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Limits of noise control over space

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Abstract

This paper investigates the best possible performance of noise cancellation over a region of space for a fixed set of secondary sources within a given acoustic environment. We first formulate the spatial active noise control (ANC) problem in a 3-D acoustic enclosure (room). Secondly, we derive a wave-domain least square method by matching the secondary sound field to the primary noise field in wave domain. Thirdly, we propose a subspace method by matching the secondary field coefficients to the projection of primary noise field in the subspace. Simulation results compare between the wave-domain least square method and the subspace method, in terms of energy of the loudspeaker driving signals, noise reduction inside the region, and residual noise field outside the region. The subspace matching method provides the limits of the ANC system for the given constraints and the acoustic characteristics of the room.

Keywords: Sound, Insulation, Transmission

1 INTRODUCTION

Spatial Active noise control (ANC) is a challenging problem as the aim is to create a large quiet zone for multiple listeners in three-dimensional (3-D) space. In spatial ANC applications, such as noise cancellation in aircraft [1] and automobiles [2, 3, 4, 5], multichannel ANC systems equipped with multiple sensors and multiple secondary sources are adopted [6]. In literature, both time-domain [7, 8] and frequency-domain [9, 10] algorithms have been implemented in multichannel ANC systems, which can cancel the noise at error sensor positions and their close surroundings [10]. Recently, ANC over space has been approached via cylindrical/-spherical harmonic-based wave-domain algorithms [11, 12, 13, 14, 15, 16], with which the noise over entire region of interest can be cancelled directly. Here onwards, we use the terminology *'wave-domain ANC'* to refer to harmonics-based wave-domain ANC.

2 PROBLEM FORMULATION

Let the desired quiet zone be a spherical region with a radius R_1 located inside a room and let there by L loudspeakers as secondary sources (see Fig. 1). The goal of a spatial noise control system is to cancel the spatial noise field within the region of interest by a secondary sound field generated by secondary loudspeakers. Let $v(\boldsymbol{x},k)$ be the noise field at a point $\boldsymbol{x} \equiv (r,\theta,\phi)$ with respect to an origin within the desired zone of interest where $k = 2\pi f/c$ is the wavenumber, c is the speed of sound and f is the frequency in Hz. Similarly, let $s(\boldsymbol{x},k)$ be the secondary sound field generated by the loudspeakers and $e(\boldsymbol{x},k)$ be the residual noise field such that

$$e(\boldsymbol{x},k) = v(\boldsymbol{x},k) + s(\boldsymbol{x},k). \tag{1}$$

The secondary sound field generated by the loudspeaker array can be written as

$$s(\boldsymbol{x},k) = \sum_{\ell=1}^{L} d_l(k) G_\ell(\boldsymbol{x},k) \tag{2}$$

where $d_\ell(k)$ is the driving signal of the ℓ^{th} loudspeaker and $G_\ell(\boldsymbol{x},k)$ represents the acoustic transfer function from the ℓth loudspeaker to the point $\boldsymbol{x}$ which includes room reflections and scattering by other objects in the room.

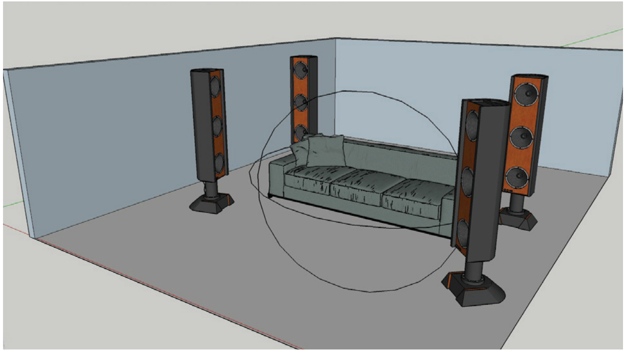

Figure 1. ANC system in a 3-D room.

The noise field at a point $\boldsymbol{x}$ can be expressed using modal decomposition [17, 18] as

$$v(\boldsymbol{x},k) = \sum_{n=0}^{N} \sum_{m=-n}^{n} \beta_{nm}(k)\, j_n(kr) Y_{nm}(\theta,\phi), \tag{3}$$

where $j_n(\cdot)$ is the spherical Bessel function of order n, $Y_{um}(\cdot)$ denotes the spherical harmonics, $\beta_{nm}(k)$ are the spherical harmonic coefficients of the sound field and the truncation limit $N = \lceil ekR_1/2 \rceil$ [19, 20, 21]. Similarly, the secondary sound field $s(\boldsymbol{x},k)$ and the residual sound field $e(\boldsymbol{x},k)$ within the quiet zone can also be represented by

$$s(\boldsymbol{x},k) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \gamma_{nm}(k)\, j_n(kr) Y_{nm}(\theta,\phi) \tag{4}$$

$$e(\boldsymbol{x},k) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \alpha_{nm}(k)\, j_n(kr) Y_{nm}(\theta,\phi) \tag{5}$$

where $\gamma_{nm}(k)$ and $\alpha_{nm}(k)$ are the spherical harmonic coefficients of the secondary and residual fields, respectively. The acoustic transfer function in (2) can also be parameterised in the spherical harmonic domain[22] as

$$G(\boldsymbol{x},k) = \sum_{n=0}^{N} \sum_{m=-n}^{n} \eta_{nm}^{(\ell)}(k)\, j_n(kr) Y_{nm}(\theta,\phi). \tag{6}$$

where $\eta_{nm}^{(\ell)}(k)$ is the harmonic coefficient of the acoustic transfer function of the ℓth loudspeaker.
We substitute (4) and (6) into (2) and decompose into spherical harmonic domain to obtain

$$\gamma_{nm}(k) = \sum_{\ell=1}^{L} d_l(k) \eta_{nm}^{(\ell)}(k) \text{ for } n = 0,\dots,N,\ m = -n,\dots,n. \tag{7}$$

Similarly, by substituting (5), (3) and (5) into (1) and decomposing into spherical harmonic domain, we obtain

$$\alpha_{nm}(k) = \beta_{nm}(k) + \gamma_{nm}(k) \tag{8}$$

for $n = 0,\dots,N$, $m = -n,\dots,n$. We combine (7) and (8) to obtain a relationship between modal components of residual field, noise field and the room modes of the acoustic transfer functions of loudspeakers as

$$\boldsymbol{\alpha}(k) = \boldsymbol{\beta}(k) + \boldsymbol{H}(k)\boldsymbol{d}(k) \tag{9}$$

where $\boldsymbol{\alpha}(k)=[\alpha_{00},\ldots,\alpha_{NN}]$, $\boldsymbol{\beta}(k)=[\beta_{00},\ldots,\beta_{NN}]$,

$$\boldsymbol{H}=\begin{bmatrix}\eta_{00}^{(1)}(k) & \eta_{00}^{(2)}(k) & \cdots & \eta_{00}^{(L)}(k)\\ \eta_{-11}^{(1)}(k) & \eta_{-11}^{(2)}(k) & \cdots & \eta_{-11}^{(L)}(k)\\ \vdots & \vdots & \ddots & \vdots\\ \eta_{NN}^{(1)}(k) & \eta_{NN}^{(2)}(k) & \cdots & \eta_{NN}^{(L)}(k)\end{bmatrix}, \tag{10}$$

and $\boldsymbol{d}(k)=[d_1(k),\ldots,d_L(k)]^T$.

In a complete spatial ANC system, there are error microphones located within or around [] the desired quite zone to obtain $e(\boldsymbol{x},k)$ as well as reference microphones to obtain a clean copy of noise signal to be fed into an adaptive algorithm to drive secondary loudspeaker signals. For the analysis in this paper, we assume that the error signal $e(\boldsymbol{x},k)$ and hence the mode coefficients matrix $\boldsymbol{\alpha}(k)$ can be estimated accurately. The goal of this paper is to analyse how much of the incident spatial noise field $\boldsymbol{\beta}(k)$ can be cancelled by a given loudspeaker array geometry within a given acoustic environment (i.e., by a given $\boldsymbol{H}$).

3 WAVE-DOMAIN LEAST SQUARE METHOD

One method for deriving the loudspeaker driving signal $\boldsymbol{d}(k)$ is to minimise error coefficients, i.e.,

$$\min_{\boldsymbol{d}(k)} \|\boldsymbol{\alpha}(k)\|^2 \quad \text{for each } k \tag{11}$$

where $\|\cdot\|^2$ is 2-norm operator. The above minimisation problem can be written as

$$\min_{\boldsymbol{d}(k)} \|\boldsymbol{\beta}(k)+\boldsymbol{H}(k)\boldsymbol{d}(k)\|^2 \quad \text{for each } k \tag{12}$$

In practical applications, the number of loudspeakers available are less than the number of active modes of the desired region of interest. Thus, (12) can be solved by least square method which results in minimum mean squared errors [23] solution. The optimal solution of this minimization problem can be written as

$$\boldsymbol{d}(k)=-\boldsymbol{H}^{\dagger}(k)\boldsymbol{\beta}(k), \tag{13}$$

where $(\cdot)^{\dagger}$ denotes the pseudoinverse of a matrix. We denote this method as 'wave-domain least square method' (WDLS) as the problem is formed in spherical harmonic (wave-domain) and solved using least squares. To this method to work, one has to obtain the acoustic transfer function coefficients between each loudspeaker and the region of interest. Also, it is assumed that the these coefficients are fixed and invariant with time.

4 SUBSPACE METHOD

In the second method, we construct the subspace spanned by the transfer function coefficient vectors of the loudspeaker array. Then, we project the noise field coefficient vector onto the subspace spanned by the loudspeakers. It can be infer that we can only cancel noise which lie in the loudspeaker subspace but the noise in the null space of the loudspeakers can not be cancelled and constitute the residual error.

4.1 Principal component analysis of the secondary path

Let $\boldsymbol{\eta}^{(\ell)}(k)$ be the wave-domain secondary-path coefficient vector of the ℓ^{th} loudspeaker. The matrix $\boldsymbol{H}(k)$ defined in (10) for the entire loudspeaker array represents the secondary paths, where

$$\boldsymbol{H}(k)=[\boldsymbol{\eta}^{(1)}(k),\ldots,\boldsymbol{\eta}^{(L)}(k)]. \tag{14}$$

In an arbitrary loudspeaker array setup in an acoustic environment, the columns of matrix $\boldsymbol{H}(k)$ are not necessarily orthogonal to each other. We use the principal component analysis (PCA) of the correlation matrix of

$\boldsymbol{H}$ to obtain an orthonormal eigen-basis for the space of the secondary path of loudspeakers in the spherical harmonic (wave) domain. Here onwards, the frequency dependent k is omitted for notational simplicity.

We take the correlation matrix $E\{\boldsymbol{H}^*\boldsymbol{H}\}$, and then decompose this matrix into a set of orthonormal eigenvectors and their corresponding eigenvalues, as follows:

$$E\{\boldsymbol{H}^*\boldsymbol{H}\} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V}, \tag{15}$$

where $\boldsymbol{U} = [\boldsymbol{u}_1, \ldots, \boldsymbol{u}_i, \ldots, \boldsymbol{u}_L]$ are the eigenvectors of the correlation matrix, $\boldsymbol{V} = \boldsymbol{U}^T$, the i^{th} column of $\boldsymbol{U}$ corresponds to the eigenvalue λ_i, and the eigenvalue are in the matrix

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_L \end{bmatrix}. \tag{16}$$

Here, the vectors $\boldsymbol{u}_i$ are written in order of descending eigenvalues λ_i [24]. Depending on the acoustic environment and the loudspeaker placement not all eigen values are significant. Let there are Q significant eigen values, then the corresponding eigenvectors are given by the matrix $\boldsymbol{U}^\diamond = [\boldsymbol{u}_1, \ldots, \boldsymbol{u}_Q]$.

We can now construct a subspace $\boldsymbol{O}$ spanned by the loudspeaker secondary-path coefficients $\boldsymbol{H}$ defined as

$$\boldsymbol{O} = \boldsymbol{H}\boldsymbol{U}^\diamond, \tag{17}$$

where the dimension of $\boldsymbol{U}^\diamond$ is $L \times Q$, and $Q \le L$. By normalizing each column of matrix $\boldsymbol{O}$, the orthonormal vectors $\boldsymbol{o}_1, \ldots, \boldsymbol{o}_B$ can be obtained. These vectors generate a subspace, which represents the loudspeaker array and the acoustic environment. The dimensions of basis $\boldsymbol{O}$ are $(N+1)^2 \times Q$.

For the ℓ^{th} loudspeaker, the average acoustic transfer coefficients can be represented in this space as

$$\bar{\boldsymbol{\eta}}^{(\ell)} = \sum_{q=1}^{Q} \kappa_a^{(\ell)} \boldsymbol{o}_q, \tag{18}$$

where $\kappa_q^{(\ell)}$ are the projection coefficients. In vector form, (18) can be written by

$$\bar{\boldsymbol{\eta}}^{(\ell)} = \boldsymbol{O}\boldsymbol{\kappa}^{(\ell)}, \tag{19}$$

where $\boldsymbol{\kappa}^{(\ell)} = \{\kappa_1^{(\ell)}, \ldots, \kappa_q^{(\ell)}, \ldots, \kappa_B^{(\ell)}\}^T$ and

$$\kappa_q^{(\ell)} = \langle \boldsymbol{\eta}^{(\ell)}, \boldsymbol{o}_q \rangle \tag{20}$$

where $\langle \cdot, \cdot \rangle$ represents inner product operator between two two vectors, $Q \times L$ matrix $\boldsymbol{\kappa}^{(\ell)}$ is the secondary-path coefficients of the ℓth loudspeaker in the subspace $\boldsymbol{O}$.

4.2 Projection from the primary noise field into the subspace

Below we project the wave-domain coefficients of the primary noise field into the subspace $\boldsymbol{O}$. For a primary noise field represented by vector $\boldsymbol{\beta}$, by projecting $\boldsymbol{\beta}$ into the subspace $\boldsymbol{O}$, we obtain

$$\text{Proj}_{\boldsymbol{O}}\boldsymbol{\beta} = \sum_{q=1}^{Q} \langle \boldsymbol{\beta}, \boldsymbol{o}_q \rangle \boldsymbol{o}_q \tag{21}$$

where $\text{Proj}_{\boldsymbol{O}}\boldsymbol{\beta}$ denotes the projection of vector $\boldsymbol{\beta}$ into subspace $\boldsymbol{O}$. The matrix form of the projection is represented by

$$\text{Proj}_{\boldsymbol{O}}\boldsymbol{\beta} = \boldsymbol{O}\boldsymbol{y}, \tag{22}$$

where $\boldsymbol{y} = \{y_1, y_2, \ldots, y_Q\}^T$ are the primary noise field coefficients in the subspace, and $y_q = \langle \boldsymbol{\beta}, \boldsymbol{o}_q \rangle$. Therefore, the primary noise field can be separated into two parts: the projected part and the remaining part,

$$\boldsymbol{\beta} = \text{Proj}_{\boldsymbol{O}} \boldsymbol{\beta} + R(\boldsymbol{\beta}), \tag{23}$$

where $R(\boldsymbol{\beta})$ is the orthogonal complement of the subspace $\boldsymbol{O}$. The projected part indicates the primary noise field which can be cancelled in this system setup, and the orthogonal complement indicates the primary noise field which can not be cancelled in this system.

If $R(\boldsymbol{\beta}) = 0$, $\boldsymbol{\beta}$ lies in the subspace, then the primary noise field can be completely cancelled by the loudspeaker array. In more general cases, $R(\boldsymbol{\beta}) \neq 0$. This indicates the limitation of noise cancellation over the region of interest, under the particular loudspeaker placement and acoustic environment.

4.3 Noise control in the subspace

In this section, we design the driving signal of loudspeaker $d_\ell(k)$ to cancel the primary noise field projected into the subspace ($\text{Proj}_{\boldsymbol{O}} \boldsymbol{\beta}$). In the subspace, matching the secondary sound field coefficients to the projected primary noise field coefficients, the optimal solution of the secondary sound field coefficients can be written by

$$\boldsymbol{\gamma} = -\text{Proj}_{\boldsymbol{O}} \boldsymbol{\beta}. \tag{24}$$

The projection from the primary noise field into the loudspeaker subspace $\text{Proj}_{\boldsymbol{O}} \boldsymbol{\beta}$ can be calculated by (22). In a given loudspeaker setup, the representation of secondary sound field coefficients can be rewritten by

$$\boldsymbol{\gamma} = \boldsymbol{O} \boldsymbol{\kappa} \boldsymbol{d}, \tag{25}$$

where $\boldsymbol{d} = \{d_1, \ldots, d_L\}^T$, $\boldsymbol{\kappa} = \{\boldsymbol{\kappa}^{(1)}, \ldots, \boldsymbol{\kappa}^{(L)}\}$.
Substituting (25) and (22) into (24), we can get

$$\boldsymbol{O} \boldsymbol{\kappa} \boldsymbol{d} = -\boldsymbol{O} \boldsymbol{y}. \tag{26}$$

Multiplying the left inverse of $\boldsymbol{O}$ on both sides, (26) becomes

$$\boldsymbol{O}^\dagger \boldsymbol{O} \boldsymbol{\kappa} \boldsymbol{d} = -\boldsymbol{O}^\dagger \boldsymbol{O} \boldsymbol{y}. \tag{27}$$

Further simplifying (27), the final equation to design the driving signal can be written as

$$\boldsymbol{\kappa} \boldsymbol{d} = -\boldsymbol{y}. \tag{28}$$

The loudspeaker driving signals can be calculated by solving the system of linear equations described by (28). The number of principal components specifies whether the linear system (28) can be solved exactly. For $Q < L$, where we use only the largest Q components to generate the subspace, the driving signals $\boldsymbol{d}$ can be derived by

$$\boldsymbol{d} = -(\boldsymbol{\kappa})^\dagger \boldsymbol{y}, \tag{29}$$

where $(\boldsymbol{\kappa})^\dagger$ is the pseudoinverse of the secondary-path coefficients in the subspace. Loudspeaker driving signals $\boldsymbol{d}$ are dependent on the secondary-path information in the subspace $\boldsymbol{\kappa}$ and the primary noise field coefficients in the subspace $\boldsymbol{y}$, as shown in (29).

5 SIMULATION RESULTS

Simulation setup: The reverberant environment is modelled as a cuboid room of 6 m ×6 m ×5 m with wall reflection coefficients a $[0.75, 0.8, 0.77, 0.85, 0.1, 0.1]$ and we use the image source method with the image order of 5. The origin of the room is on the left bottom corner. The region of interest is a sphere with a radius of 0.5 m, and the center of the region is (3, 3, 1.5) with respect to the room origin. In the following investigations, the

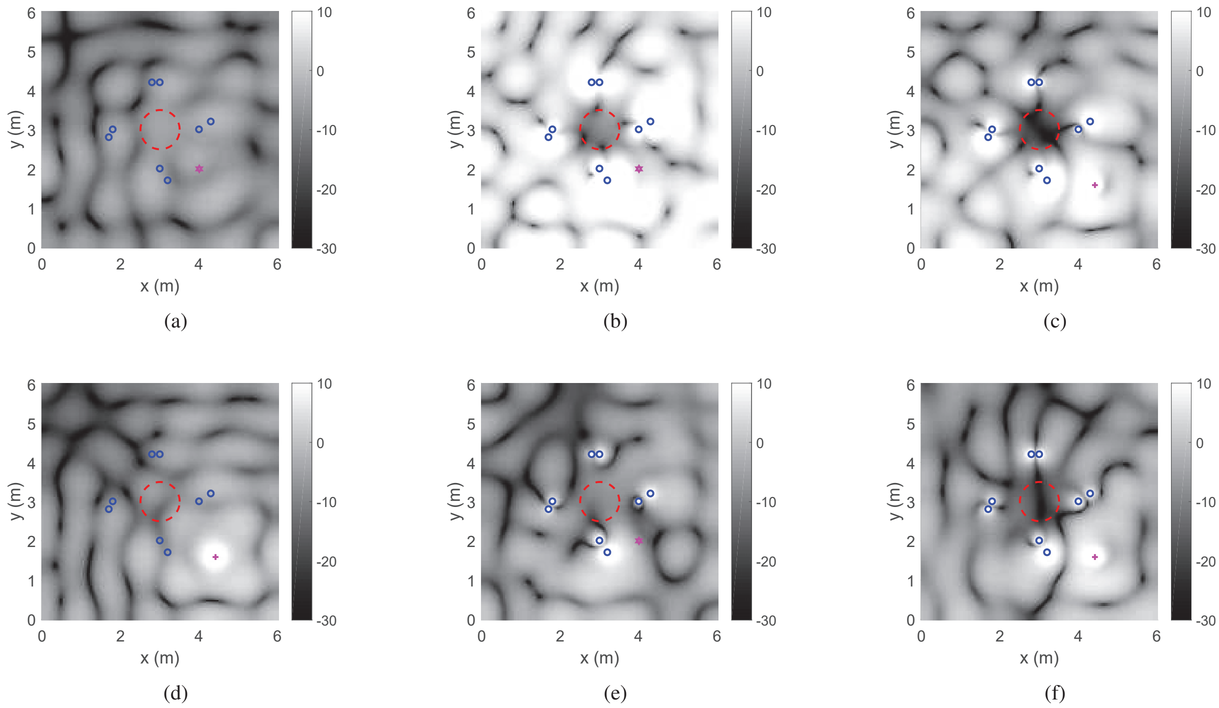

Figure 2. Energy of Case 1 (a) of the primary noise field (pink point is the location of the primary source projected on to the x-y plane, blue points are the loudspeaker locations projected on the x-y plane, and the red dashed circle is the boundary of the region of interest, (b) of the residual noise field using WDLS method and (c) the subspace method. Energy of Case 2 (d) the primary noise field (e) of the residual noise field using WDLS method and (f) the subspace method.

primary noise field is a single frequency (200 Hz) spherical wave originating from a point source located within the room, with a constant magnitude of 10. For the the given parameters, we have $N = \lceil ekR_1/2 \rceil = 3$ modes. Thus, at least $(N+1)^2 = 16$ microphones must be placed on the boundary to capture the the residual noise field. In this simulation, we place 32 microphones on the spherical boundary, following the Gauss-Legendre sampling method. White Gaussian noise (60 dB SNR[1]) is added to each microphone recording. To control all modes, 16 loudspeakers are required to be placed outside the control region. To emulate a practical scenario, however, in this simulation, only 12 loudspeakers are placed outside the region with 8 loudspeakers placed on the x-y plane with two different geometries. The loudspeaker positions for each case are shown in Table **??**.

The noise reduction inside the region of interest over Z points $N_{\mathrm{r}}^{\mathrm{in}}$ can be written by

$$N_{\mathrm{r}}^{\mathrm{in}} \triangleq 10\log_{10} \frac{\sum_z E\{|e_{\mathrm{in}\_z}|^2\}}{\sum_z E\{|e_{\mathrm{in}\_z}(0)|^2\}}, \tag{30}$$

where $E\{|e_{\mathrm{in}\_z}(0)|^2\}$ is the energy of the primary noise field at the z^{th} sample point, and $E\{|e_{\mathrm{in}\_z}|^2\}$ is the energy of the residual sound field at the z^{th} sample point. To evaluate the loudspeaker energy consumption, we compare the total energy of all the loudspeakers E_d. The loudspeaker energy consumption is given by $E_d = \boldsymbol{d}^T \boldsymbol{d}$.

[1]Here, the SNR level is with respect to the primary noise field level on virtual microphone in the center of the region.

5.1 Noise cancellation comparison

We first compare cancellation performance using the subspace method and the WDLS method in two different noise source positions: Case 1: $(2,315°,45°)$, Case 2: $(2,315°,90°)$. The energy of the primary noise field is shown in Figure 2.

Figure 2 depicts the energy of the residual noise field on the x-y plane. As we expected, since the number of loudspeakers (12) can-not cover all the modes (16) in the region, in all four figures, the primary noise field in the region of interest can not be fully cancelled. In case 2, compared with the primary noise field (Figure 2(d)), both the WDLS method and the subspace method can achieve significant noise reduction in the region of interest, which are dark areas in the middle of Figure 2(c) and Figure 2(f). In case 1, since the noise source is located in a different hemisphere as the loudspeaker array, compared with Figure 2(a), cancellation performance inside the region is fairly limited for both WDLS and the subspace method, as shown in Figure 2(b) and Figure 2(e). Compared with Figure 2(b) and Figure 2(c), in Figure 2(e) and Figure 2(f), the subspace method results in lower energy of the residual noise field outside the region of interest. The WDLS method increases the sound energy outside the region, especially when the noise reduction level is fairly limited inside the region. Using the subspace method, we reduce the sound amplification to people sitting outside the control area. A comprehensive simulation study using the theory developed here is reported in [].

6 CONCLUSIONS

This paper developed a subspace based method to obtain the best possible performance of noise cancellation over a region of space for a fixed set of secondary sources within a given acoustic environment. The subspace matching method provides the limits of the ANC system for the given constraints and the acoustic characteristics of the room.

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