edge of the finite-size crystals. The 3D-printed phononic crystal samples are experimentally tested to assess the transmission spectra of surface and edge modes in the finite-size structures. And the experimental results show good agreement with the theoretical predictions. The surface and edge modes would open up avenues for the manipulation of elastic waves in periodic structures, and show great potential for designing complex and compact phononic circuits.

Keywords: Phononic crystal, Bandgap, Elastic wave

1. INTRODUCTION

For its unique physical properties, phononic crystals (PnCs) open up a new avenue to control the propagation of acoustic or elastic waves. The existence of a phononic bandgap exhibits a lot of potential applications, such as noise reduction, waveguides, acoustic filters, energy harvesters, to name a few (1).

For realizing three-dimensional (3D) manipulation of elastic waves 3D periodic structures are indispensable. 3D phononic bandgaps, which are frequency ranges where the propagation of elastic waves is prohibited along all spatial directions, can only be achieved in 3D PnCs. The bandgaps smother the entire 3D Brillouin zone, not just any one plane for two-dimensional cases.

Just like bound states can exist at the surfaces and edges of 3D photonic crystals (2, 3), similar bound states can also be observed at the surfaces and edges of 3D PnCs. However, compared with the bulk modes, the investigations of the surface and edge modes in 3D PnCs are rather limited. In this work, we study the elastic surface and edge modes in finite-size 3D PnCs both numerically and experimentally.

2. GEOMETRICAL MODEL

The scheme of the 3D PnC is illustrated in Fig. 1(a). The cubic scatterers (resonators) are arranged periodically and connected by thin cylinders. The 3D PnC is of the simple cubic lattice and possesses the six-connected network topology (4). Thanks to the periodicity of the PnC the wave behavior (i.e., the band structure) can be calculated in one unit-cell by using the Bloch theorem. The PnC samples are formed by 3D printing with the photopolymer material (mass density $\rho = 1200 \text{ kg/m}^3$, Young's modulus $E = 3.3 \text{ GPa}$ and Poisson's ratio $\nu = 0.41$). Considering the sample fabrication and experimental activity, the unit-cell size is chosen as $a = 25 \text{ mm}$, where a is the lattice constant. In this work, all numerical results are calculated by the finite element method using the commercial software COMSOL Multiphysics. The band structure of the PnC unit-cell is shown in Fig. 1(b). A complete bandgap can be observed in the band structure, which is of the frequency range [10.7 kHz, 29.0 kHz].

¹ tianxue.ma@uni-siegen.de

Notably, a large band gap for bulk modes is beneficial for further investigations of the surface and edge modes.

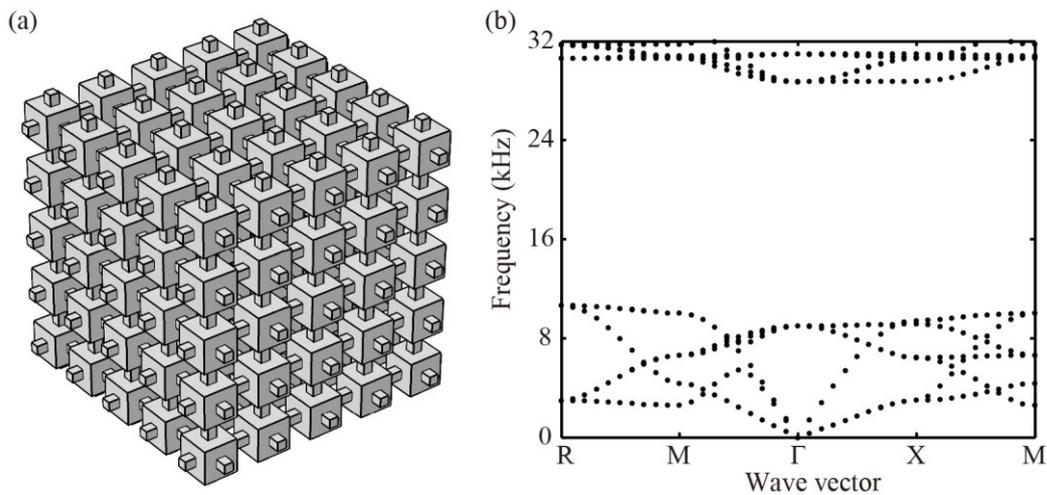


Figure 1 – (a) Scheme of the 3D PnC. (b) Band structure of the PnC unit-cell.

3. ELASTIC SURFACE AND EDGE MODES

Free surfaces can be formed by terminating an infinite 3D PnC at the top and bottom surfaces. The scheme of the super-cell for studying the surface modes is illustrated in Fig. 2(a). The band structure of the super-cell is given in Fig. 2(b), where the frequency ranges of the elastic bulk modes are indicated by the shadowed zones. Five bands can be found inside the bandgap. However, the lowest two bands are too close to the bulk modes and hardly be used in practice. Thus, the three bands in the middle of the bandgap, which correspond to the surface modes, are taken into account in the work. And the frequency range of the surface modes is [19.7 kHz, 23.0 kHz]. Due to the existence of the complete bandgap, the elastic surface modes cannot penetrate into the bulk.

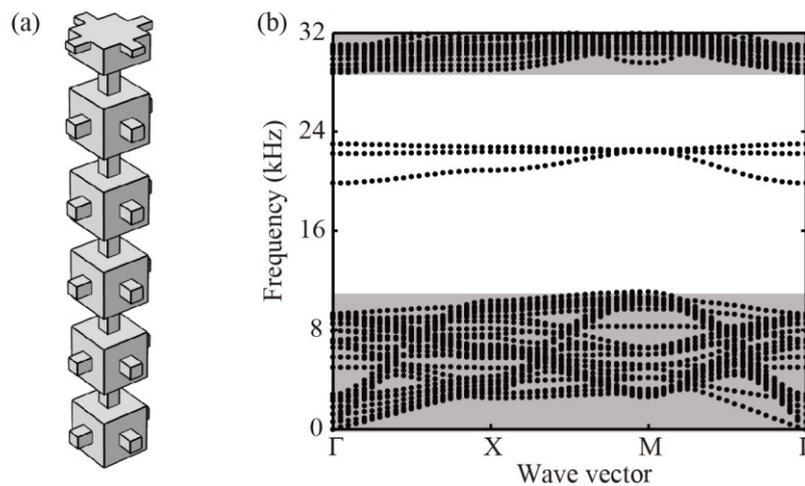


Figure 2 – (a) Scheme of the PnC super-cell for the surface modes. (b) Band structure of the PnC super-cell, where the frequency ranges of the bulk modes are indicated by the shadowed zones.

Similar to the case of surface modes, we terminate the 3D PnC at the top, bottom, front and back surfaces in order to investigate the elastic edge modes. The scheme of the super-cell is plotted in Fig. 3(a), where the super-cell consists of $1 \times 5 \times 5$ unit-cells. The band structure of the super-cell is shown in Fig. 3(b). One can see four bands corresponding to the edge modes inside the bandgap, where the lowest one is close to the bulk modes and is neglected in the following discussion. The other edge modes are of the frequency ranges [21.8 kHz, 22.0 kHz], [22.1 kHz, 24.0 kHz], and [26.7 kHz, 28.7

kHz], respectively. Such edge modes are confined at the edges of the PnC due to the bandgaps for the bulk and surface modes.

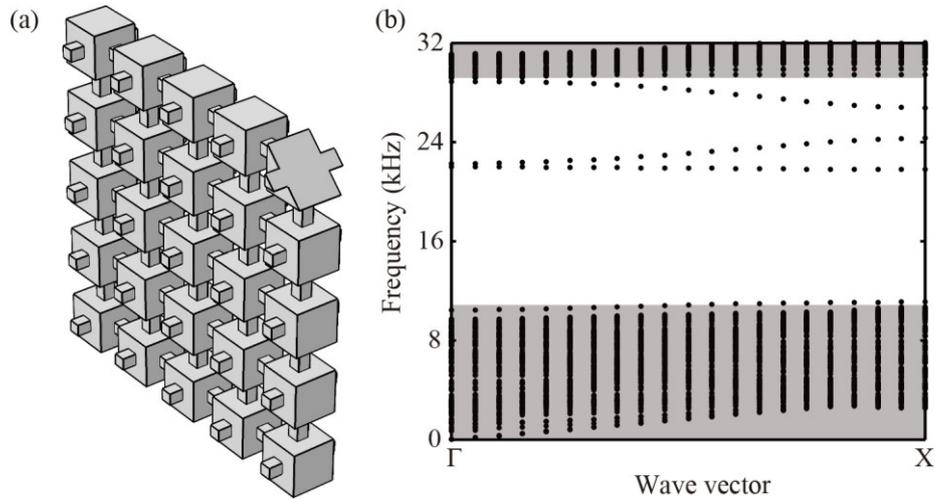


Figure 3 –(a) Scheme of the PnC super-cell for the edge modes. (b) Band structure of the PnC super-cell, where the frequency ranges of the bulk modes are indicated by the shadowed zones.

In the experiments, the samples are fabricated by 3D-printing, where the finite-size PnCs with $6 \times 7 \times 7$ and $7 \times 4 \times 4$ unit-cells are utilized for investigating the surface and edge modes, respectively. The excitations (out-of-plane polarization) are generated by the piezoelectric patches attached on the samples. And the reference points are set near the sources. The out-of-plane displacements at the reference and detection points are collected by the Polytec PSV-500 scanning vibrometer. The transmission coefficient T is evaluated as the ratio of the displacement amplitudes between the detection and reference points, i.e., $T = 20\log_{10}(|u_2|/|u_1|)$, where the subscripts 1 and 2 denote the reference and detection points, respectively. The experimental transmission coefficients are shown in Fig. 4. The pass bands for the surface(edge) modes obtained by the band structures are highlighted by the transparent blue zones. It is seen that the transmission peaks appear in the frequency ranges of the pass bands (the transparent blue zones), which correspond to the elastic surface (Fig. 4(a)) or edge (Fig. 4(b)) modes. Moreover, the transmitted bulk modes (especially below 10 kHz), as well as the bandgaps, can be identified in the transmission spectra. Therefore, the experimental results show good agreement with the theoretical predictions.

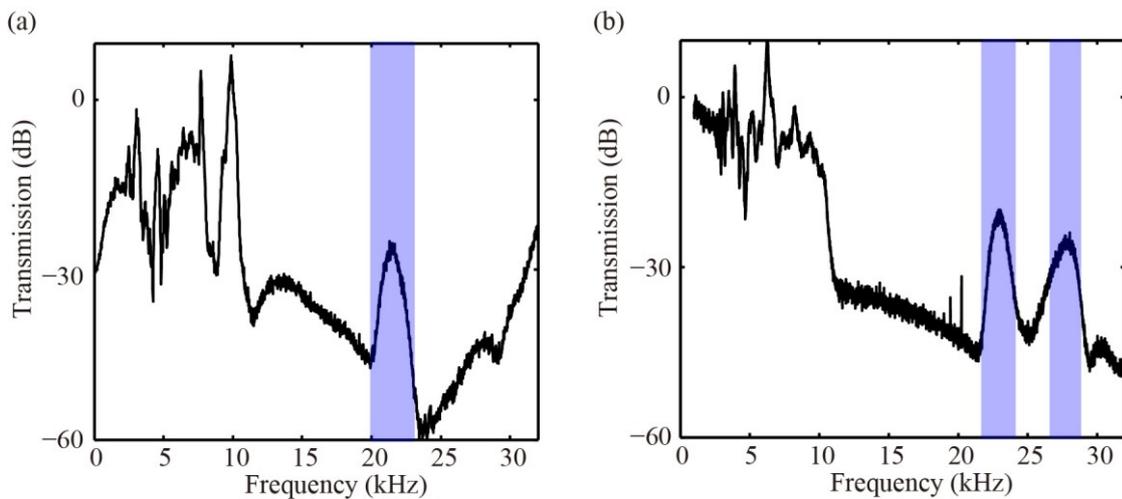


Figure 4 –Transmission spectra for the (a) surface and (b) edge modes, where the pass bands for the surface or edge modes are indicated by the transparent blue zones.

4. CONCLUSIONS

By appropriately terminating the 3D PnCs, the elastic waves can be guided at the surface and/or the edge of the finite-size crystals. The 3D-printed samples are experimentally tested to assess the transmission spectra of surface and edge modes. And the experimental results show good agreement with the theoretical predictions. In the future, the bulk, surface and edge components may cooperate with others in complex 3D PnC circuits. The surface and edge modes can also be utilized for sensing applications.

ACKNOWLEDGEMENTS

The work was supported by the German Research Foundation (DFG, ZH 15/27-1), and the Joint Sino-German Research Project (Grant No. GZ 1355).

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