

## Prediction of atmospheric sound propagation subject to parameter variability of atmospheric turbulence

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### Abstract

As part of the project „WEA-Akzeptanz“ an atmospheric sound propagation model is developed to predict the sound immission from a wind turbine. The CNPE (Crank Nicolson Parabolic Equation) method, based on the Helmholtz equation is chosen as the sound propagation model. Complex environmental conditions such as atmospheric turbulence can be implemented with a random number generator in this model. Atmospheric turbulence is characterized by temporal fluctuating wind speed and temperature, which has an impact on the sound propagation and leads to scattered sound waves due to the turbulent eddies.

Accordingly, the input of the model contains parameter variability. As a result, the sound pressure level at the point of immission also has a high variability and is not entirely deterministic. In this work, the influence of random input variables due to atmospheric turbulence on the uncertainty of the resulting sound pressure level is investigated. Using this, the probability of the predicted sound pressure level can be quantified to get a better idea of sound immission under complex atmospheric conditions.

Keywords: Sound Propagation, Atmosphere, Uncertainty

### 1 INTRODUCTION

The propagation of sound in the atmosphere is strongly influenced by turbulent fluctuations of temperature or wind speed. The impact of turbulence becomes particularly relevant in the presence of refractive shadow zones, which occur in times of negative temperature gradients or upwind directions. A reliable prediction of the sound pressure level is of interest for example to residents near industrial plants, traffic routes or wind turbines in order to keep noise pollution at a minimum. Hence, this paper analyses the probability of sound pressure levels occurring at an immission point. These sound pressure levels are subject to variability due to the randomness of turbulent fluctuations.

This paper is based on simulations with the Parabolic Equation (PE) Method to predict the sound propagation in the atmosphere. The PE method originating in electromagnetic wave propagation was transferred to the sound propagation in the atmosphere by Gilbert and White [1] in 1989. In order to predict the relationship between the variability of sound pressure level in refractive shadow zones and turbulent fluctuation, Cotté and Bland-Benon [2] investigated the coupling between turbulent scales, geometries and acoustic frequencies. Further, Ostashev and Wilson [3] analyzed the statistical properties of sound pressure levels due to the influence of temperature and wind speed variations for vertical and slanted sound propagation including the height dependence of the variance and length scale of turbulence.

As a further investigation, this paper examines the uncertainty of the prediction of sound propagation due to atmospheric turbulence in refractive shadow zones. The uncertainty in sound pressure levels and the probability of the occurrence of higher sound pressure levels due to turbulent fluctuations are investigated. Thus, the prediction of sound pressure levels in turbulent atmospheres can benefit in the future, e.g. to provide more precise information for compliance with noise protection.

## 2 ATMOSPHERIC SOUND PROPAGATION

Sound propagation in the atmosphere is affected by meteorological conditions. In particular, those conditions include changes in temperature, density, wind speeds and turbulence. The effects on the sound propagation are particularly strong when spread over large distances.

As discussed in Lerch et al. [4], in the atmosphere, the speed of sound  $c_0$  is superimposed with the wind speed  $v_w$ . Hence, the effective speed of sound can be derived. In the atmosphere, the wind speed increases with the height. This height dependence of the wind speed is transferred to the effective speed of sound. Consequently, a refraction of the sound waves occurs. In Figure 1 this geometric effect is visualized in upwind direction using sound rays. Due to subtraction of the wind speed in an upwind situation, the effective speed of sound decreases with increasing height. Accordingly, the sound waves are refracted upwards and refractive shadow zones arise near the ground. In those refractive shadow zones sound pressure levels are reduced.

As a result of unstable atmospheric stratification, turbulence arises due to buoyancy instabilities and wind shear. In addition to amplitude and phase fluctuations of the sound waves, these atmospheric turbulence leads to the scattering of sound energy into refracted shadow zones [5]. The scattering of the sound waves is shown in Figure 1. According to Rossing et al. [6], the reduction of sound pressure levels within the shadow zone is limited to 20-25 dB due to scattered sound by atmospheric turbulence.

## 3 DEVELOPMENT OF SOUND PROPAGATION MODEL

According to West et al. [7] and Salomons [8], the PE method is an appropriate choice for the numerical calculation of a sound field due to the various advantages, such as neglectation of back-scattering and good ways to implement meteorological effects. Precisely, the sound propagation model presented here is based on calculations using Crank Nicolson method of finite difference method, called the Crank Nicolson Parabolic Equation (CNPE) method. This method focuses only on the direction of propagation, so that it is efficient and also suitable for the calculation of large sound fields in free field. The PE method is based on the Helmholtz equation, which is reduced from 3D to 2D by assuming axial symmetry:

$$\frac{\partial^2 q}{\partial r^2} + \frac{\partial^2 q}{\partial z^2} + k_{\text{eff}}^2 q = 0, \quad (1)$$

with the effective wave number  $k_{\text{eff}} = \omega/c_{\text{eff}}$ , the frequency  $\omega$  and  $c_{\text{eff}} = c_0 \pm v_w$  in the sound field  $q$ . The used cylindrical coordinates  $r$  and  $z$  are shown in Figure 2. It is assumed that the sound field is excited by a monopole sound source and that the atmospheric wind is approximated by the effective speed of sound as shown in Salomons [8].

The discretization of the calculation domain is done by the use of an equidistant grid. The distance of the grid points is one tenth of the wavelength each ( $\Delta z = \Delta r = \lambda/10$ ). In Figure 2, the calculation domain is shown in a

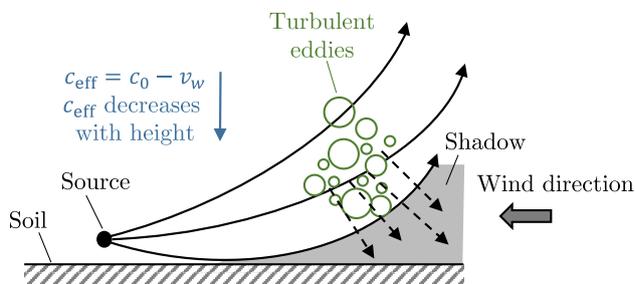


Figure 1. Refraction of sound waves and scattering at turbulent eddies into the shadow zone.

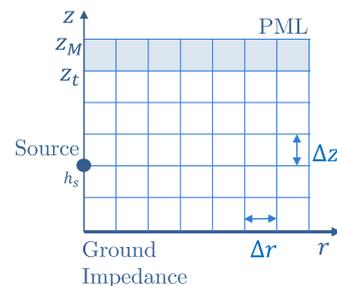


Figure 2. Sound field with discretization and boundary conditions.

principle sketch with the discretization and boundary conditions. The boundary conditions at the lower boundary of the calculation domain is defined by the acoustic impedance  $Z$ . The acoustic impedance varies between fully reflective ( $Z \rightarrow \infty$ ) and partially to fully absorbing ( $Z = \rho c_0$ , with the density  $\rho$  of the absorbing material). A Perfectly Matched Layer (PML) is used at the upper boundary of the calculation domain to ensure complete absorption of the sound waves and simulating a free field. In the PML, the wavenumber  $k$  is supplemented by the imaginary part  $k_i$ :

$$k = \frac{c_{\text{eff}}}{\omega} + k_i \quad \text{with} \quad k_i = iA_t(z - z_t)^2 / (z_M - z_t)^2. \quad (2)$$

The used heights  $z_M$  and  $z_t$  in the calculation of  $k_i$  are shown in Figure 2. The absorbance  $A_t$  is frequency dependent. The imaginary part of the wave number causes the amplitude to be attenuated. Due to neglect of backscattering, the definition of a boundary conditions at the left and right boundary of the calculation domain is not necessary. The calculation of the sound field  $q$  starts at position  $(0, z)$ . The sound field is computed by a step-wise extrapolation of  $q(r, z) \rightarrow q(r + \Delta r, z)$ . The wave number  $k$  is independent of the propagation direction  $r$  and a function of the height  $z$ . Due to that, a variation of the effective speed of sound with the height exists enabling the implementation of a refractive atmosphere. A detailed description of the sound field calculation can be found in West et al. [7] and Salomons [8]. The sound propagation model is verified by a benchmark case of Attenborough et al. [9] which is described in Hörmeyer et al. [10].

### 3.1 Implementation of Turbulence

Equivalent to the effective speed of sound  $c_{\text{eff}}$ , the (acoustic) refractive index  $n$  is calculated as follows

$$n = \bar{n} + \mu = \frac{c_0}{c_{\text{eff}}}. \quad (3)$$

The refractive index in the turbulent atmosphere varies at each point around an average value  $\bar{n}$ . The turbulent fluctuations are called  $\mu$  and  $c_0$  describes a reference value of the speed of sound.

Turbulent fields in the atmosphere can be characterized with the Gaussian correlation function  $B(R)$  as follows:

$$B(R) = \mu^2 e^{\left(\frac{-R^2}{a^2}\right)}, \quad (4)$$

$$\text{with} \quad \mu^2 = \frac{\sigma_T^2}{4T_0^2} + \frac{\sigma_w^2}{c_0^2}. \quad (5)$$

To ensure that the use of the Gaussian correlation function is valid, the turbulent spectrum is assumed to be homogeneous and isotropic and is only a „snapshot“ of the time-varying turbulent field [5].

According to Salomons [8], the correlation length in atmospheric turbulence is 1m. The two-dimensional turbulent field is described in  $R$ . As stated in equation 5, the variance of temperature  $\sigma_T^2$  and wind speed  $\sigma_w^2$  are related to reference values of temperature and speed of sound. For these reference values  $T_0 = 273$  K and  $c_0 = 330$  m/s are used. From a Fourier transformation of the Gaussian correlation function  $B(R)$  the spectral density of a homogeneous random function can be derived. The turbulence is taken into account by multiplying the sound field by a height-dependent phase factor  $e^{(ik\mu\Delta r)}$  after each calculation step.

## 4 PREDICTION OF SOUND PROPAGATION IN TURBULENT ATMOSPHERE

The randomness of turbulence in the atmosphere leads equally to a randomness of the sound pressure level at an immission point. For this reason, it is of interest to investigate the uncertainty in the prediction of the sound pressure level and the probability of reaching a certain sound pressure level at an immission point. Therefore, the randomness of the turbulence is first determined and second, the distribution of the occurring sound pressure levels is examined. The frequency dependence of the previously determined results is evaluated afterwards.

#### 4.1 Influence of Turbulence on Sound Propagation

The simulations of the sound propagation are performed at a frequency of  $f = 250\text{Hz}$  with a source height of  $h_s = 5\text{m}$ , a receiver height of  $h_r = 1\text{m}$  and up to a distance of  $D = 1500\text{m}$ . For the variance of wind speed fluctuations  $\sigma_w^2 = 1.75 \cdot 10^{-5} \text{m}^2/\text{s}^2$  is used, based on literature values of Salomons [8]. The term transmission loss (TL) is used in the following to describe the decrease of the sound pressure level over the distance. The sound source is excited with 0dB to represent the relative ratio of the decrease of the sound pressure level. Accordingly, the relative sound pressure level  $\Delta\text{SPL}$  is also used in the following.

Figure 3 shows the transmission loss in the direction of propagation in an upward refracting atmosphere. In a non-turbulent and upward refracting atmosphere, a refractive shadow zone is formed. As a result, the sound pressure level decreases sharply over short distances. If turbulence are present, the relative sound pressure level in the refractive shadow zone increases and the amplitude fluctuates over the distance.

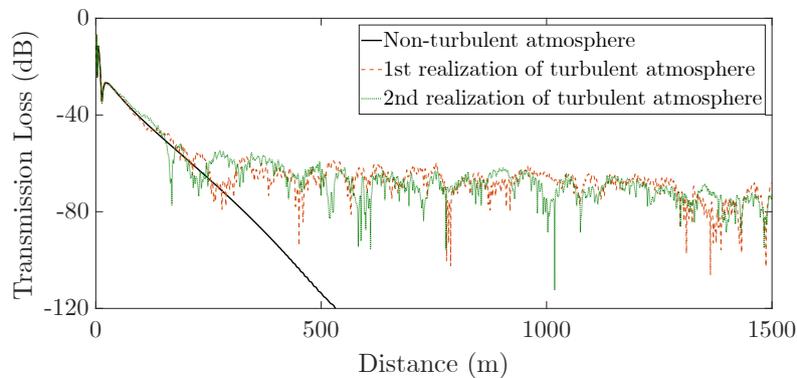


Figure 3. Transmission loss in direction of propagation in an upward refracted atmosphere.

The turbulent atmosphere is considered as frozen. Therefore, one realization of the simulation is a snapshot of the turbulent field. Due to randomness of turbulence, sound pressure level also fluctuates in time and space as introduced in Ostashev and Wilson [5]. This is shown in Figure 3 as a comparison of a first and a second realization of the randomness of turbulence. As shown, there are clear variances of the sound pressure level within the two realizations, indicating the variability of sound pressure level due to randomness of turbulence.

Figure 4 (left) shows the transmission loss in a non-turbulent, upward refracting atmosphere. The refractive shadow zone is formed at a distance of about 300m. In the refractive shadow zone, the sound pressure level is significantly lower. The sound propagation in a turbulent, upward refracting atmosphere is shown in Figure 4 (right). The sound waves are scattered clearly into the refractive shadow zone due to the atmospheric turbulence. Consequently, a significantly higher and fluctuating sound pressure level can be expected in the refractive shadow zone.

#### 4.2 Distribution of Transmission Loss

The high variability in the sound propagation and therefore in the sound pressure level at a point of immission due to the randomness of turbulence in the atmosphere results in uncertainties in the sound pressure levels in the refractive shadow zone. To quantify this uncertainty, the histogram of the frequency of occurrence of the relative sound pressure levels is shown in Figure 5 as an example for an acoustic frequency of 250Hz in a distance of 500m with a sample rate of 5000 realizations. As shown, the average relative sound pressure level of the transmission loss is about 66dB. The relative sound pressure levels higher than the average are less scattered with high occurrence probabilities. Relative sound pressure levels lower than the average, on the other hand, are widely scattered and have a lower probability of occurrence.

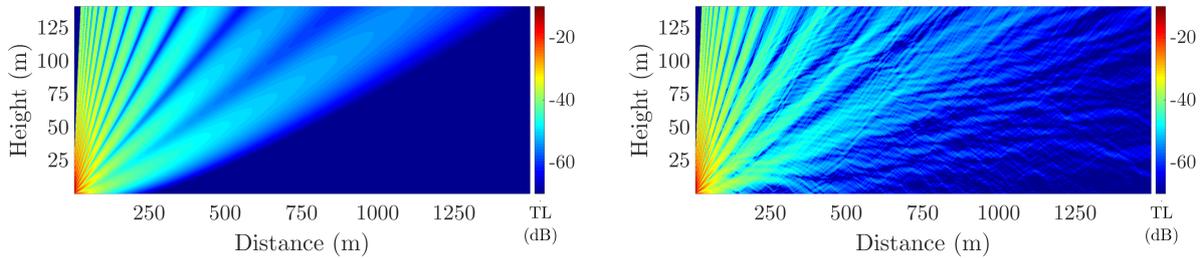


Figure 4. Transmission loss in non-turbulent (left) and turbulent (right) upward refracting atmosphere at 250Hz.

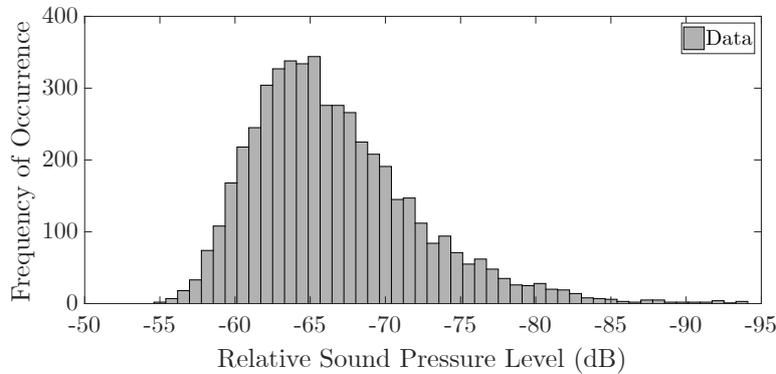


Figure 5. Frequency of occurrence of relative sound pressure level in distance of 500m in turbulent atmosphere with a sample rate of  $n = 5000$ .

Since higher relative sound pressure levels and the likelihood of reaching higher relative sound pressure levels are more relevant for predicting noise immissions, only the part of the histogram up to the mean value is considered for the following investigations. To evaluate the results, a distribution function is fitted to the histogram using a maximum likelihood estimation. Several distributions (e.g. normal, logarithmic normal, extreme value, etc.) are tested and the goodness of their fits are evaluated using a  $\chi^2$ -test. Since a logarithmic normal distribution can represent the data up to the mean value, it is chosen for further investigations. It should be mentioned that for all fits positive relative sound pressure levels are assumed to enable for example lognormal fits. The histogram truncated at the mean value of the data with the fitted lognormal distribution is shown in Figure 6 as an example for an acoustic frequency of 250Hz. The probability density function of the lognormal distribution is given by

$$f(x) = \frac{1}{\sigma x \sqrt{2\pi}} \cdot e^{-\frac{(\ln(x) - \mu)^2}{2\sigma^2}}, \quad x > 0 \quad (6)$$

with the mean value  $\mu$  and the standard deviation  $\sigma$  of the underlying normal distribution [11].

### 4.3 Frequency Dependence

To investigate the frequency dependence of scattering, the distributions of relative sound pressure levels are analyzed in frequencies from 125Hz to 2000Hz. The fitted lognormal distributions for all analyzed frequencies

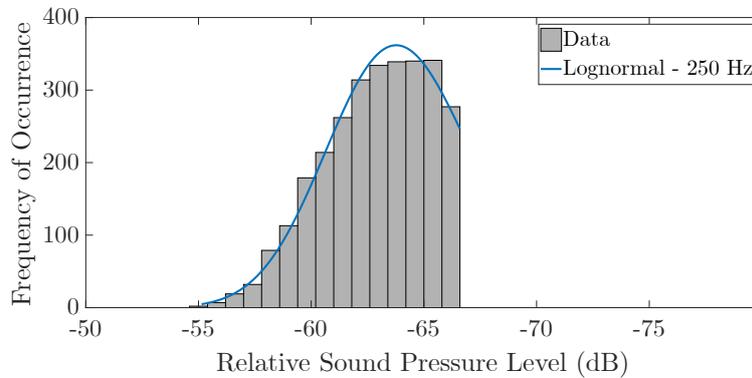


Figure 6. Frequency of occurrence of relative sound pressure level in distance of 500m in turbulent atmosphere with a sample rate of  $n = 5000$  and distribution fit of data truncated at the mean value.

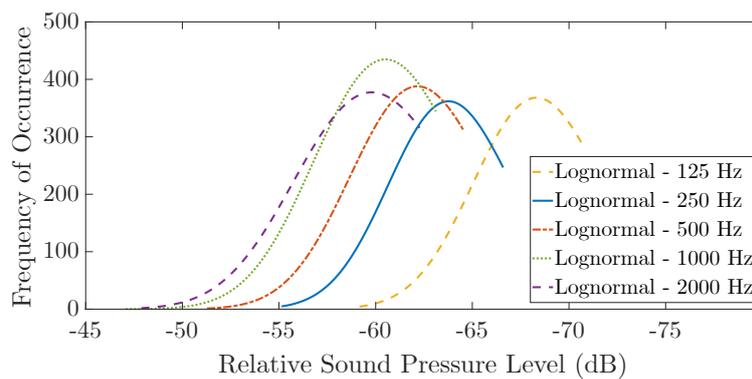


Figure 7. Frequency of occurrence of relative sound pressure level in distance of 500m in turbulent atmosphere with a sample rate of  $n = 5000$  and distribution fit of data truncated at the mean value for analyzed frequencies.

are shown in Figure 7. It can be observed that with increasing frequency the relative sound pressure level of the mode, the global maximum of the distribution function, increases from approximately  $-68$  dB for 125 Hz to about  $-59$  dB for 2000 Hz. Likewise, the distance between the mode decreases with increasing frequency thus a converging behavior can be observed.

For detailed description, the relationship between mode and standard deviation is plotted in Figure 8. The mode of the lognormal distribution is determined by  $e^{(\mu - \sigma^2)}$ . As before, the mode increases with increasing frequency and converges. In addition, the standard deviation increases with increasing frequency, indicating that the histograms of sound pressure levels are more scattered.

As shown in Figure 8, the relative sound pressure levels increase with increasing frequency. Because of shorter wavelength, the scattering of the sound waves is much greater than at a lower frequency with long wavelengths. To illustrate this, Figure 9 shows the relative sound pressure level over distance and height of the sound field for 125 Hz (left) and for 2000 Hz (right) in turbulent upward refracting atmosphere. The effect of sound scattering as

a function of wavelength is clearly visible. At 2000Hz, the relative sound propagation is much more affected by the scattering and there is a strongly inconsistent propagation influenced by the turbulence. Higher frequencies are therefore more affected by sound scattering.

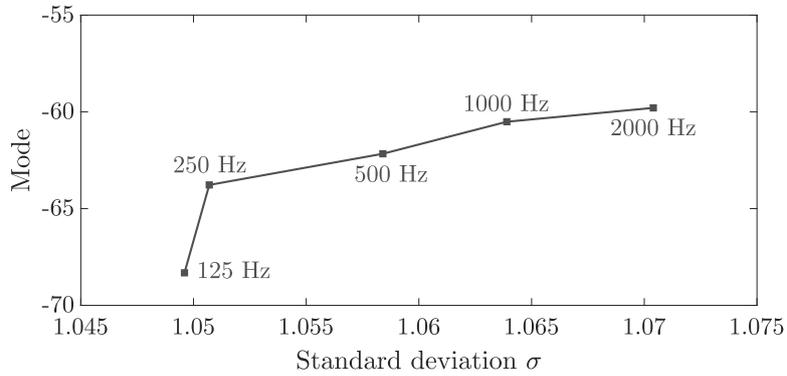


Figure 8. Ratio of mode and standard deviation of the lognormal distributions for analyzed frequencies.

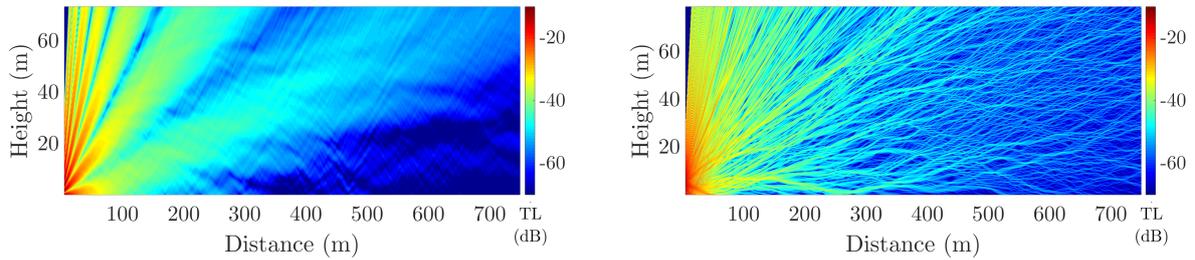


Figure 9. Transmission loss in turbulent upward refracting atmosphere at 125 Hz (left) and at 2000 Hz (right).

## 5 CONCLUSIONS

In this paper, the influence of sound scattering by atmospheric turbulences on sound propagation was investigated. In addition, the sound pressure levels varying due to turbulence in refractive shadow zones were analyzed using numerical simulations. The sound pressure levels at an immission point are subject to temporal fluctuations due to sound scattering caused by atmospheric turbulence. In order to investigate the value range of the sound pressure levels and the probability of the occurrence of high sound pressure levels, samples were represented in a histogram.

The distribution of the sound pressure levels can be approximated with a lognormal distribution. The frequency-dependent analysis of the distributions shows that the sound pressure levels increase with higher frequencies. This can be explained by the fact that sound scattering due to atmospheric turbulence has a stronger influence on higher frequencies due to shorter wavelengths. In addition to uncertainties due to the varying sound pressure levels caused by turbulence fluctuations, uncertainties in the distribution of sound pressure levels in the frequency domain exists.

In future work, the superposition of atmospheric absorption, sound scattering effects and distance attenuation will be considered in order to investigate the effects on sound propagation. In addition, the effects of different atmospheric stratifications will be examined to make a weather-dependent statement about the reliability of predicting the sound propagation.

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