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Attenuation mechanisms of high-frequency acoustic waves in piezoelectric cubic crystals

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ABSTRACT

Attenuation mechanisms of high frequency acoustic waves in piezoelectric bismuth silicate (BSO) and bismuth germanate (BGO) crystals has been studied in the frequency range of 0.4–1.6 GHz. It is shown that the main mechanisms of attenuation are the Akhiezer mechanism and the effect of dielectric losses due to the piezoelectric coupling of deformation with polarization in piezoelectric crystals. To describe the attenuation anisotropy of acoustic waves, real and imaginary elastic moduli of these crystals were determined. It is established that the most contribution of dielectric loss to the attenuation of acoustic waves is observed for the piezoactive transversal waves propagating in the (100) plane in bismuth germanate crystals. The dielectric loss can also affect the frequency dependence of the attenuation coefficient of piezoactive acoustic waves.

Keywords: Acoustic wave, Attenuation, Piezoelectric crystals

1. INTRODUCTION

The piezoelectric crystals of bismuth silicate $Bi_{12}SiO_{20}$ (BSO) and bismuth germanate $Bi_{12}GeO_{20}$ (BGO) are widely used as an active medium in different optic and acoustooptical devices [1-2]. One of the main parameters of these devices is the attenuation coefficient of the acoustic waves, which can be calculated using the real and imaginary components of the complex elasticity tensor [3, 4]:

$$c_{ijkl} = c'_{ijkl} + ic''_{ijkl}, \tag{1}$$

which enter into the expression of effective elastic constant:

$$c_{eff} = c'_{eff} + ic''_{eff} = c'_{ijkl}\kappa_j\kappa_l\gamma_i\gamma_k + ic''_{ijkl}\kappa_j\kappa_l\gamma_i\gamma_k, \tag{2}$$

where κ_j and γ_i are the direction cosines of the wave vector and the displacement vector, respectively.

Equations (1) and (2) are valid for centrosymmetric dielectric crystals, in which the Akhiezer mechanism is the main mechanism of attenuation of acoustic waves [1].

As is known, in crystals with piezoelectric properties (non-centrosymmetric crystals), piezoactive acoustic waves are accompanied by a polarization wave of the electric field [1]. In piezoelectric crystals, at large coefficients of electromechanical coupling, the dispersion of the sound propagation speed and the

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contribution of dielectric losses to the attenuation of acoustic waves due to the phonon-dipole interaction mechanism can be observed. As a result of this interaction, an additional attenuation of the acoustic wave can be occurred.

A related system of equations of elastodynamics and electrostatics for a piezoelectric crystal, describing the propagation of acoustic waves in a piezoelectric crystal, can be represented as [1, 3]:

$$C_{ijkl}\frac{\partial^2 u_k}{\partial x_j \partial x_e} + e_{nij}\frac{\partial^2 \varphi}{\partial x_n \partial x_j} = \rho\frac{\partial^2 u_i}{\partial t^2} \tag{3}$$

$$e_{mkl}\cdot\frac{\partial^2 u_k}{\partial x_m \partial xl} - \varepsilon_{mn}\cdot\frac{\partial^2 \varphi}{\partial x_m \partial x_n} = 0 \tag{4}$$

where e_{mkl} are the components of the piezoelectric coefficient tensor, φ is the potential of an alternating electric field arising during the propagation of an acoustic wave. u_i are the components of the displacement vector, $\varepsilon_{mn} = \varepsilon'_{mn} - i\varepsilon''_{mn}$ are the components of the complex tensor of dielectric constants.

Taking as the solution of the system of equations the equation of a plane damped wave:

$$u(\vec{r},t) = u_0\cdot e^{-\alpha\cdot t}\cdot e^{i(\omega\cdot t-\vec{q}\cdot\vec{r})}, \tag{5}$$

we can get the well-known Christoffel equation and expression determining the attenuation of acoustic waves in piezoelectric crystals:

$$\alpha = \pi\nu\frac{c''_{eff}}{c'_{eff}} \tag{6}$$

where

$$c'_{эфф} = c'_{ijkl}\cdot\kappa_j\kappa_l\gamma_i\gamma_k + \frac{(e_{nij}\cdot\kappa_n\kappa_j\gamma_i)^2}{\varepsilon_0\varepsilon_{rs}\kappa_r\kappa_s} \tag{7}$$

$$c''_{эфф} = c''_{ijkl}\cdot\kappa_j\kappa_l\gamma_i\gamma_k + \frac{(e_{nij}\cdot\kappa_n\kappa_j\gamma_i)\cdot(e_{mkl}\kappa_m\kappa_l\gamma_k)}{\varepsilon_0\varepsilon_{rs}\kappa_r\kappa_s}\cdot tg\delta_{mn}\cdot\kappa_m\kappa_n \tag{8}$$

In equation (8), $tg\delta_{mn}$ characterizes dielectric losses that occur in dielectrics due to polarization relaxation.

The real components of the elastic constant tensor c'_{ijkl} for these crystals were measured in a number of papers [5]. As for the imaginary components c''_{ijkl} which describe the attenuation of acoustic waves, they were practically not investigated. In the present paper, the detailed study of the complex tensor of elastic constants of BSO and BGO crystals was carried out to determine the anisotropy of attenuation of high-frequency acoustic waves at room temperature.

2. EXPERIMENTAL METHODS

The crystals $Bi_{12}SiO_{20}$ and $Bi_{12}GeO_{20}$ were grown by the Czochralski technique on a Donets-1 setup. The crystals were prepared in the form of a parallelepiped with sides 16×8×4 mm. The faces of samples have been oriented with accuracy no worse than 1° along the <001>, <110> and <111> crystallographic directions. All faces of the parallelepipeds were processed by mechanical polishing and polishing with optical accuracy.

The velocity and attenuation of acoustic waves were investigated with the help of Bragg diffraction of light on acoustic waves in frequency range 0.4–1.6 GHz. Longitudinal and transverse acoustic waves were excited by piezoelectric transducers of quartz, accordingly of the X-cut or Y-cut. The helium-neon laser with a wavelength of 632.8 nm was used as a light source. The intensities of diffracted light were measured with the help of photoelectric multiplier.

The acoustic wave velocity V was determined from the Bragg diffraction angle with the accuracy of 0.2% [1]:

$$V = \frac{\lambda \cdot \nu}{2 \cdot \sin \theta_B}, \tag{9}$$

where ν is the linear frequency of acoustic wave, θ_B – the external Bragg angle.

The acoustic wave attenuation has been investigated for longitudinal and transverse acoustic waves propagating along the investigated directions of these crystals. The attenuation coefficient was calculated with the accuracy ~ 5% by the formula:

$$\alpha = \frac{10 \ell g\left(I_1 / I_2\right)}{2L_0}. \tag{10}$$

Here L_0 is the length of the sample, I_1 and I_2 are the intensity of the light diffracted on the direct acoustic wave and the intensity of the light diffracted on the wave, reflected from the free end of the sample, respectively. These measurements were carried out at the same point of the sample along the wave propagation direction.

3. RESULTS OF EXPERIMENTS AND DISCUSSION

The experimental values of the velocity and the attenuation of acoustic waves along the <001>, <110> and <111> directions were used to determine all the independent components of the complex tensor of elastic constants c_{ijkl}, which define effective elastic constant in equation (2). The results of the calculations are presented in table1.

For example anisotropy of the velocity and attenuation of acoustic waves in BGO and BSO crystals was considered for the (001) plane, which is orthogonal to the symmetry axis of the second order. The direction cosines of the wave vector of waves propagating in this plane are equal to $\kappa_1=\cos\psi$, $\kappa_2=\sin\psi$, $\kappa_3=0$. Then the components (respectively real or imaginary) of the Green-Christoffel tensor can be written in the form [1, 4]:

Table 1 – Effective elastic constants of germanate and bismuth silicate crystals (ν = 1 GHz)

q	γ	Expression	c'_{eff}, 10^{10} N·m^{-2}		c''_{eff}, 10^{7} N·m^{-2}	
		c_{eff}	BGO	BSO	BGO	BSO
[100]	[100]	c_{11}	12.8	3.72	2.44	2.60
	[001]	c_{44}	2.57	1.67	0.11	0.60
		c_{12}	2.88		1.60	
[110]	[110]	$\frac{c_{11}+c_{12}+2c_{44}}{2}$	10.4	3.36	2.14	2.80
	[110]	$\frac{c_{11}-c_{12}}{2}$	4.96	2.32	0.42	1.17
	[001]	$c_{44}+\frac{e_{14}^2}{\varepsilon_0\varepsilon}\ tg\delta^*$	2.85	1.76	0.40	1.90
[111]	[111]	$\frac{c_{11}+2c_{12}+4c_{44}}{3}+\frac{4e_{14}^2}{3\varepsilon_0\varepsilon}\cdot tg\delta^*$	9.98	3.29	2.41	3.29
	[110]	$\frac{c_{11}-c_{12}+c_{44}}{3}$	4.15	2.12	0.32	1.05

$\Gamma_{11}=c_{11}(\cos\psi)^2+c_{44}(\sin\psi)^2$

$\Gamma_{22}=c_{44}(\cos\psi)^2+c_{11}(\sin\psi)^2$

$\Gamma_{33}=c_{44}$

$\Gamma_{12}=\Gamma_{21}=(c_{12}+c_{44})\cos\psi\sin\psi$

$\Gamma_{13}=\Gamma_{31}=\Gamma_{23}=\Gamma_{32}=0$

In these expressions, the angle ψ is the angle between the direction of the wave vector and the second-order symmetry axis [100]. Components $c_{\alpha\beta}$ are substituted real or imaginary, respectively. In particular, the real part of the effective elastic constant will be written as:

$$c'_{eff}=\frac{\Gamma'_{11}+\Gamma'_{22}}{2}\pm\frac{1}{2}\sqrt{(\Gamma'_{11}-\Gamma'_{22})^2+4\Gamma'^2_{12}}\ . \tag{11}$$

As a result, the phase velocities of the longitudinal and one of transverse waves polarized in the (001) plane of the BSO and BGO crystals are determined in terms of the real components of the elastic constants tensor by the expression:

$$V^2_{1,2}=\frac{c'_{11}+c'_{66}\pm\sqrt{(c'_{11}-c'_{66})^2\cos^2 2\psi+(c'_{12}+c'_{66})^2\sin^2 2\psi}}{2\rho}\ . \tag{12}$$

Relation (12) determines the values of the velocities of two waves, the polarization of which lies in the (001) plane. In this case, the [100] and [010] axes are equivalent. The expression for imaginary effective elastic constant is obtained; if in expressions (12) change the real components of the elastic constants tensor into suitable imaginary elastic constants.

Further the obtained values of effective elastic constants can be used for determination of the attenuation of longitudinal and transverse waves along any arbitrary crystallographic direction by the equation (6). The results of calculation are shown in Fig. 1 in the form of cross-section of characteristic surfaces of attenuation of acoustic waves by plane (001).

It can be seen that in the considered crystallographic plane a relatively small anisotropy of the attenuation of acoustic waves is observed for the longitudinal acoustic waves. At the same time it is seen that the strongest anisotropy of transversal waves is observed in BGO crystals. It can be explained by the relatively large value of the dielectric loss tangent in bismuth germanate crystals.

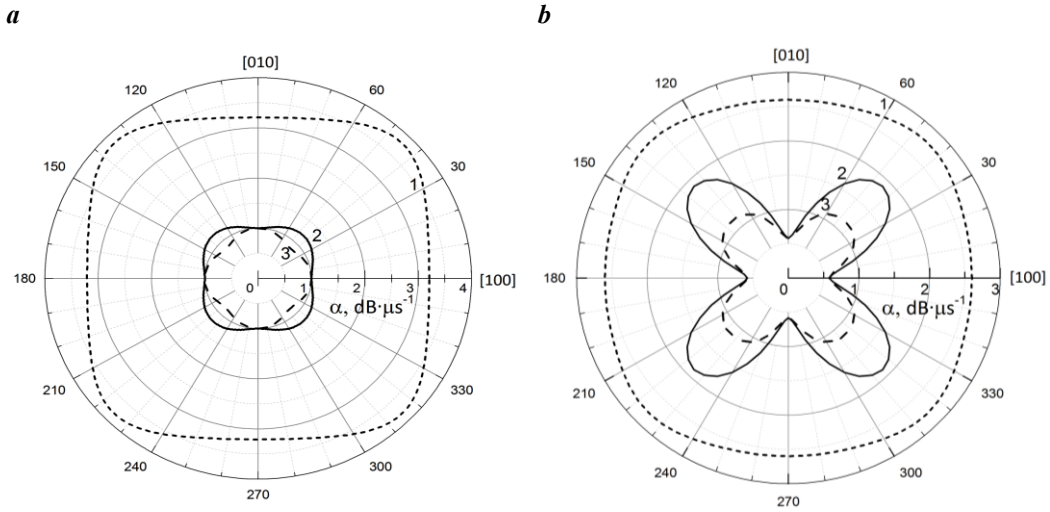

Figure 1 – Anisotropy of the attenuation of quasilongitudinal (*1*), quasitransverse (*3*) and pure transverse (*2*) waves in BSO (*a*) and BGO (*b*) crystals.

4. CONCLUSIONS

Both real and imaginary elastic moduli of the noncentrosymmetric cubic crystals of germanate and bismuth silicate (point symmetry group 23) has been determined for the first time. By the example of these crystals it is shown that in piezoelectric crystals it is necessary to take into account the contribution of dielectric losses to attenuation [8]. The calculations with help of equations (6), (7) and (8) have shown that this contribution is most significant in bismuth germinate crystals and it is about 45%.

For slow transverse waves along [110] the imaginary effective elastic constant is written in the simplest form:

$$c''_{eff} = c''_{44} + \frac{e_{14}^2}{\varepsilon_0 \varepsilon} tg\delta \tag{13}$$

Here: e_{14} is the piezoelectric coefficient, ε – the dielectric constant, and tgδ – the dielectric loss tangent

Since in the field $tg\delta_{mn} << 1$ the dielectric losses, as a rule, grow in direct proportion to the frequency, the considered mechanism of attenuation of acoustic waves gives, like the Akhiezer mechanism, a quadratic dependence of attenuation on frequency.

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