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Trained Algorithms for Mode Decomposition in Ducts

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ABSTRACT

A new method to split acoustic fields in flow-ducts into upstream and downstream propagating sound waves is presented. Goal of this new approach to modal decomposition is to create a methodology that does not rely on semi-analytical solutions of the sound propagation and is, hence, applicable to a much broader set of aeroacoustic problems. An artificial neural network is trained with data gained from a small set of numeric reference solutions of the linearized Navier-Stokes equation in the frequency domain in a straight duct. The network is tested on relevant experimental data. A very good agreement with the analytical solution is demonstrated in two applications with different flow conditions. The good results obtained for the test cases in this paper make this method a promising addition to the classical methodology;

Keywords: Flow acoustics, two-port, neural network, duct acoustics, modal decomposition

1. INTRODUCTION

Sound created in flow-ducts is a strong contributor to perceived noise disturbance from, for example, ventilation systems [1] and intake and exhaust systems [2]. In order to develop and improve noise mitigation strategies, reliable models to describe the sound propagation inside such systems are needed. Established methodologies, such as the multi-port method [3] and the mode matching method [4], use analytical and semi-analytical expressions the acoustic wave shape and the acoustic wave propagation [5], often corrected for thermo-viscous dissipation [6]. That limits the methods to hard walled duct systems with well-defined flow conditions.

In this conference contribution, we present a new approach to analyze in-duct sound without the need for semi-analytical solutions. Instead, we develop an algorithm that can learn the acoustic propagation of plane waves from a small set of numerical reference solutions of the linearized Navier-Stokes equation in the frequency domain in a straight duct.

1.1 Low Frequency Propagation of Acoustic Waves in Flow-ducts

In many relevant aero-acoustic applications where in-duct sound is important the acoustic waves are linear, time-invariant, and stationary. The acoustic field is then a superposition of many tonal components and each tonal component fulfills a convective wave equation in the frequency domain, e.g. the Helmholtz equation

$$(\nabla^2 + k(\omega, X)^2)p(\omega, X) = 0. \tag{1}$$

Here, p is the acoustic pressure, k is the acoustic wave number, ∇ is the Nabla-Operator, ω is the angular frequency, and X = (x,y,z) is a coordinate vector. For a soundwave propagating in a straight hard-walled duct with stationary, uniform background flow, Equation 1 may be solved analytically. At frequencies lower than the cut-on frequency where only planar acoustic modes can propagate, the solution can be expressed as

$$p(\omega, z) = P_+(\omega)e^{ik_+z} + P_-(\omega)e^{-ik_-z}. \tag{2}$$

Equation 2 describes a superposition of two opposed plane waves with constant pressure in a cross-section. The two waves propagate in upstream (-) and downstream (+) direction. They have a complex amplitude P_+ and P_- and the complex wave number k_+ and k_- as a function of frequency, fluid-flow conditions, and duct geometry.

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1.2 Mode Decomposition

If a sound field is sampled along the duct, e.g. with microphones mounted along the duct walls, a common task in duct acoustics is to split this sound field into its upstream and downstream propagating waves. With the analytical description of the wave, the complex wave amplitudes can be solved from at least two samples of the acoustic field (p1 and p2 at z1 and z2). Equation 2 is therefore written in vector notation

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} e^{ik_+z_1} & e^{-ik_-z_1} \\ e^{ik_+z_2} & e^{-ik_-z_2} \end{bmatrix} \begin{bmatrix} p_+ \\ p_- \end{bmatrix} = \mathbf{M}\, p_\pm\,. \tag{3}$$

The matrix **M** is called the modal matrix. More detailed information about the modal decomposition and its application can be found in Ref [7–9]. In the classical mode decomposition, **M** is inverted and multiplied with the sample's pressure to solve for the complex mode amplitudes p_+ and p_-. The system can become ill-conditioned and therefore ambiguous in its solution, for example at tones of certain wavelengths [10] or close to the cut-on frequencies of higher order modes [7]. Consequently, **M** is commonly over-determined by sampling the sound field at more than 2 positions. Challenges are still associated with the analytical solutions of the wave shape and propagation for example at high frequencies [11] and background flows with greater Mach-numbers (M>=0.3) [12]. In fact, the background flow can deform the acoustic modes and influence the propagation speed, which adds uncertainties in the form of bias errors if the classical mode decomposition (Equation 3) is used.

1.3 Learning Algorithm

Regression problems such as the mode decomposition in Equation 3 can be solved with artificial neural networks (ANN) [13]. In theory, a network with a single layer between input and output can learn any desired deterministic regression function, provided that a sufficient number of neurons and a representative set of training data is available ("universal approximation theorem" [14]). The goal of this paper is to show that an ANN can learn the modal decomposition in Equation 3. The algorithm learns the acoustic mode shapes and propagation from numerical simulations without the need for explicit knowledge of the governing equations. This allows mode decomposition in complex flows and arbitrary duct geometries. The numerical scheme can include turbulent flow and boundary layer effects and hence cover for flow-related acoustic dissipation.

The ANN is implemented as a Feed Forward Back Propagation topology with a single fully connected hidden layer between input and output. We split the complex numbers of the sampled pressure into a real and an imaginary part and add those as separate input for the model. We use the same pressure probe number and positions as in our experimental setups. The input is therefore a row vector of 7 elements, namely 3 elements for the real part of sampled pressure transfer functions at 3 microphone positions, 3 elements for the imaginary part of the sampled pressure transfer function at 3 microphone positions, and the frequency. The output is a row vector with four elements in the form of the real and imaginary part of p_+ and p_-.

To create the topology of the neural network, we have tested different numbers of neurons in the hidden layer, namely 10, 30, 50, 100, 200, 500, 1000, 2000, 4000, and 8000 and the solution started to converge at around 1000 neurons. For the results presented in the paper, we use 2000 neurons.

Equation 3 is linear only if a single frequency is considered. For a frequency-dependent modal decomposition, the regression is non-linear. We address this by adding non-linear elements in the form of normalized exponential ("softmax") activation functions to the weights of the neurons in the hidden layers. The input has no activation functions, and the output has linear activation functions.

1.4 Training Data

The data needed to train the neural network was generated from a small set of numeric solutions. We solved the linearized Navier-Stokes equation (LNSE) in the frequency domain for empty duct sections with reflection-free determinations. For each training frequency, we simulated two sound fields, one for plane wave excitation on the upstream and one for plane wave excitation on the downstream side. The training data could then be generated by superposing phase-shifted and scaled versions of the two reflection-free reference solutions. This approach allowed us to create theoretically infinitely large training and validation data sets. We trained the ANN on data in 100 Hz steps between 300 and 2600 Hz with 200 000 training samples per frequency. The method is applicable for linear and time-invariant wave propagation. This is true in many applications with sound pressure being lower than 155 dB (1% of atmospheric pressure) [15].

The numeric solutions were generated in two steps. First, the flow field was computed solving Reynolds Averaged Navier-Stokes (RANS) equations. In a second step, the acoustic field was computed linearized around the stationary mean background flow. We retained all terms containing viscosity and thermal conductivity, so that the corresponding acoustic dissipation was included in the training data. The computational domain was inherited from Ref [16] (Figure 1), where good performance of this method was shown.

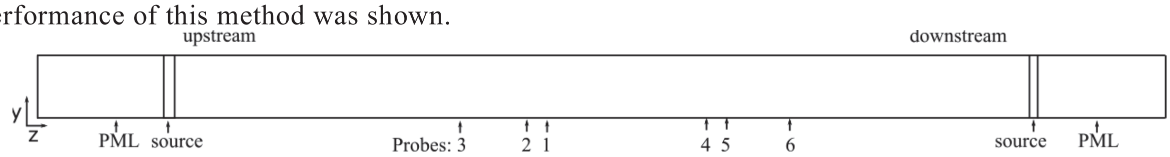

Figure 1: Computational domain for empty duct section with reflection-free perfectly matched layers (PML), sources, and six microphone probes.

1.5 Algorithm Testing

The algorithm was tested on experimental data measured as part of the IFAR Acoustic Liner Challenge at the 2018 AIAA Conference. The IFAR was a NASA initiative where 3D-printed acoustic liner configurations were measured on multiple test rigs with the aim to collect and compare the results. Data was shared with each participant in order to assess how fabrication, data acquisition and analysis approach influence the results.

Measurements of the acoustic properties of liners are common tasks in aeroacoustics. We therefore consider the data as a relevant test case for our ANN. To evaluate the results, we plot the scattering matrix, which is an anechoic representation of the passive acoustic properties of the liner. To this aim, we decompose the sound field upstream and downstream of the liner and compute the sound transmission and reflection. The scattering matrix is defined as

$$\begin{pmatrix} p_{u+} \\ p_{d+} \end{pmatrix} = \begin{bmatrix} \rho_d & \tau_{du} \\ \tau_{ud} & \rho_u \end{bmatrix} \begin{pmatrix} p_{u-} \\ p_{d-} \end{pmatrix} \quad , \tag{4}$$

where p_{u-} and p_{d-} are amplitudes of plane pressure waves incident of the lined section, p_{u+} and p_{d+} are amplitudes of palne pressure waves outgoing from the sample, and ρ_d and ρ_u are reflection coefficients with $_u$ and $_d$ denoting upstream and downstream, respectively. τ_{du} and τ_{ud} are transmission coefficients from the downstream to the upstream and the upstream to the downstream side. The procedures to extract the scattering with the multi-port method and to perform the necessary measurements are explained in Ref [7].

The experiments for this paper were carried out at the KTH modular advanced flow-rig facilities, using 6 microphones and 4 loudspeakers. Sound fields were excited upstream and downstream of the liner using the four sources simultaneously, each playing a different sinusoidal tone between 300 and 2600 Hz in 20 Hz steps and we measured the frequency response functions between the microphones and the loudspeaker reference signals. Note, that we have chosen a finer frequency resolution for the experiment than for the data included in the training data set. We can therefore evaluate the ability of the ANN to learn the actual regression functions by testing frequency points that were not included in the training data set.

In this paper, we compare scattering data gained from the experiments, using i) the two-port method with mode decomposition based on analytical solutions of the wave equation and ii) the learning algorithm trained on numerical data. We test two different flow conditions. Firstly, a case without flow (M=0) and secondly a case with moderate flow (centerline Mach number M=0.3).

2. Results and Discussion

The scattering matrix of the tested liner is presented in Figure 2 and 3 for the no flow (M=0) case, and in Figure 4 and 5 for the moderate (M=0.3) flow. The performance of the neural network compared to the analytical modal decomposition is presented.

For the no flow (M=0) case, the wave propagation inside the straight duct is governed accurately by the Helmholtz equation and the analytical solution can therefore be used for quality assessment. We found, that the agreement of the analytical solution and the neural network is distinguished in the mid-frequency range between 750 Hz and 2100 Hz for both, the amplitude and the phase (Figure 2 and 3). For low and high frequencies, the accuracy of the neural network decreases for the amplitude, but remains of good quality for the phase. This indicates a general good decomposition of incident and outgoing waves on each side over the entire frequency range, but a bias in the amplitude of both p+ and p- waves on each side towards the limits of the trained frequencies.

We could find similar agreement for the moderate flow case (M=0.3) depicted in Figure 4 and 5. All important features of the scattering matrix are captured with good accuracy. The neural network furthermore approximated the scattering matrix smoother than the analytical solution which becomes apparent for the noisier upstream reflection coefficients.

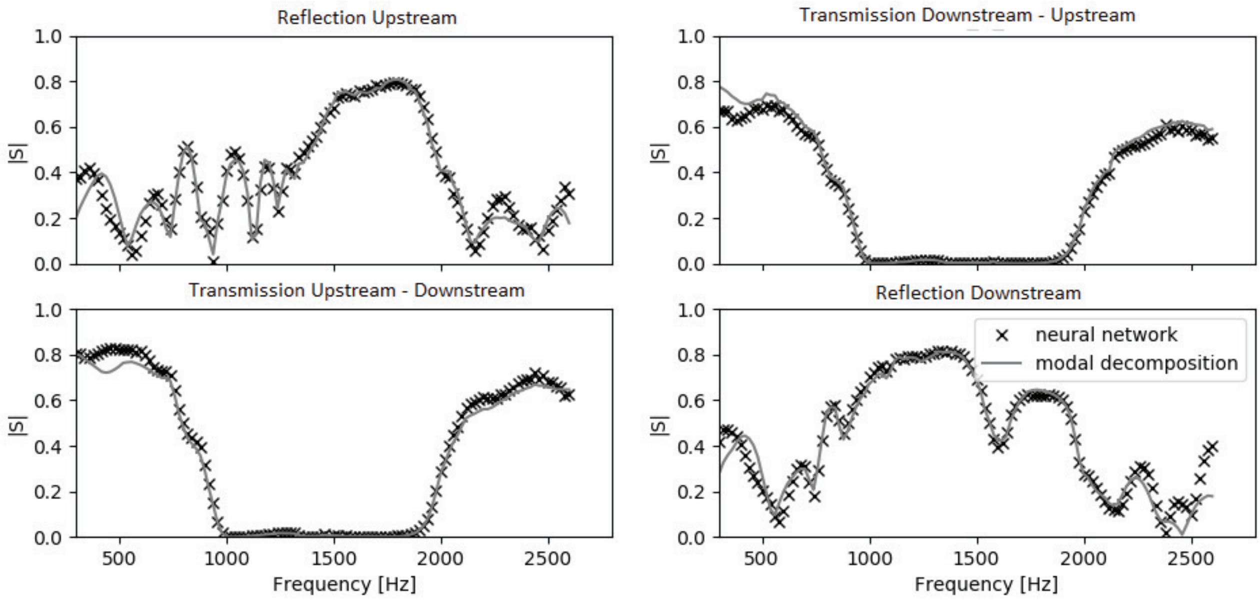

Figure 2: Performance of neural network compared to analytical solution for M= 0. Amplitude of the scattering matrix of the tested liner.

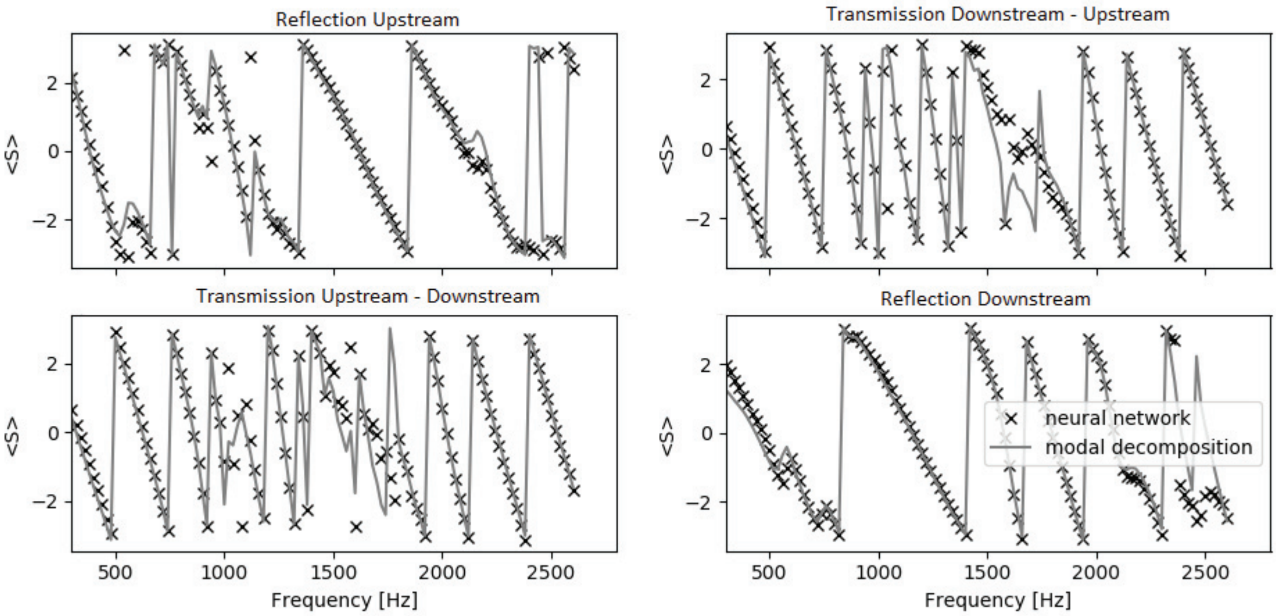

Figure 3: Performance of neural network compared to analytical solution for M= 0. Phase of the scattering matrix of the tested liner.

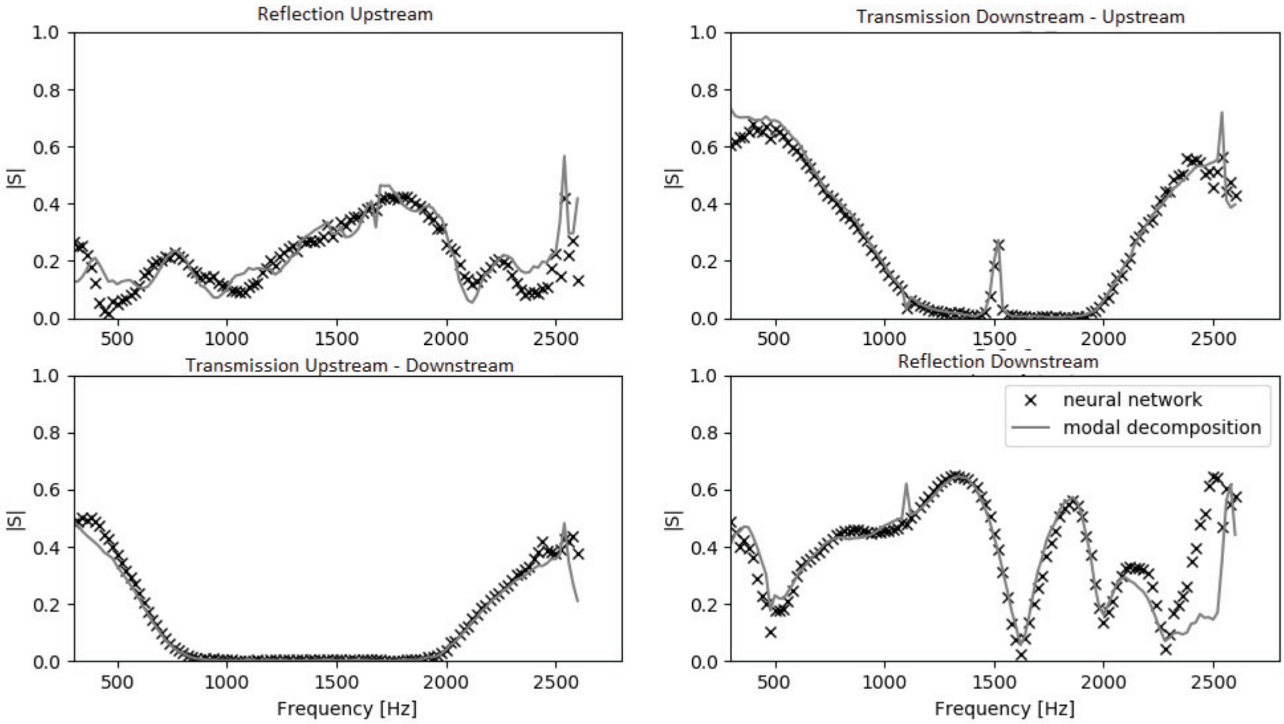

Figure 4: Performance of neural network compared to analytical solution for M= 0.3. Amplitude of the scattering matrix of the tested liner.

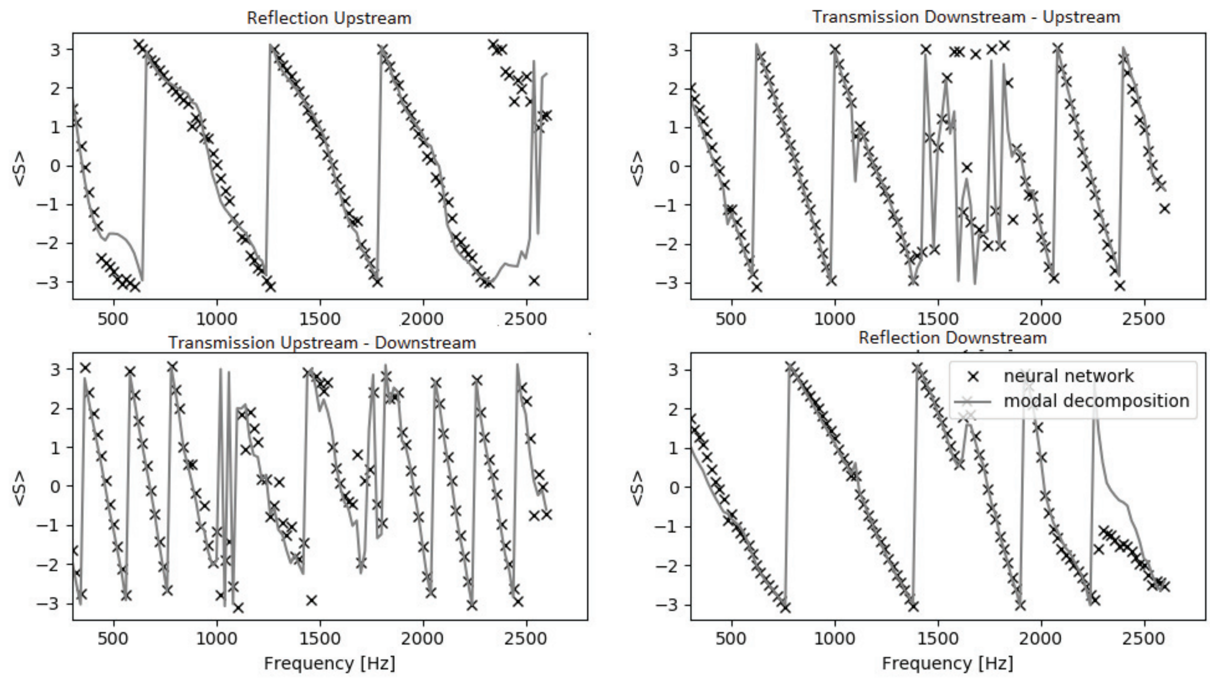

Figure 5: Performance of neural network compared to analytical solution for M= 0.3. Phase of the scattering matrix of the tested liner.

3. Conclusion

We present a new method to decompose acoustic fields in ducts into upstream and downstream propagating plane waves. The novelty of the presented methodology is the algorithm's ability to learn the decomposition without explicit knowledge of the governing equations. This makes the method applicable to experiments where the analytical solution of the wave propagation is unavailable. This applies for example to duct systems with complicated cross-sections, soft walls, and high flow velocities.

We developed a learning algorithm that can be trained with numerical data. The algorithm contains a single layer feed forward - back propagation artificial neural network. The data needed to train the network is created from two reference solutions of the Linearized Navier-Stokes equation in a straight duct, solved for a number of training frequencies. Such computations are generally computational inexpensive although they contain the significant sound dissipation mechanisms.

We tested the trained network on experimental data measured for an acoustic liner, which represents a relevant application. By comparing the liner's scattering matrix extracted with the neural network and the classical two-port method for a case without flow, we found that the learning algorithm performs well in the mid-frequency range of the training area, whereas the accuracy decreases slightly towards the higher and lower training frequencies. In the mid-frequency range, the algorithm reached very good performance for most of the frequencies even though the network was only trained on 20 % of the tested frequencies. This indicates a good ability of the presented network to learn the non-linear regression functions without overfitting the trained points. A more complex neural network with several layers might be able to increase the accuracy towards the borders of the trained frequencies.

To improve the method, the network could be trained to also predict the wave numbers of the extracted duct modes. This would allow to move the modal data from the reference cross section to arbitrary cross sections, as commonly done in the two-port method.

4. References

1. IdealVent (Integrated Design of Optimal Ventilation Systems for Low Cabin and Ramp Noise). https://www.idealvent.eu/, (online; accessed 10-April-2017).

2. Åbom M, Kabral R. Turbocharger Noise - Generation and Control. SAE Tech Pap [Internet]. SAE International; 2014. Available from: http://dx.doi.org/10.4271/2014-36-0802

3. Sack S, Åbom M, Efraimsson G. On Acoustic Multi-Port Characterisation Including Higher Order Modes. Acta Acust United with Acust. 2016;102:834–50.

4. Gabard G, Astley R. J. A computational mode-matching approach for sound propagation in three-dimensional ducts with flow. J Sound Vib. 2008;315:1103–24. Available from: http://www.sciencedirect.com/science/article/pii/S0022460X0800148X

5. Davies P O A L. Practical flow duct acoustics. J Sound Vib. Academic Press Limited; 1988;124:91–115. Available from: http://www.sciencedirect.com/science/article/pii/S0022460X8881407X

6. Kirchhoff G. Über den Einfluss der Wärmeleitung in einem Gase auf die Schallbewegung. Ann Phys [Internet]. WILEY-VCH Verlag; 1868;210:177–93. Available from: http://doi.wiley.com/10.1002/andp.18682100602

7. Sack S, Åbom M, Efraimsson G. On acoustic multi-port characterisation including higher order modes. Acta Acust united with Acust. 2016;102.

8. Lavrentjev J, Åbom M. Characterization of Fluid Machines As Acoustic Multi-Port Sources. J Sound Vib. 1996;197:1–16. Available from: http://linkinghub.elsevier.com/retrieve/pii/S0022460X96905143

9. Holmberg A, Åbom M, Bodén H. Accurate experimental two-port analysis of flow generated sound. J Sound

Vib. 2011;330:6336–54. Available from: http://linkinghub.elsevier.com/retrieve/pii/S0022460X11006225
10. Åbom M. Modal decomposition in ducts based on transfer function measurements between microphone pairs. J Sound Vib. 1989;135:95–114.
11. Sack S, Åbom M. Investigation of orifice aeroacoustics by means of multi-port methods. J Sound Vib. 2017;407:32–45. Available from: http://www.sciencedirect.com/science/article/pii/S0022460X17304923
12. Weng C, Boij S, Hanifi A. On the calculation of the complex wavenumber of plane waves in rigid-walled low-Mach-number turbulent pipe flows. J Sound Vib. 2015;354:132–53. Available from: http://www.sciencedirect.com/science/article/pii/S0022460X15004952
13. Bishop CM. Neural Networks for Pattern Recognition. 1995th ed. Oxford: CLARENDON PRESS;
14. Kurt Hornik, Maxwell Stinchcombe, Halbert White. Multilayer Feedforward Networks are Universal Approximators. Neural Networks. 1989;2:359–66. Available from: https://pdfs.semanticscholar.org/f22f/6972e66bdd2e769fa64b0df0a13063c0c101.pdf
15. Davies P ~O. ~A. ~L., Bhattacharya M, Coelho J ~L. ~B. Measurement of plane wave acoustic fields in flow ducts. J Sound Vib. 1980;72:539–42.
16. Sack S, Shur M, Åbom M, Strelets M, Travin A. Numerical eduction of active multi-port data for in-duct obstructions. J Sound Vib. 2017;411.