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Measurement and modelling of the Japanese *koto*: Problems and solutions

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ABSTRACT

There are few studies of the acoustical properties of the Japanese *koto* (zither) resonances. As part of the measurement of a hand-crafted professional-grade koto as the basis of the construction of a high-resolution finite element model using COMSOL Multiphysics® software, three major problems had to be resolved and are discussed. First, issues relating to measuring the complex geometry were only resolved by obtaining 2300 cross-sections of the koto in a high-resolution computer tomography scan. These revealed hitherto unseen details such as the roughhewn interior. These were then used to create a finite element mesh for modelling with up to 34 million degrees of freedom. This significantly improved the accuracy of the modelling. Second, lack of information on the paulownia wood for key model inputs required scientific evaluations of physical properties by microscopy and independent testing. Model accuracy was further improved when this data was combined with knowledge of the traditional Japanese grade system relating to grain structures used in different instrumental components. Third, identifying the peaks from the results of modelling also proved to be an ongoing problem that required a number of solutions. Results of the modelling of the acoustic characteristics of the koto are also briefly reported.

Keywords: finite-element, modelling, koto

1. INTRODUCTION

The *koto* (plucked zither) has deep cultural significance in Japan and instruments dating from up to 1000 years ago are still extant. It is made from paulownia wood, is 1.83m long and has thirteen strings with movable bridges. It is recognized for its complex resonances, but the acoustics of the koto have not been investigated to the same extent as the stringed instruments of Europe. Research by Ando (1, 2) remains a principal source while other information is found as part of more general discussions of wooden stringed musical instruments (3,4) including those of Asia (5). Finite element studies of Asian instruments are still rare (6) even though studies of European stringed instruments such as the violin (7) using these methods are increasing.

More recently, the construction and validation of a high-resolution finite element model of this author's hand-crafted professional grade koto using COMSOL® Multiphysics Versions 5.2 to 5.4a has been reported (8). The basis of this model was 2300 DICOM images of cross-sections of the koto obtained from a high-resolution computer tomography CT scan. This model (hereafter the CT model) was the final outcome of a series of models including simple box models which remain an important reference point (9). A multi-faceted validation process including a number of experiments using equipment such as an acoustic camera, a laser scanning vibrometer (LSV) and techniques such as Chladni patterns was undertaken. When appropriate, model results were compared to the actual koto including Fourier Transforms of the waveform as played by the author. Few details of the challenges associated with obtaining measurements prior to the CT scan and constructing the finite element model of the koto were provided previously. This paper identifies three key problems and discusses solutions found that enabled the delivery of the CT model. These are presented not only to provide a basis for repeatability of the koto studies by other scholars, but it is also believed that such solutions are more generally relevant to advancing finite element modelling of stringed, wooden instruments.

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Before proceeding to discussion of the problems, some additional details of the specifications of the CT model are needed. The computer used for the finite element modelling had two Intel Xeon Gold 6134 CPU @ 3.20GHz, 3201 MHz processors, with 8 cores each and required substantial memory (1 TB RAM). The physical properties of paulownia were provided by Chris Waltham from the University of British Columbia (10). The geometry used was based on a CT scan of the whole instrument, converted from DICOM images to a Simpleware mesh consisting of 966,311 elements with a minimum quality of 0.07064 and average quality of 0.6586, and imported directly into COMSOL.

No arbitrarily adjusted constants were used.

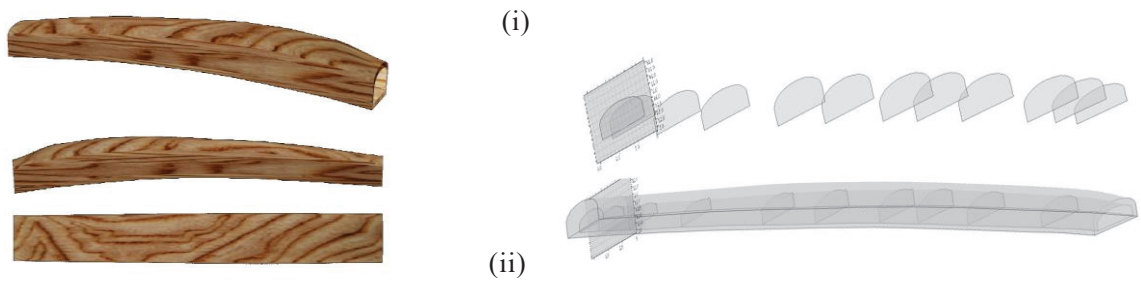
2. PROBLEMS AND SOLUTIONS

This section focuses on three problems and their solutions for the measurement and finite element modelling of the koto. Many solutions are not self-contained and often included a number of solutions found over time. For example, cases such as the development of the mesh or considerations of grain coordinates arose as a result of identifying solutions to related problems. The section discusses not only their implications for constructing the CT model and interpreting results to characterize the acoustics of the koto in particular, but also reflects on their potential application to finite modelling of stringed, wooden instruments more generally. The issues of modelling the strings will be reported separately (11).

2.1 Measuring the Geometry and its Implications for the Modelling

The instrument used as the basis for the study is hand-crafted using centuries old methods. The external geometry has curves in all directions and tapered along its length. The interior was produced by using an adze to hollow out the tree trunk and then planed to smooth some surfaces. Initial construction of the model was based on the assumption that a box model would be adequate, but it was not. Furthermore, it was found that the geometry cannot be constructed from simple primitives in general and those in Comsol in particular. Such factors have major implications for constructing a finite element model.

A number of solutions to address these problems were investigated (see Figure 1). The external shape of the koto was initially created using NURBS (non-rational B-splines) in Autodesk 3DSMax Design. However, it required extensive reworking of the mesh, which was mostly unsatisfactory. A lofted model was therefore used to increase the accuracy. Lofting is a technique available within Comsol's Design Module. Cross-sections based on external measurements were formulated and an external shape built around them in the finite element software to make an outer-casing based on these measurements. A partial loft to make the inner shell and end plates was followed by Boolean subtraction to create a hollow shell with end plates. Final touches were added as upper and lower sound holes were punched in the base by shaped cylinders, then internal struts constructed. As the model was constructed totally within the same software platform there were no issues importing shapes from other software and meshing was less of a problem. Modelling was thus able to proceed.



(a) Solution 1: NURBS (b) Solution 2: Lofting (i) Outer loft (ii) Inner loft by subtraction

Figure 1 – Two solutions for creating the external geometry of the koto for modelling

The lofted model and box models remain useful tools for rapid analysis. Such analysis of results not only can be instructive for acoustic studies, but also help to gauge the accuracy of the CT model. For example, comparison with experimental work during the validation process suggested that internal

details such as the thickness of the side walls or measurement of the hand-made interior which were invisible by standard techniques were needed for optimization of the model.

The use of non-destructive methods such as x-rays to address the problem of measuring the internal geometry were investigated, but the length of the instrument presented difficulties. However, the commissioning of a new, high-resolution computer tomography CT scanner (Toshiba Aquilion ONE) at the Royal Adelaide Hospital offered an unexpected solution. As a result, 2300 DICOM images of the koto were made in both horizontal and vertical planes. The cross-sections successfully captured the irregularities of the hand-crafted instrument (see Figure 2) and images were readable in MATLAB for visual inspection. JPEGs were then produced for detailed measurement and analysis. We could now not only see inside the koto in minute detail, but there was also a bonus — the DICOM images could be used to produce a mesh which included both internal and external geometry. The extra details of the internal measurements were also incorporated into a revised lofted model for greater accuracy of those simulations. Thereafter two koto finite element models have been routinely used; the lofted model with a resolution suitable for standard computer specifications for rapid analysis and a high-resolution CT model.

(a) Horizontal perspective

(b) Vertical perspective

Figure 2 – Examples of DICOM images from CT scan of the koto

While the scanning did resolve one problem, it raised two new problems associated with the finite element mesh. First, the 2300 DICOM cross-sections could not be transferred directly into Comsol to create a mesh. This was resolved by using Simpleware ScanIP, the standard program for conversion of medical CT scans for finite element modelling. It was able to segment the koto into seven components: its top plate, base plate, internal partitions, struts, bridges, feet, and end plates. Despite difficulties associated with separately segmenting several components comprised of the same wood, and local greyscale variation in wooden parts due to the wood's grain, accurate segmentation was achieved. In regions where the image data was distorted due to metal artefacts associated with the bridges, the interpolation toolbox was used to ensure these artefacts did not affect the quality of the segmentation. Using the Simpleware FE module, a quality mesh of the *koto* of 966,311 elements with a minimum quality of 0.07064 and average quality of 0.6586. The +FE free meshing algorithm ensured that the model was true to the image geometry while keeping the element count to appropriate limits. The mesh was then exported to Comsol and boundary conditions were applied to allow simulations.

The second problem was the significant increase in computing power now needed for the new mesh This was resolved in two ways. First, the shorter (duration up to two days) simulation runs were made with the instrument placed at the centre of a 2m diameter sphere of air. This assembly typically consists of 8.9 million domain elements and 34 million degrees of freedom. For longer runs, typically transient studies of greater than 10 days, the model size is reduced by placing the koto along the long axis of a cylinder that reduces the number of domain elements to 4.2 million and the degrees of freedom to 10 million with a proportional reduction in run time. The mesh element size always encloses a minimum of six elements per wavelength. The second solution was to upgrade the computer.

The CT model has been used to carry out a range of simulations based on the solid mechanics of the koto's natural vibrations and pressure acoustics (sound production in air), measured at twelve locations at 1/48,000 second intervals. Figure 3 shows the koto's first ten eigenmodes with the lowest frequency of 99 Hz and more complex modes that contribute to the koto's distinctive resonances.

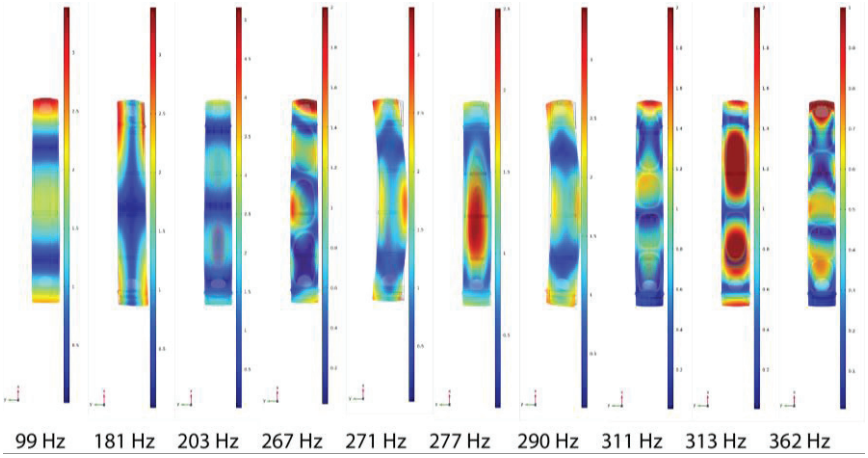
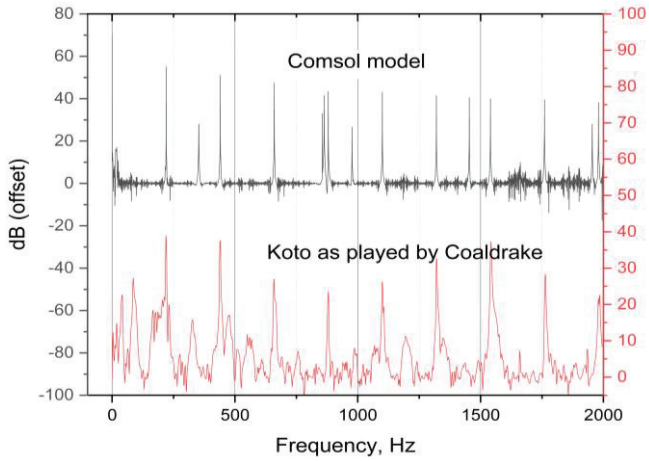


Figure 3 – The first ten eigenmodes of the koto

Modelling and verification from the acoustic camera also identified and established that two frequencies dominate the koto's acoustic characteristics —the 99 Hz as (0,1), 84 Hz as an air cavity resonance as well as details of other resonances. Comparison between Fourier Transform of a recording of the koto and the CT model played at 220 Hz is shown in Figure 4.

(a) Japanese koto as played at 220Hz (12)



(b) CT model results

Figure 4 – Comparison of the koto as played and CT model results

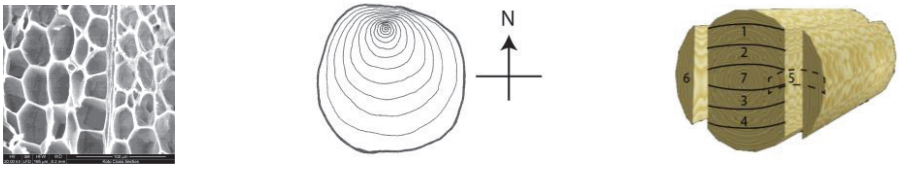
Importantly, the increasing resolution of the CT model has revealed greater activity in the lower frequencies with a number of different techniques, for example 50 Hz with extremely high displacement, but it is not yet possible to explain its cause. The problems arising for modelling due to

the difficulty of obtaining accurate physical properties of the paulownia wood remains a critical issue and this is now discussed.

2.2 Modelling the Physical Properties of Paulownia Wood

2.2.1 Paulownia wood and its grain structure

The koto is made of *paulownia tomentosa,* (*kiri* in Japanese), also known as the empress tree because of its historical connections with the Japanese Imperial family. It is a wood commonly used in East Asian stringed musical instruments. There are, however, more than thirty names for various paulownia trees, but only *p. tomentosa, p. kawakamii, p. elongata, p. fotrunei* and *p. fargesii* appear on most botanical classifications lists as a separate species. The scanning electron microscopy (SEM) images of the author's koto (see Figure 5 (a)) reveal that the wood is highly anisotropic. It is categorised as a soft hardwood with low density (250-280kg/m^2) and is highly resonant. The top plate of the instrument is cut from the natural curve of the side the vertical trunk and the cross-grain structures often used for the bottom plate or struts are cut from the internal section of the trunk (see Figure 5 (b)). Grades of koto allocated in a hierarchical system during construction are determined by the cut of the wood from the tree construction techniques and materials for accessories in accordance with traditional practice (12). For example, a 7-grade system (see Figure 5 (c)) designates Grades 1 to 4 as average, Grade 5 as cross-grain and Grades 6 and 7 as the highest. The grain of the top plate (soundboard) is generally longitudinal along the length of the instrument.



(a) (b) (c)

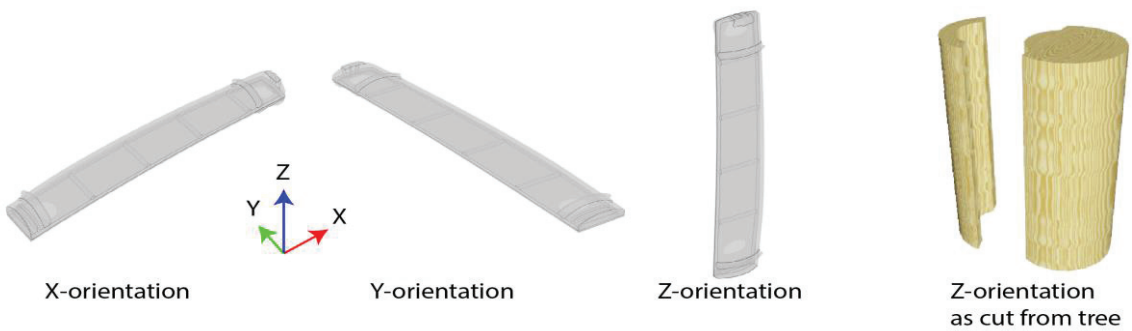
(a) SEM of paulownia from the author's koto
(b) Compression of wood grain rings on the northern side of the tree
(c) Dissections of tree used as basis for the traditional hierarchical koto grading system

Figure 5 –Understanding the grain structure of paulownia wood

2.2.2 Constitutive Equations and Grain Coordinates

Two problems arose for the modelling of the physical properties associated with the constitutive equations that related the stress and strain through a matrix which, for this purpose, was the Voigt matrix.

<u>*Problem1*</u>*: Orientation in the Cartesian coordinate system*. The physical properties of the paulownia wood are entered as an anisotropic Voigt matrix within Comsol and applied to the koto shape. This in turn is placed into a 3D sphere (or cylinder) of air in any of the normal orientations, x, y and z. While there was no *a priori* reason to prefer one orientation over another, as the main body of the instrument is cut from the vertical aspect of the tree there could be a case made for the instrument to be aligned in the z orientation as shown in Figure 6.



(a) (b)

Figure 6 – X, Y and Z orientations in relation to (a) the koto and (b) a paulownia tree

The consequence of a wrong assignment, however, is major as can be seen from the distribution of eigenfrequencies given by Comsol for each orientation as shown in Figure 7. The first eigenfrequency for x, y and z is 100 Hz, 35 Hz and 24 Hz respectively with no rational way to resolve the difference. Simple methods such as tapping the instrument gave frequencies as low as 24 Hz but they were routinely dismissed as spurious electrical interferences, even low flying aircraft on the approach to an international airport. The same frequencies kept being observed however and could not be ignored but they did not seem reasonable.

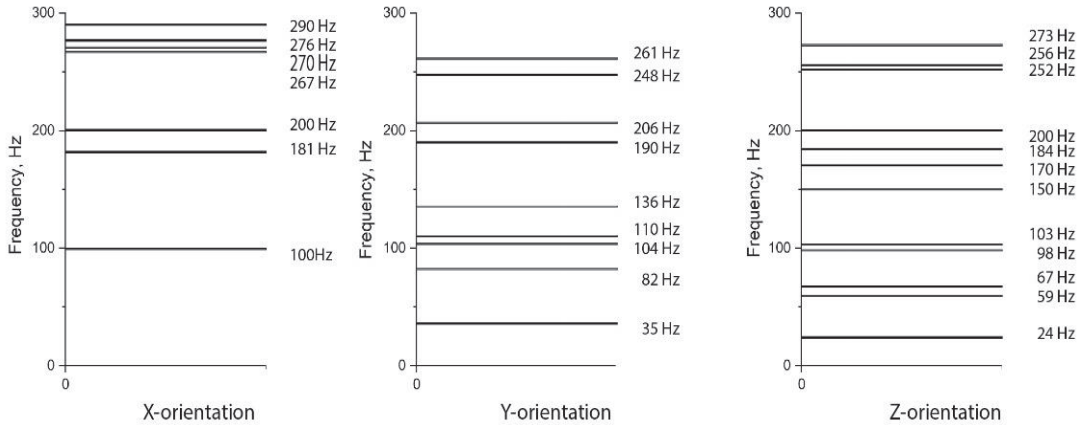
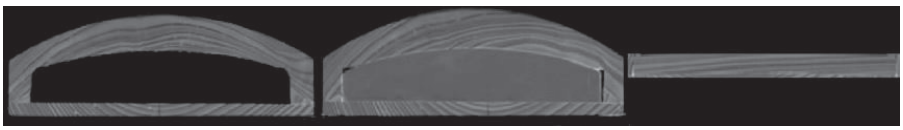


Figure 7 – Comparison of the eigenfrequencies for the three primary orientations of the koto to ascertain the orientation that relates best to the actual instrument as played

The study was at an impasse for an extended period until data became available from, firstly Chladni patterns showing clearly that the (0,1) mode was at 100Hz and the (0,0) mode at 85Hz which the acoustic camera confirmed. However even it continued to show the existence of the low frequency components. They would appear to be real, albeit very confusing. The x-orientation was taken and the project rapidly advanced from there.

<u>Problem 2</u>: *Observations of grain orientation based on the CT scan*. The CT scan of the koto was set up by the technician to image soft tissue. It consequently produced clear images of the grain pattern of the interior cavity that were much clearer than human observation. This highlighted two issues. First, the wood grain was not homogeneous so some averaging would be needed, but the fraction of each component over which the average would need to be calculated varied along the length of the instrument across the 2300 cross-sections. Second, it was clear that different components of the instrument had grain at different angles (see Figure 8). This in turn meant that whatever the matrix used for one direction would need to be rotated for the other two directions.

The components for the Voigt notation for paulownia used in the CT model was established by reference to the Waltham data used in the CT scan. It now remained to fit the remaining two orientations of the wood to the existing data. The CT scan showed that one single constitutive equation for the wood was insufficient. Leaving aside the issue of non-homogeneity, three orientations were observed— the top plate/body, an internal plate and the struts with the corollary that three Voigt matrices would be required.



(a)Top plate/body b) Internal plate c) Struts

Figure 8 – Three orientations of wood grain identified by the CT scan

After assuming that the wood used for the construction of the instrument was from the same or similar trees, it was concluded that the original Voigt matrix was valid and only needed to be rotated. This proved difficult even after consultation with experts. Finally, Comsol support in Stockholm reported that it was unnecessary to rotate the matrix. Rather, the curvilinear coordinates function could be used and a local rotation of the coordinate system applied to any chosen domain then rotated at any arbitrary angle— the so-called Euler angles— allowing a rotation of the coordinate system by $\pi/2$ radians in the Y-plane (see Figure 9) to be applied to the struts and a rotation of $\pi/2$ to the other identified components in the Z-plane. These rotations were applied successfully.

	α	β	γ
Main koto body	0	0	0
Struts	0	π/2	0
Internal plate, other	0	0	π/2

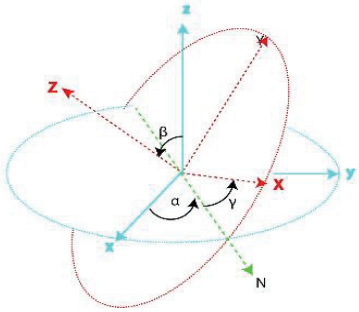


Figure 9 – Euler angles for local rotation of physical properties

The success of the rotation by this method shows that the problem in this case is not a conceptual one, but the pragmatic problem of obtaining information on most woods and especially when more obscure wood such as paulownia are required. It still leaves unanswered questions. For example, it is generally assumed that Hooke's Law which establishes elastic constants in three dimensions is an absolute constant for deformation. This is correct for metals and many other materials which are isotopic. However, for organic materials such as wood it may not hold as rigorously because of other factors such as anisotropy, geographical variation, humidity and surface treatments. Given this variability, a question remains, is a Voigt matrix adequate for describing extra non-linear properties associated with the highly anisotropic paulownia wood?

2.3 Peak Identification

Identifying the peaks from the results of the CT model has been an ongoing problem. With no reference points from other studies other than Ando's Chladni patterns, identification of the peaks was often perplexing so a number of solutions were investigated

First, from the outset it was realised that understanding the properties of paulownia was critically important to the overall results of the modelling. The highly anisotropic nature of paulownia as confirmed by the SEM of the koto's wood and subsequently highlighted by studies of the grain components mean that isotropic parameters alone are inappropriate for koto modelling even though there may be a smaller impact on results with less anisotropic woods. The interpretation of modelling results has only emerged over time as the problems associated with physical properties and the measurement of complex geometry have been incrementally applied.

Second, as part of the validation, it was only possible to identify peaks when results from the CT model were compared with experimental data from the acoustic camera, laser scanning vibrometer and Chladni patterns. In this process, overtones are obvious so these can be manually removed from the spectrum while the remaining peaks are assumed to include beats or are a possible result of amplitude modulation. However, many others are not so obvious. Nevertheless, the identification of the first eigenmode was resolved by cross-reference to results from the acoustic camera, LSV and Chladni patterns. Air cavity resonances were determined by using the CT model as an experimental tool.

Identifying peaks below 85 Hz are still far more problematic. Interestingly, many areas of the biological and chemical sciences commonly use vast digital libraries and specialized equipment developed over an extended period to identify peaks. The data for the wood for many musical instruments such as paulownia wood, is under-studied and would appear to need a corresponding data base. In the next phase of work, the objective is to continue to use the CT model as an experimental tool, for example, to make selective substitutions or by switching components of the components or off or on in order to assign the origin of some frequencies, something that can be readily done in the model, but not easily in a real instrument.

3. CONCLUSIONS

The necessity to find solutions for the myriad problems faced during the construction process of any finite model is well-known and inevitably arises from the specific details of the object chosen for study. The measurement and modelling of the koto however presented a surprising number of problems. The lack of information on the physical properties of paulownia wood, the measurement of the organic complex shape and the challenges to interpret peaks encountered are notable. The solutions offered here are ones that have been found to work and believed to be more generally applicable for finite element modelling. Clearly obtaining reliable physical properties of wood remains one of the biggest challenges. While software programs such as Comsol typically have libraries of materials and models, there are still major gaps in the characterization of lesser-known woods such as paulownia. This is particularly important for the study of Asian stringed instruments and the vast heritage of instruments that are found in cultures along the Silk Road. It suggests that there is a need for a centralized data base for finite element modelling

The use of a range of emerging technologies such as the latest high resolution tomography CT scans and the acoustic camera, was critical to solving seemingly intractable problems. These solutions enabled higher levels of accuracy for the measurement and modelling of the acoustics of the koto. It is expected that many of these techniques will in future become more readily accessed by researchers. This study demonstrated the extraordinary advantages and insights that they can offer. Nevertheless, many challenges still remain.

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