PROCEEDINGS of the
23rd International Congress on Acoustics

9 to 13 September 2019 in Aachen, Germany

Optimal Design of Symmetric and Asymmetric Beampatterns with Circular Microphone Arrays

Xudong ZHAO[(1)], Gongping HUANG[(1)], Jacob BENESTY[(2)], and Jingdong CHEN[(1)]

[(1)]CIAIC, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China, E-mail: xudong.zhao@mail.nwpu.edu.cn, gongpinghuang@gmail.com, jingdongchen@ieee.org.

[(2)]INRS-EMT, University of Quebec, 800 de la Gauchetiere Ouest, Suite 6900, Montreal, QC H5A 1K6, Canada, E-mail: benesty@emt.inrs.ca.

Abstract

This paper is devoted to the study of the beamforming problem with circular microphone arrays (CMAs). It presents an approach to the design of beamformers with asymmetric and symmetric frequency-invariant beampatterns. We first discuss how to express a desired target directivity pattern, either symmetric or asymmetric, into a linear weighted combination of sine and cosine functions as well as circular harmonics of different orders. Using the hypercardioid pattern as an example, we show how to determine the weighting coefficients by maximizing the directivity factor (DF) with different constraints. Then, by using the Jacobi-Anger expansion, an approximation of the beamformer's beampattern is presented. A linear system is subsequently formed by forcing the approximated beampattern to be equal to the target asymmetric or symmetric directivity pattern. The optimal beamforming filter is finally determined by identifying the linear system. In comparison with symmetric beampatterns, the use of asymmetric beampatterns can offer more flexibility for practical application, such as a high DF or better null positions for interference rejection, which is demonstrated by simulations.

Keywords: Microphone arrays, circular microphone arrays, frequency-invariant beamforming, differential microphone arrays, symmetric beampattern, asymmetric beampattern.

1 INTRODUCTION

Frequency-invariant beamforming with small microphone arrays has been extensively studied as it has the great potential in processing broadband audio and speech signals [1–5]. Several frequency-invariant beamforming techniques have been developed over the last few decades. Among those, the differential beamformers, which respond to the differential of the acoustic pressure field, have attracted much research and engineering interest [6, 7] as they can achieve frequency-invariant beampatterns and high directivity factors at the same time.

Traditional differential beamformers are designed with a multistage structure [7,8]. However, this structure lacks flexibility in dealing with the problem of white noise amplification, which is inherent to differential microphone arrays (DMAs). Recently, a method to design DMAs in the short-time-Fourier-transform (STFT) domain was developed [9], which provides a minimum-norm beamforming approach, which basically circumvents the white noise amplification problem by maximizing the white noise gain (WNG) while making the order of the differential beamformer smaller than the number of microphone sensors minus one. This method is then extended to the design of differential beamformers with circular microphone arrays (CMAs) [11], which have better steering flexibility as compared to linear microphone arrays (LMAs) [12–14].

Another widely used beamforming method with CMAs is the modal beamformer, which is based on decomposing the sound field into a series of circular harmonics and then combining those circular harmonics with appropriate weighting coefficients to form the beamformer's output [4, 5, 11, 15]. In our recently work, we developed a differential beamforming method with CMAs based on the Jacobi-Anger expansion and showed that the modal beamformer is a particular case of this method [12].

However, in all the aforementioned works [11–14], the designed beampatterns are assumed to be symmetric.

The underlying reason for this is that the conventional DMAs were based on LMAs and any DMA pattern for LMAs is a linear combination of cosine functions of different orders, which is symmetric in nature [7]. Recently, a more general expression of directivity patterns, including both symmetric and asymmetric patterns, was developed [16]. With such an expression, a beamforming approach were developed based on solving linear equations formed using constraints from the target directivity pattern, which can design beamformers with symmetric and asymmetric beampatterns.

This paper is an extension of the principle presented in [12, 16]. We develop an approach to the design of beamformers with both symmetric and asymmetric beampatterns based on the use of the Jacobi-Anger expansion. We first discuss how to express a desired target directivity pattern, either symmetric or asymmetric, into a linear weighted combination of sine and cosine functions as well as circular harmonics of different orders. Then, by using the Jacobi-Anger expansion, an approximation of the beamformer's beampattern is presented. A linear system is subsequently formed by forcing the approximated beampattern to be equal to a target asymmetric or symmetric directivity pattern. The optimal beamforming filter is subsequently determined by identifying the linear system. In comparison with the existing methods that can only design symmetric beampatterns, the developed method has the flexibility in forming either asymmetric or symmetric beampatterns for benefits such as higher DF, better null steering for interference rejection, etc. In comparison with the method in [16], the developed beamforming approach offers better performance since the resulting beampattern is an optimal approximation of the target beampattern from the least-squares error perspective.

2 SIGNAL MODEL AND PERFORMANCE MEASURES

Consider a uniform circular microphone array (UCMA) with radius r, which consists of M omnidirectional microphones. Suppose that we want to steer the beamformer to the direction θ, the steering vector of length M is written as [17, 18]

$$\mathbf{d}_{\theta}(\omega) = \begin{bmatrix} e^{\jmath\varpi\cos(\theta-\psi_1)} & e^{\jmath\varpi\cos(\theta-\psi_2)} & \cdots & e^{\jmath\varpi\cos(\theta-\psi_M)} \end{bmatrix}^T, \tag{1}$$

where $\jmath$ is the imaginary unit with $\jmath^2 = -1$, $\varpi = \omega r/c$, $\omega = 2\pi f$ is the angular frequency, $f > 0$ is the temporal frequency, $\psi_m = 2\pi(m-1)/M$ is the angular position of the mth array element, c is the speed of sound in the air, and the superscript ^{T} is the transpose operator. The interelement spacing is $\delta = 2r\sin(\pi/M)$. In order to avoid spatial aliasing, it is assumed that this interelement spacing is much smaller than the minimum acoustic wavelength of interest.

Assume that the source signal of interest (i.e., the desired signal) comes from the direction $\theta_{\rm s}$. The objective of beamforming is then to design a frequency-invariant beampattern, which is as close as possible to a desired (symmetric or asymmetric) directivity pattern, with its main beam pointing in the direction $\theta_{\rm s}$. For that, we apply a beamforming filter of length M:

$$\mathbf{h}(\omega) = \begin{bmatrix} H_1(\omega) & H_2(\omega) & \cdots & H_M(\omega) \end{bmatrix}^T, \tag{2}$$

to the UCMA observation signals.

In order to evaluate the designed beamformer, we will use the three commonly used measures. They are

- the beampattern, which describes the sensitivity of the beamformer to plane waves arriving at the array from different directions, is defined as [1, 9]

$$\mathcal{B}_{\theta}[\mathbf{h}(\omega)] = \mathbf{h}^H(\omega)\,\mathbf{d}_{\theta}(\omega) = \sum_{m=1}^{M} H_m^*(\omega) e^{\jmath\varpi\cos(\theta-\psi_m)}, \tag{3}$$

  where the superscript ^{H} is the conjugate-transpose operator,

- the white noise gain (WNG), which reflects the robustness of the beamformer, is defined as [9]

$$\mathcal{W}\left[\mathbf{h}(\omega)\right] = \frac{\left|\mathbf{h}^H(\omega)\,\mathbf{d}_{\theta_s}(\omega)\right|^2}{\mathbf{h}^H(\omega)\,\mathbf{h}(\omega)}, \tag{4}$$

- and the directivity factor (DF), which quantifies the signal-to-noise ratio (SNR) gain of the beamformer in the spherically isotropic noise field, is defined as [1, 9, 10]

$$\mathcal{D}\left[\mathbf{h}(\omega)\right] = \frac{\left|\mathbf{h}^H(\omega)\,\mathbf{d}_{\theta_s}(\omega)\right|^2}{\mathbf{h}^H(\omega)\,\boldsymbol{\Gamma}_{\mathrm{d}}(\omega)\,\mathbf{h}(\omega)}, \tag{5}$$

  where the elements of the $M \times M$ matrix $\boldsymbol{\Gamma}_{\mathrm{d}}(\omega)$ are given by

$$\left[\boldsymbol{\Gamma}_{\mathrm{d}}(\omega)\right]_{ij} = \mathrm{sinc}\left(\frac{\omega\delta_{ij}}{c}\right) \tag{6}$$

  with $i, j = 1, 2, \ldots, M$, $\mathrm{sinc}(x) = \sin x / x$, and

$$\delta_{ij} = 2r\left|\sin\left[\frac{\pi(i-j)}{M}\right]\right| \tag{7}$$

  being the distance between sensors i and j.

3 DESIRED DIRECTIVITY PATTERN

We know from the array literature that any desired (symmetric or asymmetric) beampattern of order $N \geq 1$ can be written as

$$\begin{aligned}\mathcal{B}_\theta(\mathbf{a}_N, \mathbf{b}_N) &= \sum_{n=0}^{N} a_{N,n}\sin(n\theta) + \sum_{n=0}^{N} b_{N,n}\cos(n\theta) \\ &= \mathbf{a}_N^T\mathbf{p}_{\mathrm{s},\theta} + \mathbf{b}_N^T\mathbf{p}_{\mathrm{c},\theta},\end{aligned} \tag{8}$$

where $a_{N,n}, b_{N,n},\ n = 0, 1, \ldots, N$ are real coefficients and

$$\begin{aligned}\mathbf{a}_N &= \begin{bmatrix} a_{N,0} & a_{N,1} & \cdots & a_{N,N}\end{bmatrix}^T, \\ \mathbf{p}_{\mathrm{s},\theta} &= \begin{bmatrix} 0 & \sin\theta & \cdots & \sin(N\theta)\end{bmatrix}^T, \\ \mathbf{b}_N &= \begin{bmatrix} b_{N,0} & b_{N,1} & \cdots & b_{N,N}\end{bmatrix}^T, \\ \mathbf{p}_{\mathrm{c},\theta} &= \begin{bmatrix} 1 & \cos\theta & \cdots & \cos(N\theta)\end{bmatrix}^T.\end{aligned}$$

The values of the coefficients $a_{N,n}, b_{N,n},\ n = 0, 1, \ldots, N$, determine the shape of the directivity pattern. In the particular case of symmetric beampatterns, i.e., $\mathbf{a}_N = \mathbf{0}$, (8) degenerates to

$$\mathcal{B}_\theta(\mathbf{b}_N) = \mathbf{b}_N^T\mathbf{p}_{\mathrm{c},\theta}. \tag{9}$$

Another very useful way to express (8) is

$$\begin{aligned}\mathcal{B}_\theta(\mathbf{a}_N, \mathbf{b}_N) &= \frac{1}{2\jmath}\sum_{n=0}^{N} a_{N,n}\left(e^{\jmath n\theta} - e^{-\jmath n\theta}\right) + \frac{1}{2}\sum_{n=0}^{N} b_{N,n}\left(e^{\jmath n\theta} + e^{-\jmath n\theta}\right) \\ &= \frac{1}{2\jmath}\sum_{i=1}^{N} a_{N,i}e^{\jmath i\theta} - \frac{1}{2\jmath}\sum_{i=-N}^{-1} a_{N,-i}e^{\jmath i\theta} + b_{N,0} + \frac{1}{2}\sum_{i=1}^{N} b_{N,i}e^{\jmath i\theta} + \frac{1}{2}\sum_{i=-N}^{-1} b_{N,-i}e^{\jmath i\theta} \\ &= \sum_{n=-N}^{N} c_{2N,n}e^{\jmath n\theta} = \sum_{n=-N}^{N} c_{2N,n}^{*}e^{-\jmath n\theta} = \mathbf{c}_{2N}^H\mathbf{p}_{\mathrm{e},\theta} = \mathcal{B}_\theta(\mathbf{c}_{2N}),\end{aligned} \tag{10}$$

where the superscript * is the complex-conjugate operator,

$$\begin{cases} c_{2N,0} = b_{N,0} \\ c_{2N,i} = \dfrac{b_{N,i} - \jmath a_{N,i}}{2}, \ \ i = 1, 2, \ldots, N \\ c_{2N,-i} = \dfrac{b_{N,i} + \jmath a_{N,i}}{2} = c_{2N,i}^*, \ \ i = 1, 2, \ldots, N \end{cases}, \tag{11}$$

and

$$\mathbf{c}_{2N} = \begin{bmatrix} c_{2N,-N} & \cdots & c_{2N,0} & \cdots & c_{2N,N} \end{bmatrix}^T,$$
$$\mathbf{p}_{\mathrm{e},\theta} = \begin{bmatrix} e^{\jmath N\theta} & \cdots & 1 & \cdots & e^{-\jmath N\theta} \end{bmatrix}^T.$$

The vector $\mathbf{p}_{\mathrm{e},\theta}$ of length $2N+1$ is composed of circular harmonics of different orders.
In order to take the important steering information into account, let us rewrite $\mathcal{B}_\theta(\mathbf{c}_{2N})$ as

$$\mathcal{B}_{\theta-\theta_{\mathrm{s}}}(\mathbf{c}_{2N}) = \mathbf{c}_{2N}^H \mathbf{p}_{\mathrm{e},\theta-\theta_{\mathrm{s}}} = \left(\boldsymbol{\Upsilon}_{\theta_{\mathrm{s}}} \mathbf{c}_{2N}\right)^H \mathbf{p}_{\mathrm{e},\theta} = \mathbf{c}_{2N,\theta_{\mathrm{s}}}^H \mathbf{p}_{\mathrm{e},\theta} = \mathcal{B}_\theta(\mathbf{c}_{2N,\theta_{\mathrm{s}}}), \tag{12}$$

where

$$\boldsymbol{\Upsilon}_{\theta_{\mathrm{s}}} = \mathrm{diag}\left(\mathbf{p}_{\mathrm{e},\theta_{\mathrm{s}}}\right) = \mathrm{diag}\left(e^{\jmath N\theta_{\mathrm{s}}}, \ldots, 1, \ldots, e^{-\jmath N\theta_{\mathrm{s}}}\right) \tag{13}$$

is a diagonal matrix and

$$\mathbf{c}_{2N,\theta_{\mathrm{s}}} = \boldsymbol{\Upsilon}_{\theta_{\mathrm{s}}} \mathbf{c}_{2N}. \tag{14}$$

We see that our general desired directivity pattern is $\mathcal{B}_\theta(\mathbf{c}_{2N,\theta_{\mathrm{s}}})$. In practice, the shape of the (symmetric or asymmetric) beampattern is determined by $\mathbf{a}_N, \mathbf{b}_N$ (which are given by design) and the steering angle θ_{s}; then, $\mathbf{c}_{2N}$ is obtained from (11) and $\mathbf{c}_{2N,\theta_{\mathrm{s}}}$ from (14). Note that we should normalize $\mathbf{a}_N, \mathbf{b}_N$ such that $\mathcal{B}_{\theta_{\mathrm{s}}}(\mathbf{a}_N, \mathbf{b}_N) = 1$ and $\mathcal{B}_{\theta_{\mathrm{s}}}(\mathbf{c}_{2N,\theta_{\mathrm{s}}}) = 1$.

3.1 Asymmetric Hypercardioid

Now, we give an example of the desired, target directivity pattern: the hypercardioid pattern. The asymmetric hypercardioid pattern is obtained by maximizing the DF:

$$\mathcal{D} = \frac{|\mathcal{B}_{\theta_{\mathrm{s}}}(\mathbf{c}_{2N})|^2}{\int_0^{2\pi} |\mathcal{B}_\theta(\mathbf{c}_{2N})|^2 d\theta}, \tag{15}$$

with several attenuation constraints:

$$\mathbf{D}\mathbf{c}_{2N} = \mathbf{g}, \tag{16}$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{p}_{\mathrm{e},\theta_{\mathrm{s}}} & \mathbf{p}_{\mathrm{e},\theta_1} & \cdots & \mathbf{p}_{\mathrm{e},\theta_L} \end{bmatrix}^H,$$
$$\mathbf{g} = \begin{bmatrix} 1 & g_1 & \cdots & g_L \end{bmatrix}^T.$$

The linear system of equations (16) includes the distortionless constraint and $L \leqslant 2N$ additional attenuation constraints, where $0 \leqslant g_l < 1, \ l = 1, 2, \ldots, L$. The optimization problem can be formed as

$$\min_{\mathbf{c}_{2N}} \ \ \mathbf{c}_{2N}^H \boldsymbol{\Gamma}_p \mathbf{c}_{2N} \ \ \text{subject to} \ \ \mathbf{D}\mathbf{c}_{2N} = \mathbf{g}, \tag{17}$$

where

$$\boldsymbol{\Gamma}_p = \int_0^{2\pi} \mathbf{p}_{\mathrm{e},\theta} \mathbf{p}_{\mathrm{e},\theta}^H d\theta = 2\pi \mathrm{diag}\left\{\mathbf{1}_{2N+1}\right\} \tag{18}$$

is a diagonal matrix of size $(2N+1) \times (2N+1)$. Using the method of Lagrange multipliers, we get

$$\mathbf{c}_{2N,\mathrm{opt}} = \boldsymbol{\Gamma}_p^{-1} \mathbf{D}^H \left(\mathbf{D} \boldsymbol{\Gamma}_p^{-1} \mathbf{D}^H\right)^{-1} \mathbf{g}. \tag{19}$$

Note that without the L extra constraints, the solution will reduce to the traditional symmetric hypercardioid.

4 OPTIMAL DESIGN

It is well known that the mth element of the steering vector, $\mathbf{d}(\omega, \theta)$, can be expressed as

$$\begin{aligned} e^{\jmath\varpi\cos(\theta-\psi_m)} &= \sum_{n=-\infty}^{\infty} \jmath^n J_n(\varpi) e^{-\jmath n(\theta-\psi_m)} \qquad (20) \\ &= \sum_{n=-\infty}^{\infty} \jmath^n J_n(\varpi) e^{\jmath n(\theta-\psi_m)} \\ &= J_0(\varpi) + 2\sum_{n=1}^{\infty} \jmath^n J_n(\varpi) \cos\left[n(\theta-\psi_m)\right], \end{aligned}$$

where $J_n(\varpi)$ is the Bessel function of the first kind [19] and we have $J_{-n}(\varpi) = (-1)^n J_n(\varpi)$. The series in (20) is known as the Jacobi-Anger expansion [19].

Substituting (20) into (3), we obtain

$$\begin{aligned} \mathcal{B}_\theta\left[\mathbf{h}(\omega)\right] &= \sum_{m=1}^{M} H_m^*(\omega) e^{\jmath\varpi\cos(\theta-\psi_m)} \\ &= \sum_{m=1}^{M} H_m^*(\omega) \sum_{n=-\infty}^{\infty} \jmath^n J_n(\varpi) e^{\jmath n(\theta-\psi_m)} \\ &= \sum_{n=-\infty}^{\infty} e^{\jmath n\theta} \left[\sum_{m=1}^{M} \jmath^n J_n(\varpi) e^{-\jmath n\psi_m} H_m^*(\omega)\right]. \qquad (21) \end{aligned}$$

If we limit the expansion to the order $\pm N$, $\mathcal{B}_\theta\left[\mathbf{h}(\omega)\right]$ can be approximated by

$$\begin{aligned} \mathcal{B}_{\theta,2N}\left[\mathbf{h}(\omega)\right] &\approx \sum_{n=-N}^{N} e^{\jmath n\theta} \left[\sum_{m=1}^{M} \jmath^n J_n(\varpi) e^{-\jmath n\psi_m} H_m^*(\omega)\right] \\ &= \sum_{n=-N}^{N} e^{\jmath n\theta} \jmath^n J_n(\varpi) \boldsymbol{\psi}_n^T \mathbf{h}^*(\omega), \qquad (22) \end{aligned}$$

where

$$\boldsymbol{\psi}_n = \left[\begin{array}{cccc} e^{-\jmath n\psi_1} & e^{-\jmath n\psi_2} & \cdots & e^{-\jmath n\psi_M} \end{array}\right]^T. \tag{23}$$

Expression (22) is the best approximation of the beampattern from a least-squares error perspective.

Now, by forcing the beampattern from the beamforming filter $\mathbf{h}(\omega)$ to the desired directivity pattern, i.e.,

$$\mathcal{B}_{\theta,2N}[\mathbf{h}(\omega)] = \mathcal{B}_{\theta}(\mathbf{c}_{2N,\theta_s}), \tag{24}$$

we find that

$$\mathbf{\Psi}^{*}\mathbf{h}^{*}(\omega) = \mathbf{J}(\varpi)\,\mathbf{c}_{2N,\theta_s} \tag{25}$$

or, equivalently,

$$\mathbf{\Psi}\mathbf{h}(\omega) = \mathbf{J}^{*}(\varpi)\,\mathbf{\Upsilon}_{\theta_s}^{*}\mathbf{c}_{2N}^{*}, \tag{26}$$

where

$$\mathbf{J}(\varpi) = \mathrm{diag}\left[\frac{1}{\jmath^{-N}J_{-N}(\varpi)}, \ldots, \frac{1}{J_0(\varpi)}, \ldots, \frac{1}{\jmath^{N}J_{N}(\varpi)}\right] \tag{27}$$

is a $(2N+1)\times(2N+1)$ diagonal matrix and

$$\mathbf{\Psi} = \left[\begin{array}{ccccc} \boldsymbol{\psi}_{-N} & \cdots & \boldsymbol{\psi}_{0} & \cdots & \boldsymbol{\psi}_{N} \end{array}\right]^{H} \tag{28}$$

is a $(2N+1)\times M$ matrix. Since $\mathbf{\Psi}^{H}$ is a full-column rank matrix, we can take the minimum-norm solution of (26), which gives

$$\mathbf{h}_{\mathrm{MN}}(\omega) = \mathbf{\Psi}^{H}\left(\mathbf{\Psi}\mathbf{\Psi}^{H}\right)^{-1}\mathbf{J}^{*}(\varpi)\,\mathbf{\Upsilon}_{\theta_s}^{*}\mathbf{c}_{2N}^{*}. \tag{29}$$

The beamformer $\mathbf{h}_{\mathrm{MN}}(\omega)$ maximizes the WNG [9, 20]. So, it is also called the maximum WNG beamformer.

5 SIMULATIONS

In this section, we study the performance of the developed beamformer through simulations. We consider a UCMA with $M = 8$ and $r = 1.5$ cm. The desired directivity pattern is chosen as the second-order hypercardioid with two nulls at $\theta_1 = 90°$ and $\theta_2 = 120°$ and the desired look direction is set to $\theta_s = 0°$. Then, the coefficients vector $\mathbf{c}_{2N,\mathrm{opt}}$ is computed according to (19), where for the asymmetric second-order hypercardioid is

$$\mathbf{c}_{2N,\mathrm{opt}} = \left[\begin{array}{ccccc} 0.1736 - 0.0208\jmath & 0.1976 + 0.0448\jmath & 0.2576 & 0.1976 - 0.0448\jmath & 0.1736 + 0.0208\jmath \end{array}\right]^{T}, \tag{30}$$

and for the symmetric second-order hypercardioid is

$$\mathbf{c}_{2N,\mathrm{opt}} = \left[\begin{array}{ccccc} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{array}\right]^{T}. \tag{31}$$

Substituting all the parameters into (29), we obtain the minimum-norm beamforming filter. Figure 1 (a.1) and (a.2) plot the beampatterns as a function of the frequency. It is seen that in the studied frequency range from 0 to 4000 Hz, the beampatterns of the developed beamformer do not change much with frequency for both the asymmetric and symmetric cases. In both cases, the designed beampatterns are very close to the respective target directivity patterns.

Figure 1 (b.1) and (b.2) plot the DF and the WNG of the developed beamformer, both as a function of the frequency. It is seen that the asymmetric and symmetric hypercardioid beamformers have similar WNG but the asymmetric hypercardioid beamformer has higher DF, which shows the advantage of asymmetric beampatterns. In practical applications, if we need to place nulls at some directions and avoid nulls at some other directions for interference rejection, using asymmetric beampatterns may offer much more flexibility than using symmetric ones.

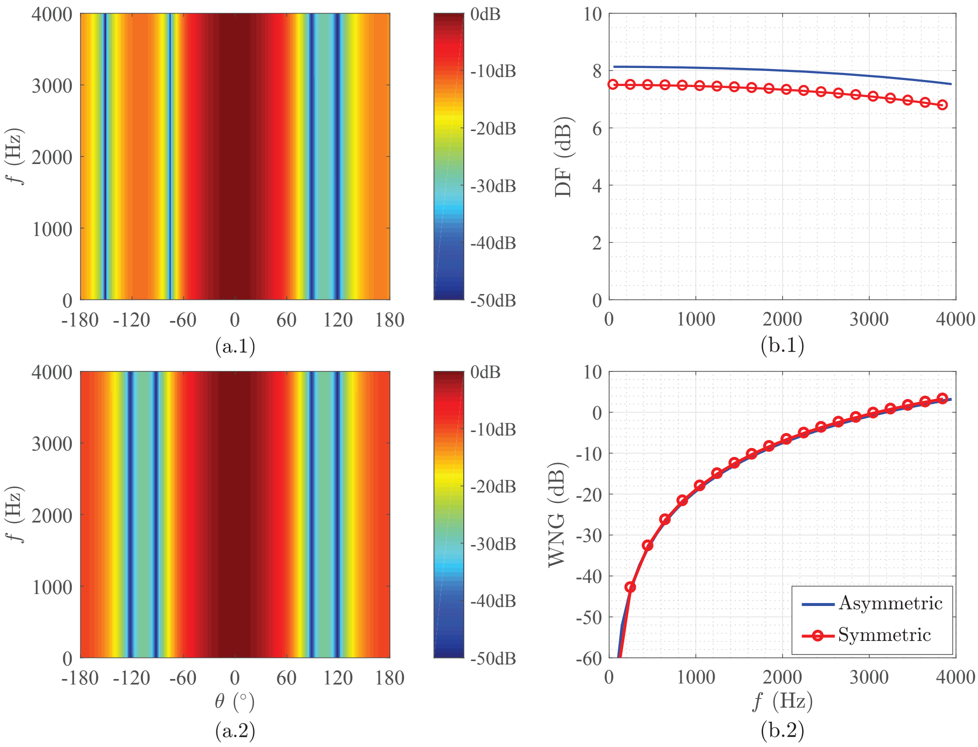

Figure 1. Performance of the designed beamformers: (a.1) asymmetric beampattern, (a.2) symmetric beampattern, (b.1) DF, and (b.2) WNG. Conditions of simulation: $M = 8$ and $r = 1.5$ cm.

6 CONCLUSIONS

In this paper, we studied the problem of beamforming with CMAs and showed how to design beamformers with symmetric and asymmetric frequency-invariant beampatterns. We started by defining the symmetric and asymmetric directivity patterns from the Fourier series expansion. Then, an asymmetric hypercardioid pattern was obtained by maximizing the DF with different attenuation constraints. By using the Jacobi-Anger expansion, the beamformer's beampattern and the desired, target directivity pattern were related, based on which a linear system is formed and the beamforming filter was derived by solving the linear system. The developed method can form beamformers with both symmetric and asymmetric frequency-invariant beampatterns, which gives much flexibility for real applications such as achieving high DF, placing nulls in proper positions, etc.

ACKNOWLEDGEMENTS

This work was supported in part by the National Science Foundation of China (NSFC) and the Israel Science Foundation (ISF) joint research program (grant No. 61761146001), the NSFC key program (grant No. 61831019), the NSFC Distinguished Young Scientists Fund (grant No. 61425005).

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