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Exploration of the pseudospectral method for auralizations

Jorge PETROSINO[1]; Lucas LANDINI[2]; Andrés BONINO RETA[2]; Georgina LIZASO[2]

[1,2] Universidad Nacional de Lanús, Argentina

ABSTRACT

Sound propagation in auralizations can be simulated with geometric or wave-based models. Each one offers particular advantages and disadvantages. Geometric models lead to the ray tracing method and the image source method. Both methods achieve good results for high frequencies and are efficient in large rooms with complex structures, but are not able to represent in a simple manner specific wave phenomena, such as diffraction. Typical wave-based models such as the finite element method, the boundary element method, or the finite difference method are characterized for achieving very precise results for individual frequencies applied to small and moderately sized rooms. A typical disadvantage is the need for high computing power, mainly due to the necessity of running separate simulations for individual frequencies because of numerical dispersion. In this paper, auralizations with the k-space pseudospectral method are explored since it is possible to reduce or eliminate the problem of numerical dispersion with this technique. For certain conditions, complete range impulse response can be obtained from a single simulation execution. The room response, including diffraction and diffusion is analyzed for different scenarios.

Keywords: Auralization, Pseudospectral Method, Numerical Methods

1. INTRODUCTION

Auralization is defined as the technique of creating audible signals from numerical methods based on simulated or measured data that describes an acoustic space, sources and listener positions (1). A first approach to auralization included physical scale models (2). The process is based on obtaining the impulse response of an acoustic space. This requires establishing a point in the acoustical space where the source will be. A mathematical model is also necessary to represent the acoustic space in which to simulate the sound propagation as well as the information related to the location of the receiver and its characteristics.The impulse response of the model is usually obtained from processes based on either geometrical or wave-based models, each with its own particular advantages and disadvantages. Strictly speaking, the ecogram resulting from geometrical models does not include all the expected characteristics of an impulse response, since specifically wave-based phenomena are not straightforwardly taken into account.

Among wave-based models, the finite element method (FEM) and the boundary element method (BEM) are worth mentioning. These are characterized for achieving very precise results for single frequencies on small or medium sized rooms (1). These methods solve differential equations by discretizing continuous variables, but, in turn result in unwanted numerical dispersion. In practice, this hinders the study of cases in which the source radiates a complex audio wave (for example, spoken word) or an impulse, therefore, they are limited to individual frequencies. To obtain the impulse response, the complex task of independently solving individual frequencies or narrow frequency bands is required, to later be combined into the total impulse response. This entails a considerable increase in the total computation time.

This paper proposes the usage of a different method for solving wave propagation differential equations in order to produce auralizations. The k-space pseudospectral method could make possible simulations for both complex audio waves and an impulse in a single run, deeming it an interesting alternative to traditional wave-based methods.

[1] jorgepetrosino@gmail.com

2. COMMONLY USED METHODS

2.1. Geometrical acoustics

For geometric models to be considered adequate wavelengths are required to be smaller than the surfaces on which sound bounces. Therefore, the assumption that sound waves can be treated as energy rays establishes considerable restrictions on the frequency range, since for the influence of small irregularities to be taken into account such simulation would be limited to very high frequencies. The turning point between wave and ray behavior is gradual and difficult to determine for a single frequency number but complementary processes to simulate phenomena such as diffraction and diffusion can be additionally employed.

The assumption that a range close to Schroeder's frequency can be considered as a lower limit for geometric methods is widely accepted. Due to the great superposition of room modes and diffuseness, certain variables can be considered of a statistical nature (4, 5, 6). The image source method is not suitable for curved surfaces, diffuse reflection, diffraction, or for complex rooms that represent hidden image sources. Interference can be simulated using additional running time phase, nonetheless, this is not precise on low frequencies where geometric methods are not valid.

For characteristics that depend on frequency (like absorption or diffusion) ray tracing must be done for each frequency band of interest separately to then be convolved with any audio file. This produces a signal representative of what a listener would perceive on the location of choice.

2.2. Wave-based models

On the other hand, wave-based models are not frequency limited, but difficulties arise when managing low computational times and defining physical elements with accuracy. FEM and BEM models are not limited on room geometry, diffraction, diffusion or interference but the elements represented on the models are based on their physical properties, such as medium density, medium propagation speed or compressibility. Simulation efficiency and accuracy is strongly related to discretization resolution and equation solver choice. In regards to auralization, these methods are more commonly used for simulating low frequencies in small and medium-sized rooms; due to the computational effort they require (7).

One of the main problems of numerical methods lays on the fact that differential equation solvers exhibit numerical dispersion resulting from wavelength related discretization error (8). This, in turn produces the unwanted and un-physical effect of propagation speed varying with frequency. This forces simulations to be carried out for single frequencies at a time and later be combined into the final room impulse response.

It is noteworthy that either, geometrical or the formerly mentioned wave based models, usually require, for different reasons, multiple simulation runs to produce a complete impulse response. As Aretz and Cols clearly state (9): "…simulation results were calculated for frequencies ranging from 200Hz to 2.5kHz in frequency steps of 1Hz…". Since computation time is directly proportional to the number of frequency steps and assuming a computation time of 5 minutes per frequency (1) this would take 190 continuous computation hours.

From a computational point of view, a viable method that made this possible in a single run could be an interesting alternative to already in use methods, even if such method did not offer any particular advantage in a strictly computational sense.

3. K-SPACE PSEUDOSPECTRAL METHOD

Spectral methods are a class of numerical techniques that solve differential equations. They are based on the principle that a solution can be obtained as the sum of certain basic functions (for example, sine functions) to then calculate suitable coefficients for the differential equation (10).

They are closely related to finite element and finite difference time domain methods (FDTD). The principal difference is that spectral methods use basic functions that are solved for the whole domain, while for other methods functions are solved across small subdomains. It could be argued that spectral methods use a global approach instead of a local approach (11, 12).

Pseudospectral methods are a sub-class of spectral methods that have been adapted to represent functions on a uniform rectangular grid, which simplifies how certain operators are evaluated and increases computation speed by applying the Fast Fourier Transform. The level of precision between spectral and pseudospectral methods is very similar (13, 14).

For acoustic wave propagation, variations of pressure, density, temperature, and particle velocity among others take place. Those variations can be described by coupled first order differential equations related to conservation of medium mass, energy and momentum. In acoustics it is common to combine these equations in relation to a variable (sound pressure or particle velocity) resulting in a second order differential equation.

The k-space pseudospectral method can be applied to simulate sound wave propagation by solving the coupled first order differential equations instead of the second order version (15). There are two reasons behind this. Firstly, it makes possible the inclusion of mass and force sources quite straightforwardly. In second place, because it enables the use of a special anisotropic Perfectly Matched Layer (PML) in order to greatly reduce computation time by reducing grid size, in comparison to other similar methods (16).

The k-space pseudospectral method has been greatly utilized on the last few years on biomedical applications related to ultrasonic wave propagation on tissue (17, 18). It has also been used to study alterations on mechanical wave propagation produced by the variation on atmospheric temperature (19). Not many examples of this method applied to room acoustics are to be found (20).

By solving the coupled first order differential equations separately, different operators can be used to reduce numerical dispersion enough for broadband simulation to be practical. Pseudospectral methods replace the spatial gradient calculation by FDTD usually used in other methods with the Fourier collocation method. This, in itself, reduces numerical dispersion for the spatial gradient but not for the temporal gradient. Nonetheless, this can be corrected for a homogeneous medium up to the Nyquist frequency employing a different operator (21).

About less than a decade ago Treeby, Cox (University College London) and Jaros (Brno University of Technology) developed a MATLAB toolbox named k-Wave that efficiently utilizes the k-space pseudospectral method. The toolbox was designed for acoustic wave propagation on the time domain for complex media. Specifically, simulates ultrasonic waves in biological tissue for medical imaging (22). An unlimited number of sound sources can be arbitrarily defined using any kind of MATLAB function with relative ease. Simulations can be carried out in one, two and three dimensions, combining pressure and velocity sources interchangeably. Pressure and particle velocity sensors can be arbitrarily placed in any and all grid points without noticeable computation impact, since the method solves the equations for the entire grid anyway. Besides, medium density, propagation speed and attenuation coefficients can be defined for any and all grid points independently.

These aspects are sufficiently interesting to begin an exploratory analysis on pseudospectral method oriented auralizations based on producing a complete impulse response with a single simulation.

4. SIMULATION PARAMETERS AND CONSTRAINTS

Similarly to FEM, pseudospectral methods solution stability is evaluated by the CFL (Courant - Friedrichs - Lewy) coefficient shown in Eq. (1) that describes the rate at which a wave travels across two contiguous grid points over two interleaved time points.

$$CFL = c_{max} \cdot \frac{\Delta t}{\Delta x} \tag{1}$$

Different numerical methods require different CFL values. An example of CFL = 0.3 with 2 grid points per wavelength on pseudospectral methods would require a Δt step five time less stringent than FDTD of equal CFL value with 10 grid points per wavelength (23, 24). Simulations were carried out with a value of CFL = 0.3 for managing a reasonable balance between precision and computation time for homogeneous and slightly heterogeneous media. Internally K-Wave utilizes a Δt obtained by Eq. (2).

$$\Delta t = \frac{CFL \cdot \Delta x}{c_{max}} \tag{2}$$

Considering CFL = 0.3, Δx = 1 cm, c_{max} = 343 m/s, results in Δt = 8.75 µs. This Δt is not the same sample period usually used in audio processing. The maximum valid frequency for temporal functions corresponding to sources and simulated recordings is obtained by Eq. (3), where c_{min} is the minimum medium propagation speed.

$$f_{max} = \frac{c_{min}}{2.\Delta x} \quad (3)$$

5. IMPULSE PROPAGATION IN 3D SPACES

A three-dimensional space of 128 x 128 x 128 grid points was defined for simulations with more than 16 million nodes in total, representing a cubic acoustic space of 2.56 m. A spatial discretization resolution of 2 cm for each spatial dimension was defined. The maximum valid frequency obtained by Eq. (3) was 8575 Hz. The center of the space is considered the coordinate origin in all three dimensions.

Propagation of an impulsive signal was simulated for a free field open acoustic space. An acoustic pressure point source was positioned at the coordinates (0 m; -1 m; 0 m), and an omnidirectional point pressure sensor located at (0.5 m; 0.5 m; 0.5 m). The distance between source and sensor was of 1.66 m. The source was defined in the k-Wave toolbox as a *dirichlet* type source. This means that the source output is forced to have a time evolution that exactly matches the given signal, in this case, an impulse.

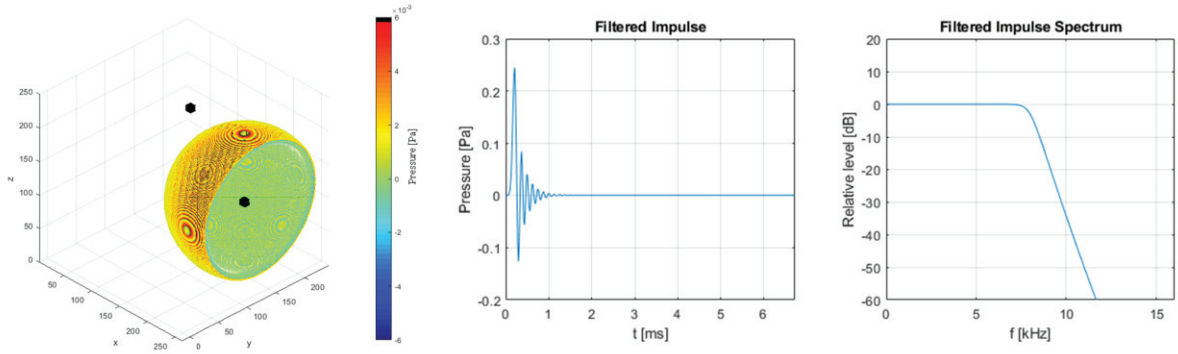

Figure 1 – a) Impulse propagation. Point sensor (black dot on the left) and point source (right); b) Time plot of the impulse response; c) Spectrum of the impulse response

Computation time on a personal computer (i7 series 3.4 GHz processor with 16 GHz of RAM) was of 17 min. Figure 1b shows the frequency response of the pressure recorded at the sensor. Ideally the recording would have a constant spectral modulus for all frequencies. The former equations predicted a maximum valid frequency of 8575 Hz for the simulation. Above this value simulation stability lowers at a great rate as frequency increases. Filtering the emitter beforehand is recommended and results in a very smooth frequency response with a marginal variation close to f_{max} (Figure 1c). Since f_{max} is proportional to discretization level different choices of Δx and Δt directly relates to higher or lower values of f_{max}. This must be taken into account as a simulation parameter and chosen accordingly to what is being simulated since it has a direct impact on total computation time.

The computation time reported above only considers enough time for the simulated impulsive wave to reach the sensor as proof of concept compared to FEM. For an actual auralization a higher computation time is expected. More simulation time would be required for multiple reflections to reach the sensor from surfaces across the room. In any case, computation time is less than the expected by individually simulating such a great range as 8.5 kHz with other methods.

6. SIMULATION

6.1. Complex wave propagation in 2D

As previously stated, the auralization process is based on obtaining the room response to then convolve with any audio signal. This produces an audio file with similar characteristics as to what a listener would perceive at the simulated receiver's position.

Clearly, simulating just the propagation of an impulse is far more computationally efficient than simulating the propagation of an entire musical or spoken program. For an impulse, it is only necessary to simulate enough time for reflections to decay to an acceptable level. This is independent of the musical program duration. Convoluting the desired signal with an impulse response generates similar final results. For doing this, the entire system is required to be linear time-invariant (LTI) to the input signal. System input is the signal emitted by the source, the output being the record and the system the entire space on which waves propagate.

With a k-space pseudospectral method it is possible to evaluate the propagation linearity of the system. This was tested by obtaining an auralization of the spoken word *'hello'* previously recorded on an audio file by simulating the propagation in 2D of the entire signal directly as the source's input and then comparing it to the auralization produced by impulse response convolution.

Propagation of complex audio waves is not possible with traditional methods but it is feasible with pseudospectral methods. Simulating the propagation of an entire complex program has an unreasonably high computational cost, for this reason, it was decided to simulate only two dimensions of propagation for the test. In the 2D simulations space resolution was set to 1 cm, rising the maximum frequency for audio signals.

6.2. Direct propagation of an audio file

A homogenous medium (air) was defined with c_{min} and c_{max} being equally 343 m/s. The resulting f_{max} = 17150 Hz. The simulated space was of 2.56 m by 2.56 m. (256 by 256 elements). A source was located at coordinates x_0 = 0 m; y_0 = - 1 m and a pressure sensor at at x_0 = 0 m; y_0 = 1 m. The source was defined as type *dirichlet* (forcing the output to match as closely as possible the input signal). Only propagation was tested, therefore no obstacles are present.

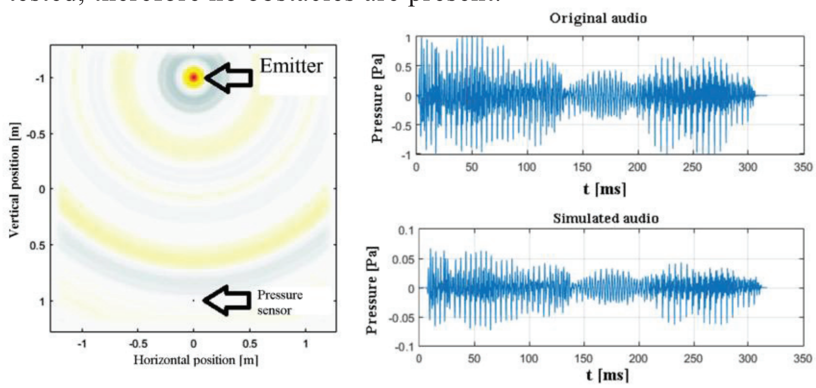

Figure 2 – a) Wave propagation for a given instant in the simulation; b) Time plot of original emitted audio and wave recorded at the sensor

The source time evolution was defined by the spoken word file. The recording was comprised of a brief word (hello) with a time span of 317 ms and a sample rate of 48 kHz. Since k-Wave works with a Δt much lower than the audio, sample period resampling was required to match the simulation parameters. The MATLAB function *resample* was used to sample it up to 114 kHz.

The simulation time required for the propagation of the entire program to reach the sensor 2 meters away took 4 min of computation time for a personal dektop computer (i7 series 3.4 GHz processor with 16 GHz of RAM). Figure 2a shows shows wave propagation. Figure 2b displays the time plots of the emitted and received waves. It can be noted a brief delay at the beginning of the recorded wave which agrees with the propagation time required to travel the 2m distance between source and sensor. The resulting signal was played directly in MATLAB. Reproduction was audible clear with a slight increase in the lower frequencies, as it is expected by the use of *dirichlet* type source in 2D spaces.

6.3. Audio convolution with the impulse response

For the second simulation the former setup was maintained but an impulse source was used instead. A slightly longer than the necessary for propagation was chosen for simulation time (6.3 ms). In this case computation time was of 6 seconds. Figure 3 shows the time plot of the impulse response. The amplitude raise at low frequencies is related to propagation in 2D spaces but not on 3D ones as seen in the proof-of-concept simulation.

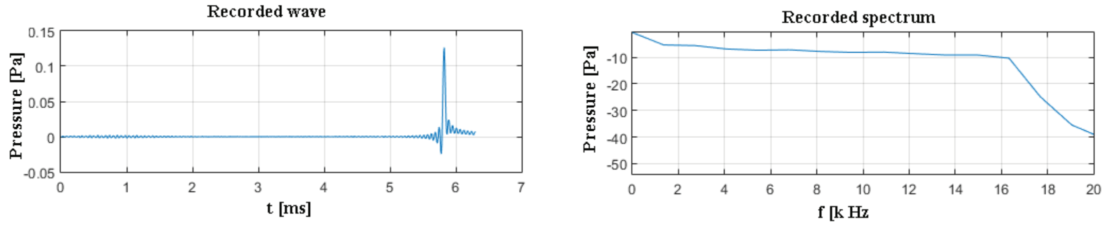

Figure 3 – Time plot and spectrum of 2D impulse response

The impulse response was convolved with the original audio sampled up to 114 kHz (to maintain the k-Wave internal Δt). The resulting signal was again clear and perfectly comprehensible audio with the same slight increase on the lower frequencies. As expected, a short delay of about 6 ms identical to the previous simulation is present.

6.4. Wave phenomena

Simulations were carried out to test the performance of k-Wave to simulate wave propagation in 3D spaces. Two scenarios were used to simulate wave phenomena in non-suitable contexts for geometrical models. First, a rigid surface with a single hole, and second, the reflection pattern on a Schroeder diffuser.

6.4.1. Scenario a - Diffraction

An ideally rigid surface with a square aperture was simulated. To achieve a compromise between computation time and simulation accuracy a spatial resolution Δx = 2 cm was defined for a grid of 96 x 96 x 64 points that represented a volume of 1.84 m by 1.84 m by 1.28 m. The resulting f_{max} = 8575 Hz. The aperture of width W = 12 cm was 4 cm deep. Multiple cases for incident waves of different frequency were simulated.

An incident plane wave of f = 4 kHz was generated. The diffraction pattern is shown in Figure 4. The effects of increase λ in relation to width W are show in Figure 4c (on the rigth).

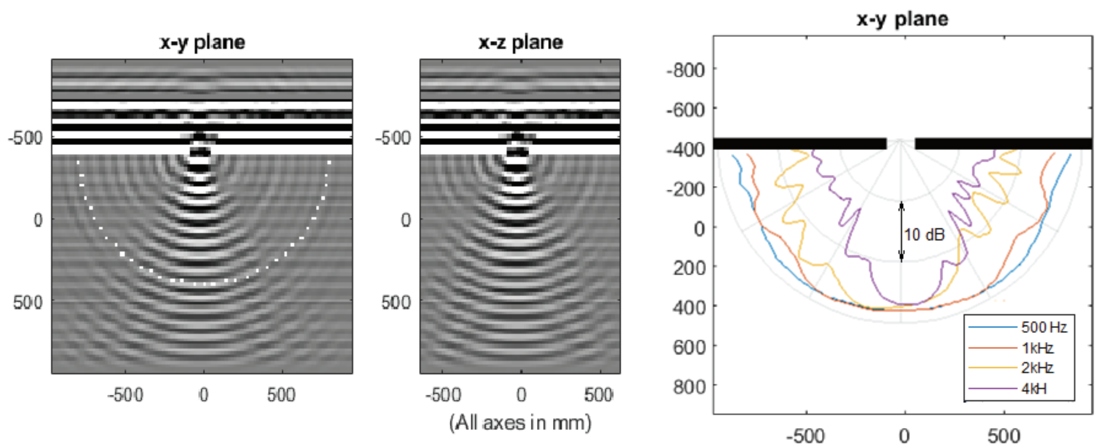

Figure 4 – Horizontal and vertical projection of the model.

6.4.2. Scenario b - Diffusion

A Schroeder diffuser of N = 7 for 800 Hz was simulated using the same parameters as in the diffraction scenario. The diffuser was defined as an ideally rigid body shaped. It was 24 cm wide by 12 cm deep and infinitely long to avoid diffraction on the edges other than the axis of diffusion. Multiple cases for incident waves of different wavelength were simulated (λ/W=5.7; λ/W=2.8; λ/W=1.4; λ/W=0.7). Incident energy is scattered in different directions as the wave reflects on the irregular surface of the diffuser. Diffusion results are based entirely on the geometric distribution of wells, their length and shape. Figure 5 shows a horizontal projection of the waves and a polar plot of the energy distribution obtained.

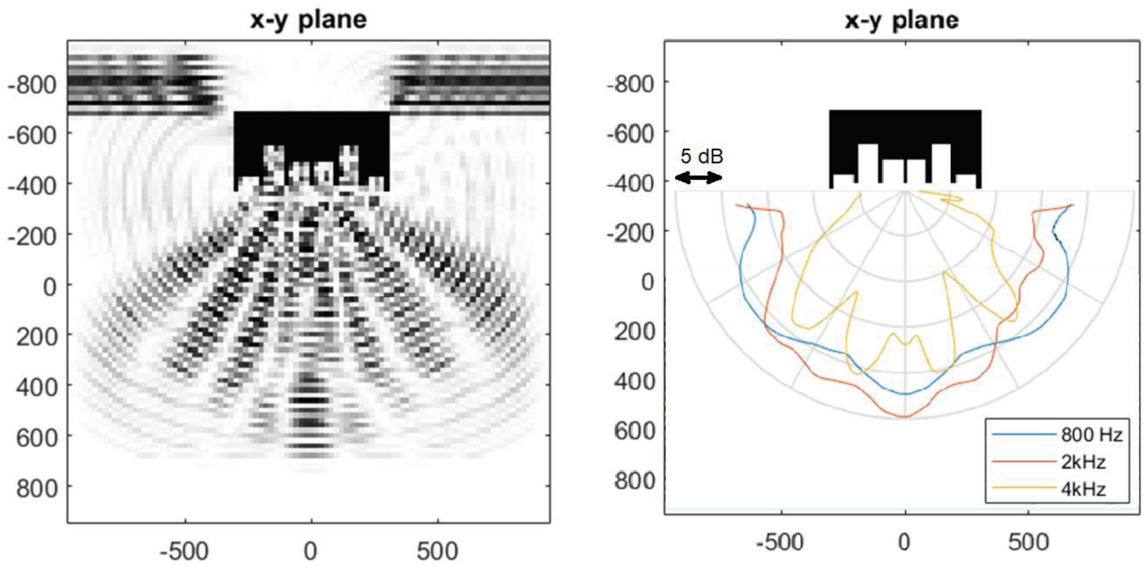

Figure 5 – Wave reflection on a Schroeder diffuser

7. CONCLUSIONS AND FUTURE WORK

Results suggest that the k-space pseudospectral method may be suitable for use in auralizations. The principal advantage relies on the use of impulsive sources with broad range of frequencies. The tests carried out show that it is possible to obtain an impulse response in just one computation run. The propagation of a complete audio wave compared with the propagation of an impulse and later convolution is consistently to what is expected of LTI system. The transfer function of the simulation system is practically flat for three dimensions with a frequency range adequate for audio application. For simulation on two dimensions an increase in level for low frequencies is found but could be compensated by filters.

The method is capable of representing wave phenomena but a more detailed analysis must be carried out to determinate how much agrees with physical behaviour under simulation constraints.

Preliminary tests have been carried out in larger room including a variety of reflecting surfaces configurations that render interesting results. This must be verified and submitted to careful examination before being reported.

Different materials could be simulated based on density, propagation speed and absorption to simulate wave propagation in more complex conditions. A greater exploration of method limitations must be carried out to verify if this does not hinder the advantage of simulating impulses on just one simulation run.

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