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Subjective evaluation of Head-Related Transfer Functions reconstructed with Spatial Principal Component Analysis and their domain dependency

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Abstract

In this paper, the subjective evaluation of the HRTFs reconstructed with the SPCA were mentioned. As a result, it was found that the reconstruction of the HRTFs with subjectively inaudible difference from the original ones are possible with smaller number of PCs than those Takane dicussed in his paper[8]. This brings about a expection that the spatial variation of HRTFs can be represented more compactly. Throught the 3AFC hearing experiment, it is found that the domain C and L have the relatively small number of conditions with significantly large number of correct answers. Especially, there were no subjects having significantly large number of correct answers when the number of PCs is 6 in the domain C.

Keywords: Subjective evaluation, Head-Related Transfer Function, Principal Component Analysis, Domain

1 INTRODUCTION

Principal Component Analysis (PCA) is well-known as one of effective methods for compact representation of data, and it is also applicable to a set of Head-Related Transfer Functions (HRTFs) obtained for various subjects in various sound source positions. Maybe the first research in which the PCA is applied to the HRTFs is Martens' one [1], and then Kistler *et al.* analyzed the properties when the PCA is applied to the HRTFs via computation and subjective experiment[2]. On the other hand, Chen *et al.*[3] and Wu *et al.*[4] called such analysis as "spatial feature extraction method," and their research purpose was to numerically analyze the HRTFs via the PCA and to effectively reconstruct the HRTFs using relatively small number of eigenvectors. The procedures of analyses in these researches were almost the same. In such kind of methods, the spatial variation and inter-subject difference of HRTFs are modeled by using small number of principal components or eigenvectors. In this paper, this method is called the Spatial PCA (SPCA) of the HRTFs/HRIRs, as Xie did[5].

Although the researches on the compact representation of HRTFs based on their SPCA exist as mentioned above, there are some differences in which domain was used for the SPCA of HRTFs among these studies. Kistler *et al.* used the logarithm of the HRTF amplitude[2], and Xie used the linear amplitude of the HRTF[5]. In these two studies, the HRTFs were assumed to have only minimum-phase component and the phase component were reconstructed by using the Hilbert transform[6]. Chen *et al.* used the complex-valued frequency spectrum of the HRTFs[3], and Wu *et al.* used the Head-Related Impulse Responses (HRIRs)[4], the inverse Fourier Transform of the corresponding HRTFs. Additionally, Takane proposed a new domain for the SPCA of HRTFs, complex logarithm of the HRTFs[9]. No minimum-phase approximation was required in these studies. The SPCA procedure can be successively and commonly adopted with the use of each parameter, and the modelling accuracy of the HRTFs was regarded sufficient in both objective and subjective viewpoints in all of these studies, with linear combination of relatively small number of principal components. However, it is known that difference in the modelling accuracy by choosing each of the four domains. Liang *et al.* compared the modelling accuracy between the linear and the logarithmic magnitude of the HRTFs in the horizontal plane, assuming that the minimum phase approximation was satisfactory[7]. Takane compared the reconstruction ac-

curacy of the HRTFs among the results of the SPCA with each of the four domains[8]. When the accuracy was compared in the same number of principal components, both researches reached the same conclusion that the SPCA of HRTFs in the domain of linear amplitudes of the HRTFs has the relatively better accuracy than that using the corresponding logarithmic ampitude under the assumption that the minimum phase approximation is accepted. Takane also concluded, in the case that the minimum phase approximation is not accepted, that the SPCA using the complex frequency spectrum of the HRTFs brings about better accuracy. In other words, the more compact representation of the spatial variation of the HRTFs is possible with their SPCA using the complex frequency spectrum of the HRTFs. Takane also mentioned that the accuracy in frequency domain, observed from the Spectral Distortion (SD) between the reconstructed and the original HRTFs, is exceptionally small when the logarithmic amplitude was chosen as the domain of the SPCA of HRTFs[8]. This property has a potential that the HRTFs are reconstructed by using the relatively small number of principal components when only the accuracy in frequency domain is focused.

Takane's discussion in his paper[8] was, however, based on the objective properties and its correspondence to the subjective evaluation is not clear. While the subjective evaluation of the reconstructed HRTFs using the SPCA was executed in some researches[2, 5, 10, 11], none of them did not compare the reconstruction results of the SPCA in multiple domains. Therefore, the subjective evaluation of the reconstructed HRTFs was carried out, and the correspondence between the results of the objective[8] and the subjective evaluations is to be discussed.

2 SPCA OF HRTFs IN MULTIPLE DOMAINS

For details of the procedures for the SPCA of HRTFs, please refer to the other researches (for example, [8, 9]). As Takane's previous research[8], the domain for the SPCA of HRTFs is expressed as a symbol as listed in Table 1.

Table 1. Correspondence between domains and symbols.

Domain	Symbol
HRIR	I
HRTF	C
Amplitude of HRTF	F
Real log-amplitude of HRTF	L

A database of HRIRs of KEMAR HATS (Head And Torso Simulator) presented by Media lab. of MIT[12] was used. This database involves 710 pairs of HRIRs (total: 1420) with sampling frequency of 44.1 kHz. For the subjective evaluation concerning the SPCA of HRTFs, various researchers used various set of HRTFs. Kistler *et al.* used the HRTFs of 10 subjects measured in 265 sound source directions per a subject, and the SPCA was carried out in the domain L[2]. Xie used those of 52 subjects, 493 directions, and the SPCA was in the domain F[5]. Matsui *et al.* used those of 108 subjects, 793 directions, and their SPCA was done in the domain L. Fink *et al.* used those of 34 subjects, 50 directions (in horizontal plane), and their SPCA was done in the domain I[11]. Generally the PCA using large amount of data brings about the PCs to reconstruct the data well, but we selected the HRTF database of a single HATS since we placed great importance on consistency between this and Takane's previous studies[8], and comparison among the results of the subjective evaluation of the reconstructed HRTFs with the SPCA of HRTFs in multiple domains.

2.1 Conditions of analysis

The initial delay in each response was extracted, then 256 sample points were taken as the data for the analysis, windowing with latter half of 512-points Blackman-Harris window function adjusting its peak at that of the HRIR. The SPCA was executed by constructing the covariance matrices from HRIRs (called as domain I),

HRTFs (domain C), amplitude of HRTFs (domain F), log-amplitude of HRTFs (domain L). The HRTFs/HRIRs in all directions (710 directions×2 ears) were used, and the average vector and the covariance matrix were calculated in each domain.

2.2 Cumulative Proportion of Variance (CPV)

When the PCA is utilized for some data, the cumulative proportion of variance $R^2(K)$, defined as follows, is used for the reference indicating how much variance in data is covered by using the first K principal components.

$$R^2(K) = \frac{\sum_{k=1}^{K} \lambda_k}{\sum_{k=1}^{N} \lambda_k}, \tag{1}$$

where λ_k denotes k-th eigenvalue of the covariance matrix. Change of the cumulative proportion of variance in each domain is plotted in Fig. 1. It is found out from this figure that the CPV is monotonically increased and converges to 1.0 as the number of components is increased in all domain. Among four domains, the domain C has the fastest increase as the number of PCs increases. This indicates that the the smallest number of principal components can cover a certain proportion of variance in the domain C.

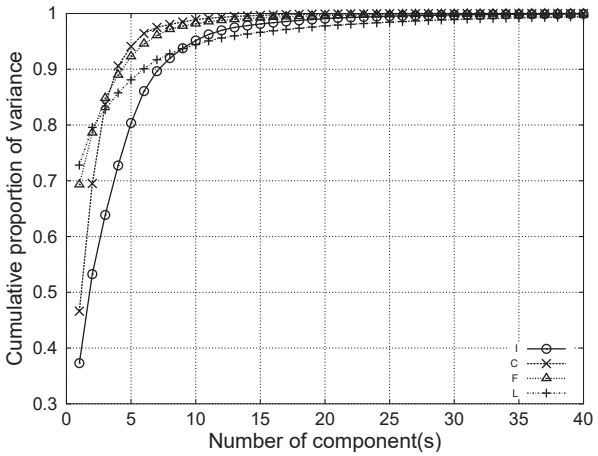

Fig. 1. Change in cumulative proportion of variance (CPV) with number of component in PCA.

The previous studies had various reference values in the proportion of variance, more than which the corresponding principal components are omitted. Kistler *et al.* set this value to 0.90[2], Chen *et al.* and Wu *et al.* set this value to 0.999[3, 4], and Xie set this to around 0.98[5]. Since the amount of data and analyzing purposes are different among these studies, direct comparison among these values is impossible. However, it can be said that the value more than 0.9 is set in these studies. After these studies, the least numbers of component to cover four values of the CPV, 0.90, 0.95, 0.99 and 0.999 are indicated in Table 2.

Table 2. The least number of component to cover the cumulative proportion of variance in each case.

Case	Variance			
	0.90	0.95	0.99	0.999
I	8	10	20	39
C	4	6	11	20
F	5	7	14	31
L	6	11	32	78

3 SUBJECTIVE EVALUATION OF RECONSTRUCTED HRTFs

In order to subjectively evaluate the reconstructed HRTFs with the SPCA of HRTFs in multiple domains, a hearing experiment was carried out.

3.1 Conditions

As for the SPCA of HRTFs, the HRTF database presented by MIT was used, and the properties of the SPCA is as mentioned in the previous section. In order to determine the numbers of PCs for the reconstruction in four domains, we executed an preliminary experiment using five male subjects with normal hearing acuity. The procedures were the same as those mentioned after this subsection, the numbers of PCs for the reconstruction were determined. For each domain, the stimuli made from the reconstructed HRTFs with the smallest and the largest number of PCs had respectively the obviously and hardly audible difference from the corresponding original ones for the subjects. Including an intermediate value between them, totally three numbers of PCs were set in each domain. The preliminary experiment was required for decreasing the number of conditions in the experiment. The determined numbers of the PCs are depicted in Table 3.

Table 3. Numbers of PCs adopted for the experiment for each domain.

Domain	Numbers of PCs
I	4, 7, 10
C	2, 4, 6
F	1, 4, 6
L	1, 3, 6

Table 4. Azimuths and elevations evaluated in the experiment.

Elevation [°]	Azimuth [°]
0	0~315 (45° inverval)
30	0~300 (60° interval)
60	0~240, (120° interval)
90	0

Number of evaluated sound source directions was decreased, and the result is shown in Table 4. Definition of azimuth and elevation is illustrated in Fig. 2. 18 directions of HRTFs were used for each domain, totally 216 ($18 \times 3 \times 4$) pairs of the reconstructed and original HRTFs were subjectively evaluated.

Participated subjects were four young males (aged 21-22) with normal hearing acuity. They are denoted as Subject 1, 2, 3 and 4, respectively.

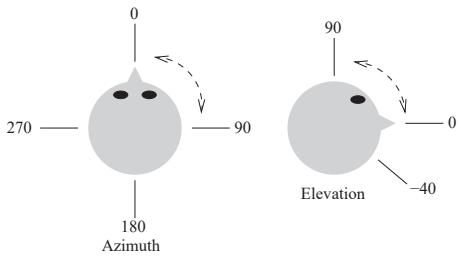

Fig. 2. Definition of azimuth and elevation.

3.2 Procedures

In the experiment, 3AFC (3 Alternative Forced Choice) was adopted. Time pattern of the stimuli is illustrated in Fig. 3. "Original" indicates the white noise filtered with the original HRTFs in a certain direction, and "reconstructed" indicates the one filtered with the reconstructed HRTF for the right ear and the original HRTF for the left ear. This means that the reconstructed HRTFs were used only for the right ear and the those for the left ear were always the original. The reason for this treatment is to avoid multiple factors affecting the audible difference between the original and the reconstructed HRTFs when the HRTFs for both ears are changed to the reconstructed ones. The stimuli were presented binaurally from the audio interface (RME Fireface UCX) via headphone (Sony MDR-MD900) with the inverse of the headphone transfer functions designed and filtered for each subject.

Subjects were asked to answer which one is audibly different from the other two out of the stimulus A~C. In the case of Fig. 3, the answer is correct if a subject answers B. The order of a "reconstructed" and two "original" stimuli was set random so that the "reconstructed" stimulus appears at A~C in equal numbers. The stimuli involving the reconstructed HRTFs with a certain number of PCs in a certain direction were presented 12 times, so the total number of judgments per a subject was $12 \times 216 = 2592$, taking about 4 hours. All subjects carried out the tasks in around 8 hours having the appropriate number of rests.

The above-mentioned procedures are adopted after Minnaar *et al.*, who investigated their HRTF interpolation method via hearing experiment[13]. The main reason is that the utilized set of HRTFs is acquired for a KEMAR HATS, and it differs from the HRTFs of every subject participated in the experiment. In that case, sound

localization experiment conducted in Kistler's group[2] cannot be done because non-individual HRTFs bring about some localization error such as front-back confusion[14]. Therefore we decided to focus on the perceptual difference between the original and the reconstructed HRTFs. The procedures in this study is also similar as one of the experiments done in Xie's research[5]. He used the same set of stimuli as ours (Fig. 3), but he asked his subjects to answer which of B and C perceives the same as A. Thus the subject's answer in his experiment was a 2AFC.

3.3 Results

The numbers of correct answers are listed in each domain for all subjects. The results are shown in Table 5~Table 8. Since each subject answered for 3AFC tasks in each cell in these tables, χ^2 test with chance level equal to 1/3 may be applied. However, number of testing was numerous per a subject (= 216), hence the Benjamini-Hochberg procedure[15] was applied to the obtained p-values from the cells in those tables. In this procedure, the false discovery rate was set to 0.15. As a result, number of correct answers is regarded as significantly large when its value is equal to and more than 10 for all directions and subjects.

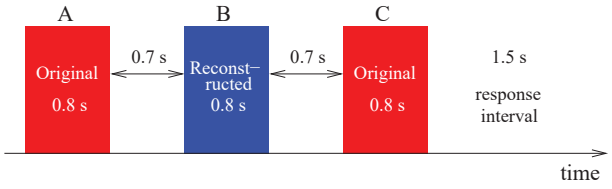

Fig. 3. Example time pattern of a combination of three stimuli presented in the experiment. A combination consists of two white noises filtered with the original HRTFs (illustrated in red rectangles) and a white noise filtered with the reconstructed HRTF for the right ear and the original for the left ear (illustrated in blue). The order of a blue and red stimuli was random.

Table 5. Numbers of correct answers obtained for the reconstructed HRTFs in various domains (Subject 1). The values with significantly large number of correct answers according to the Benjamini-Hochberg procedure (false discovery rate = 0.15) are indicated with bold font.

(a) Domain I

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	4	8	6	5	4	9	8	**11**	9	2	8	6	5	5	5	6	5	6	4
	7	8	5	2	3	5	5	**11**	4	3	4	5	7	6	4	6	5	3	2
	10	3	3	4	2	**10**	7	6	4	5	4	5	7	3	5	2	3	5	3

(b) Domain C

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	2	7	3	3	3	7	6	8	7	7	2	5	9	4	6	7	2	5	9
	4	8	2	4	5	6	2	5	4	7	3	9	5	5	6	3	6	4	5
	6	5	5	3	4	5	5	6	6	5	2	5	5	0	3	4	6	3	4

(c) Domain F

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	1	6	5	5	3	6	8	5	6	2	4	5	9	9	6	6	3	6	3
	4	3	9	5	3	7	6	2	5	3	6	4	2	6	6	2	1	6	4
	6	3	3	2	3	4	6	5	4	4	6	3	3	6	5	3	4	4	6

(d) Domain L

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	1	6	5	7	7	2	7	4	6	5	7	2	2	4	3	4	3	3	3
	3	6	5	2	6	2	4	7	5	5	6	1	3	2	3	2	3	6	3
	6	7	7	3	4	5	5	4	6	2	5	4	2	5	1	4	5	1	3

Table 6. Numbers of correct answers obtained for the reconstructed HRTFs in various domains (Subject 2). The values with significantly large number of correct answers according to the Benjamini-Hochberg procedure (false discovery rate = 0.15) are indicated with bold font.

(a) Domain I

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	4	**10**	4	3	6	7	7	**10**	8	4	8	6	3	4	9	4	6	6	3
	7	**11**	6	4	2	6	4	**10**	**11**	5	4	5	5	3	9	4	6	5	5
	10	7	6	3	5	7	5	5	3	3	5	5	2	5	4	5	7	3	5

(b) Domain C

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	2	**11**	5	6	4	7	6	9	5	4	6	7	8	5	7	7	5	3	**10**
	4	**10**	3	4	6	5	5	5	9	4	3	7	7	3	8	1	5	5	4
	6	8	8	6	8	4	4	5	8	5	4	5	4	4	7	4	5	6	3

(c) Domain F

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	1	9	5	6	9	7	5	**10**	8	**12**	8	7	**11**	**12**	10	8	5	5	9
	4	8	5	5	7	5	8	8	4	9	6	7	**10**	6	**10**	5	7	3	7
	6	5	6	4	2	4	9	8	7	5	6	7	7	5	8	2	7	7	6

(d) Domain L

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	1	**10**	6	2	11	5	9	6	9	8	5	6	6	3	8	5	3	6	7
	3	4	6	5	**10**	6	4	5	9	3	4	5	2	4	3	3	8	1	4
	6	4	4	3	5	**10**	6	2	7	5	8	6	6	4	5	6	2	5	6

3.4 Discussions

At first, it must be specified that the numbers of PCs used in the experiment for all domains are very small comparing with those stated in Takane's paper[8]. This means that the number of required PCs is relatively small so as to reconstruct the HRTFs which do not bring about the significantly audible difference. Although the range of the number of PCs was determined in the preliminary experiment, the results of the experiment support such determination. Seeing Table 5~Table 8, there exist few conditions with significantly large number of correct answers. This indicates that the reconstructed HRTFs did not bring about the audible difference in most of conditions of the experiment. Relatively small numbers of PCs in each domain may be owing to the data for the SPCA in this paper. A database of HRTFs of a single HATS were used in this study, while many researches carried out the SPCA of HRTFs with multiple subjects [2, 5, 10, 11].

Comparing among the results for four domains, the domain F has the most cells with significantly large number of correct answers. However, such cells are distributed in the smaller number of PCs except a few conditions. The domains C and L have relatively small number of cells, especially there were no subjects having significantly large number of correct answers when the number of PCs is 6 in the domain C. there are seldom conditions For all domains, such cells are located at two types of sound source directions. One of them is completely front or directions with slight ITD and/or ILD (Table 6(a), (b), (d); Table 7(a),(c); Table 8(a), (b), (c)). The reason for this tendency may be that the reconstructed HRTF was used only for the right ear. Slight change in ILD and/or ITD may help discriminating the stimulus with the reconstructed HRTF from that with the original one. The other one is when the sound source is located on the left side (azimuth ranged 225° to 315°). In this condition, the reconstruction accuracy for the contralateral (right) HRTFs may be degraded[3, 5, 8].

Table 7. Numbers of correct answers obtained for the reconstructed HRTFs in various domains (Subject 3). The values with significantly large number of correct answers according to the Benjamini-Hochberg procedure (false discovery rate = 0.15) are indicated with bold font.

(a) Domain I

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	4	**11**	7	7	7	4	8	**11**	8	4	6	7	7	6	7	4	5	6	7
	7	9	7	6	7	6	4	8	4	3	5	7	4	8	6	5	4	5	7
	10	8	4	4	7	5	5	7	7	3	3	4	3	5	6	6	3	6	4

(b) Domain C

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	2	9	8	6	7	9	7	9	5	5	6	5	5	6	5	6	6	2	9
	4	8	3	7	7	7	6	4	9	5	3	5	3	7	4	2	2	7	5
	6	4	6	4	3	5	1	7	8	6	3	6	4	4	7	3	4	6	6

(c) Domain F

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	1	7	7	4	6	7	**11**	4	**10**	**10**	**10**	7	6	9	8	5	9	7	6
	4	4	5	6	8	6	**10**	6	5	9	6	6	5	5	**12**	5	4	3	7
	6	6	4	5	7	6	9	**10**	4	7	4	8	6	7	5	3	7	3	6

(d) Domain L

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	1	7	7	2	7	9	**11**	8	**11**	6	8	7	7	5	7	8	6	2	5
	3	7	5	8	7	8	8	7	5	5	4	6	3	2	6	6	3	3	6
	6	7	5	5	3	7	8	7	6	3	4	5	6	2	5	5	4	4	4

4 CONCLUDING REMARKS

In this paper, the subjective evaluation of the HRTFs reconstructed with the SPCA were mentioned. As a result, it was found that the reconstruction of the HRTFs with subjectively inaudible difference from the original ones are possible with smaller number of PCs than those Takane dicussed in his paper[8]. This brings about a expection that the spatial variation of HRTFs can be represented more compactly. Throught the 3AFC hearing experiment, it is found that the domain C and L have the relatively small number of conditions with significantly large number of correct answers. Especially, there were no subjects having significantly large number of correct answers when the number of PCs is 6 in the domain C.

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Table 8. Numbers of correct answers obtained for the reconstructed HRTFs in various domains (Subject 4). The values with significantly large number of correct answers according to the Benjamini-Hochberg procedure (false discovery rate = 0.15) are indicated with bold font.

(a) Domain I

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	4	**11**	3	5	6	6	6	9	7	3	3	3	5	5	4	7	5	0	5
	7	9	6	4	1	4	6	4	7	5	3	3	5	3	2	4	4	5	2
	10	7	4	6	7	4	7	8	6	4	3	4	6	4	3	4	3	3	4

(b) Domain C

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	2	8	5	3	7	6	6	9	4	5	7	7	5	5	4	7	3	6	9
	4	**12**	6	5	6	6	4	5	7	2	5	6	4	8	4	6	4	5	4
	6	6	8	4	6	2	3	8	4	4	5	6	8	3	3	2	4	4	5

(c) Domain F

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	1	8	7	5	5	**11**	**10**	8	9	5	4	6	8	**12**	**10**	3	5	5	9
	4	7	5	6	8	5	**10**	6	5	6	6	4	5	7	9	2	3	1	4
	6	8	6	4	8	8	**12**	6	9	7	5	5	7	7	2	6	3	7	10

(d) Domain L

Elev. [°]		0								30						60			90
Azim. [°]		0	45	90	135	180	225	270	315	0	60	120	180	240	300	0	120	240	0
PCs	1	5	7	8	**11**	7	**11**	8	3	**11**	6	5	7	7	**10**	8	4	4	5
	3	4	4	8	4	5	6	6	8	2	5	4	5	5	4	3	4	1	2
	6	5	5	5	5	7	1	9	5	3	4	6	6	7	4	6	1	5	3

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