PROCEEDINGS of the
23rd International Congress on Acoustics

9 to 13 September 2019 in Aachen, Germany

Sound field reproduction in a cabin using loudspeakers

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ABSTRACT

To evaluate the interior sound quality of an aircraft during flight, multiple loudspeakers are used to reproduce the sound field inside a mock-up cabin which is used for the passengers to experience on ground. The reproduction is based on the principle of the frequency-domain pressure matching approach. To reduce the influence of measurement errors in acoustic transfer functions, the case of more loudspeakers than the matching points is studied, which is different from most studies before. Some important aspects, including the number of loudspeakers and the loudspeakers placement are discussed. Both numerical and experimental examples are both described in this paper.

Keywords: Sound field reproduction, Pressure matching, Inverse problem, Aircraft noise

1. INTRODUCTION

The sound field reproduction discussed in this paper, is to use the secondary sound sources to make the same sound field as the preliminary sound sources in a given space. Sound field reproduction could be used to represent the real sound field to make the 'passengers' feel as if they were personally on the real plane to evaluate the sound effects and sound quality. Besides, the produced sound field could be useful for the design and test of the noise control system.

There are three major categories of sound field reproduction methods commonly researched in literature. The first category is based on the Huygens' principle, which reproduces the sound field inside the target area through reproducing the sound pressure and particle velocity on the its boundary, such as the wave field synthesis (WFS) method (1, 2). The second category is through the mode matching approach, such as ambisonics method (3, 4). In this approach, the sound field is described using the basis functions in spherical coordinate and their coefficients. The drive signals of the secondary sound sources are calculated through minimizing the difference of the mode coefficients between the preliminary and secondary sound field. The third category is the sound pressure matching method (5), in which the sound pressure difference of the control points inside the target area between the preliminary and secondary sound field is minimized to calculate the drive signals. Compared with the first two categories, there is no limit for the location and array form of the secondary sound sources in the sound pressure matching method, so it is more convenient for engineering practice. In literature, this method is adopted by most practical cases (6, 7).

2. THEORY BASIS FOR SOUND FIELD REPRODUCTION

2.1 Principle of sound pressure matching

As in Figure 1, several target points are arranged in the reproduction area. The sound pressure of these target points has been measured in the preliminary sound filed for acting as the reproduction objectives. The sound sources, which could be loudspeakers or shakers (excite the structure to produce sound) are arranged around the target area. In our opinion, loudspeakers are preferable because the sound characteristic of shakers is coupled with the structure.

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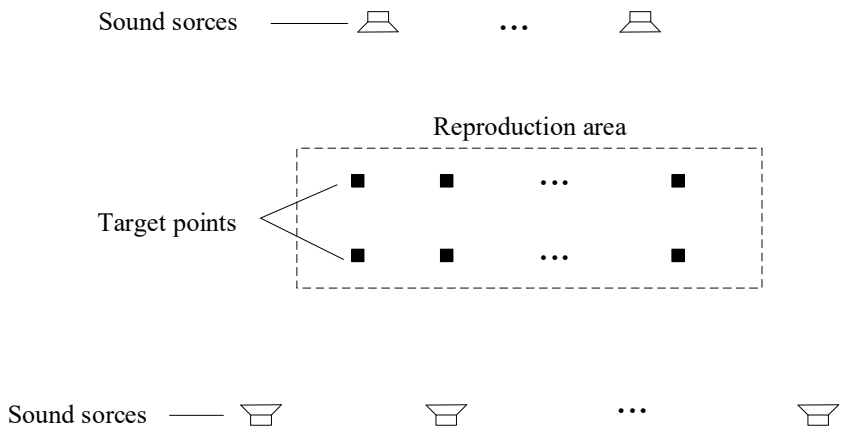

Figure 1 – Diagram for sound field reproduction

For a linear and time-invariant system, the relationship between the drive of the sound sources and the sound pressure of the target points could be described in frequency domain as:

$$P(f) = G(f)S(f) \tag{1}$$

where f is the frequency points, and is omitted for clarity later. P is a $M\times1$ column vector of the sound pressure of the target points, which has been given as the reproduction target. M is the number of the target points. S is a $N\times1$ column vector of the drives. N is the number of the sound sources. G is a $M\times N$ transfer function matrix.

In most frequencies, the transfer function matrix G is full rank.

if M=N, Eq. (1) has the exclusive solution.

if M>N, Eq. (1) has the least square solution.

if M<N, Eq. (1) has infinite solutions, and the general solution is:

$$S = G^{+}P + \left(I - G^{+}G\right)Y \tag{2}$$

where G^{+} is the Moore-Pseudo inverse of G. Y is an arbitrary $N\times1$ vector.

The case of $M>N$ is commonly researched in literature. It is treated as an optimization problem by introducing in constraint conditions, such as the number of loudspeakers, the maximum power, the total power. And the reproduction error is minimized by methods of least-square (LS), Lasso, Lasso-LS (8, 9).

In the application background of this paper, there is no quantity restriction for loudspeakers, and high-power loudspeakers could be used to avoid the power restrictions, so the situation of $M<N$ is studied here.

2.2 Sound Pressure Matching Method when *M<N*

When $M<N$, Eq. (1) has infinite solutions, among which the drive of sound sources could be arbitrarily chosen to meet the reproduction requirement accurately. However, in the measurement of the transfer function matrix G, measurement error is inevitable. Besides, the real transfer function may change with environment. To cope with the influence of errors in G on the reproduction effect, proper test design should be implemented.

Suppose the real transfer function is $\hat{G}$, and its relationship with G is:

$$\hat{G} = G + \Delta G \tag{3}$$

where ΔG is the measurement error.

In Eq. (2), the drive S is calculated with the measured transfer function. When it is applied to the actual system, the sound pressure of the target points is:

$$\hat{P} = \hat{G}S \tag{4}$$

The difference between the real sound field and the desired sound field is:

$$\Delta P = \Delta G S \tag{5}$$

The elements in ΔP should be reduced. The 2-norm is a measure of length of a vector. So it could be used to evaluate the reproduction accuracy. The following aspects would be discussed to reduce $\|\Delta P\|_2$.

2.2.1 Choice of solutions

For ΔG is a random matrix, the way to decrease $\|\Delta P\|_2$ is to decrease the element in vector S. Among all solutions in (2), the minimal 2-norm solution is:

$$S=G^{+}P \tag{6}$$

This solution is recommended in the reproduction process. It should be noted that, for a given ΔG, this solution may not be the best choice to minimize $\|\Delta P\|_2$, however, when many probabilities of ΔG are given, the usage of (6) leads to a minimized average value of $\|\Delta P\|_2$.

2.2 The number of secondary sound sources

The singular value decomposition of G is:

$$G=U\Sigma V^{\mathrm{H}} \tag{7}$$

where U is a $M\times M$ unitary matrix. V is a $N\times N$ unitary matrix, and the superscript H indicates conjugate transpose. Σ is a $M\times N$ matrix, and only the diagonal elements of the first M columns, which are called the singular value of G, are none-zeros.

The Moore-Penrose generalized inverse matrix of G could be expressed as:

$$G^{+}=V\Sigma^{*}U^{\mathrm{H}} \tag{8}$$

where Σ^{*} is a $N\times M$ unitary matrix, and only the diagonal elements of the first M lines are none-zeros. The non-zero elements of Σ^{*} and Σ are reciprocal.

Substituting (8) into (6):

$$S^{+}=V\Sigma^{*}U^{\mathrm{H}}P \tag{9}$$

V and U^{H} are unity matrix. When a vector is multiplied with a unity matrix, its 2-norm would not change. So, in Eq. (9), the change of 2-norm happens only when multiplying Σ^{*}. Σ^{*} better has small elements to reduce the 2-norm of S.

When an extra sound source is added, G has a new column. As a result, the value of all singular values of G increase, while the number of singular values of G remains the same. So it is better to have more sound sources.

2.3 Location of secondary sound sources

Substitute Eq. (3) and Eq. (6) into (5), and notice GG^{+}=I and $\hat{G}\hat{G}^{+}=\mathrm{I}$, we have:

$$\Delta\hat{P}=\left(\hat{G}-G\right)G^{+}P=\hat{G}\left(G^{+}-\hat{G}^{+}\right)P \tag{10}$$

If matrix G is badly conditioned, G^{+} is greatly varied from $\hat{G}^{+}$. The condition number of G is related to the location of the sound sources. The location with smaller condition number is preferable.

3. SOUND FIELD REPRODUCTION PROCEDURE

Flowchart of an open-loop sound field reproduction method based on sound pressure matching is shown in Figure 2.

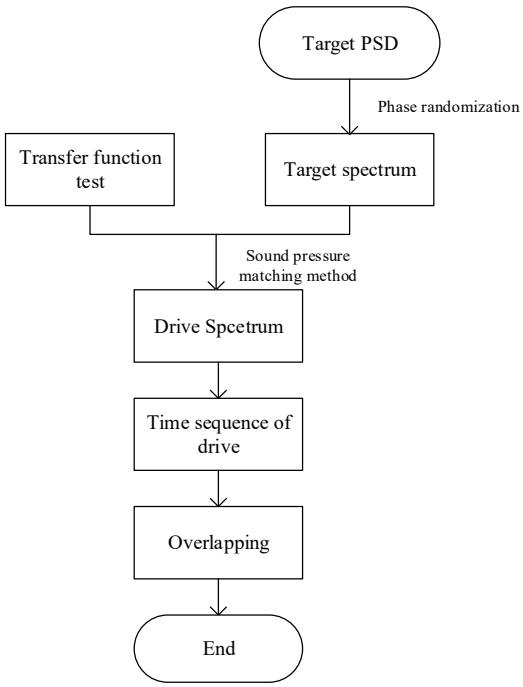

Figure 2 – Flowchart of sound field reproduction

(1) Transfer function test

The transfer function from the driving voltage of each sound source to the sound pressure of each target point shall be measured. As mentioned above, it is better to have more sound sources than the target points. There were mature methods for transfer function measurement. For the measurement of driving voltage could be accurate, the H_2 estimator could be used:

$$G(m,n) = S_{\mathrm{P}_m\mathrm{P}_m} / S_{\mathrm{U}_n\mathrm{P}_m} \tag{11}$$

where $G(m,n)$ is the (m,n) element of G, $S_{\mathrm{P}_m\mathrm{P}_m}$ is the averaged auto-power of sound pressure of the m-th target point, and $S_{\mathrm{U}_n\mathrm{P}_m}$ is the averaged cross-power of sound pressure of the m-th target point and driving voltage of the n-th sound source.

(2) Target spectrum calculation

For the noise inside a cabin is close to random signal, the target sound field is usually described by the power spectral density (PSD) of the target points. However, the phase information of the target sound field is used in (2). So firstly, the PSD spectrum should be transformed to Fourier spectrum by introducing random phase.

The amplitude spectrum is calculated as:

$$|P(m)| = \sqrt{s_m \Delta f} \tag{12}$$

where | | indicates the amplitude, s_m is the target PSD spectrum of the m-th target point, Δf is the frequency spacing of s_m. (s_m is defined on discrete frequency points.)

Then random phase is introduced into (12) to have:

$$P(m) = |P(m)| \mathrm{e}^{\mathrm{j}\varphi} \tag{13}$$

where, e is the natural logarithm. j is the imaginary unit. φ is the random phase, which is a uniform distribution between $-\pi$ to π.

(3) Drive spectrum calculation

Based on the sound pressure matching method, the drive spectrum S is calculated by (6). In the results, there may be one or more drive has an amplitude too big for the output channels of the equipment used. Each of these drives could be multiplied by a factor smaller than 1 to make its amplitude equal to the threshold. Then these drives are treated as known drives. Substitute them into (6), the other drives are calculated.

(4) Time sequence of drive

The drive spectrum could be transformed into time signal by inverse Fourier transform. The length of this time signal is $T=1/\Delta f$.

(5) Windowing and overlapping

The length of one frame is usually not long enough for reproduction practice. Repeat step (2) – (4) for several times, then the time sequences are overlapped to form a long-time drive signal in time domain.

To avoid saltation at the splice points, windowing and overlapping are necessary. Figure 3 shows this process.

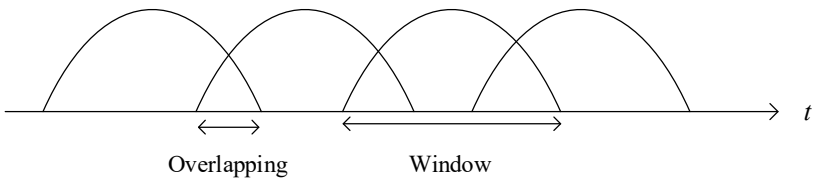

Figure 3 – Windowing and overlapping

Suppose x_1, x_2… are time sequences form step (4), w is the window function. The time signal x after overlapping is:

$$x = x_1 w_1 + x_2 w_2 + \cdots \tag{14}$$

Different window functions could be used. Half sine window is recommended:

$$w(t) = \begin{cases} \sin \pi \dfrac{t}{T} & 0 \le t \le T \\ 0 & t > T, t < 0 \end{cases} \tag{15}$$

The time interval of w_1, w_2… is T/2.

4. EXAMPLES

4.1 Numerical example

The conclusions in section 2.2 are verified through Monte Carlo simulation.

A random 6×8 complex matrix (both the real and image part of each element are between -100 to 100) is generated as the transfer function matrix G_1, and a random 6×1 complex vector is generated as the reproduction target p.

(1) Choice of infinite solutions

Step 1: Using Eq. (6), the minimal 2-norm solution S_1 is calculated. Besides, 100 different solutions S_2, S_3 , …S_{101} using Eq. (2) are also calculated.

Step 2: $\hat{G}_1$ is generate by adding uniformly distributed noise to each element of G_1.

Step 3: Substituting G_1, $\hat{G}_1$, S_1, S_2, …, S_{101} to Eq. (3) and Eq. (5), $\|\Delta P_1\|_2$, $\|\Delta P_2\|_2$,…, $\|\Delta P_{101}\|_2$ are calculated.

Repeat step 2 and 3 above for 1000 times. The result reveals that $\|\Delta P_1\|_2$ is not always the smallest, but the average value of $\|\Delta P_1\|_2$ is the smallest, as in Figure 4. So solution (6) is the best choice.

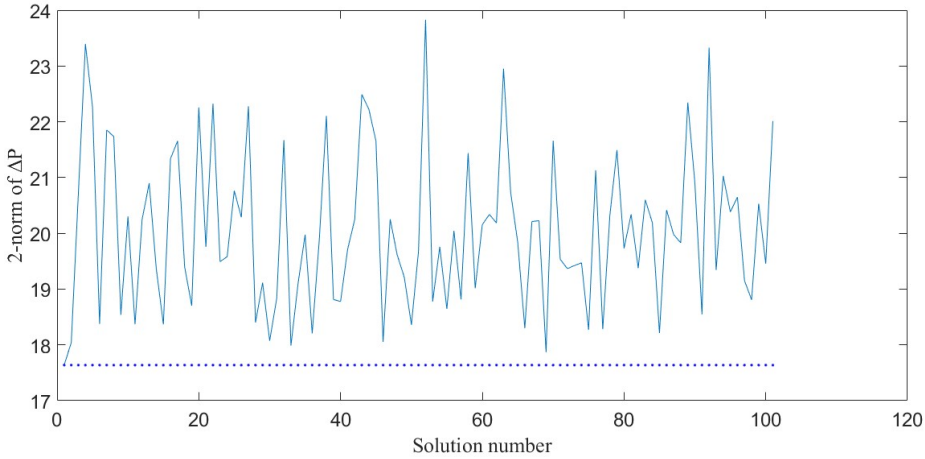

Figure 4 – Averaged reproduction errors with different solution of S

(2) Number of sound sources

Step 1: G_2 is generated by add four random columns to G_1.

Step 2: Substituting G_1 and G_2 into Eq. (6) respectively, and solving for S yields S_1 and S_2.

Step 3: ΔG_1 and ΔG_2 are generated, and the first 8 columns of ΔG_2 are the same as ΔG_1. The elements are also uniformly distributed noise.

Step 4: Substituting ΔG_1 and S_1 into Eq. (5) yields $\|\Delta P_1\|_2$, and substituting ΔG_2 and S_2 into Eq. (5) yields $\|\Delta P_2\|_2$.

Repeat step 3 and 4 above for 1000 times. The result reveals that $\|\Delta P_2\|_2$ is smaller than $\|\Delta P_1\|_2$ for most times as shown in Figure 5, and the average value of $\|\Delta P_2\|_2$ is the smaller than that of $\|\Delta P_1\|_2$. So it's better to use more sound sources.

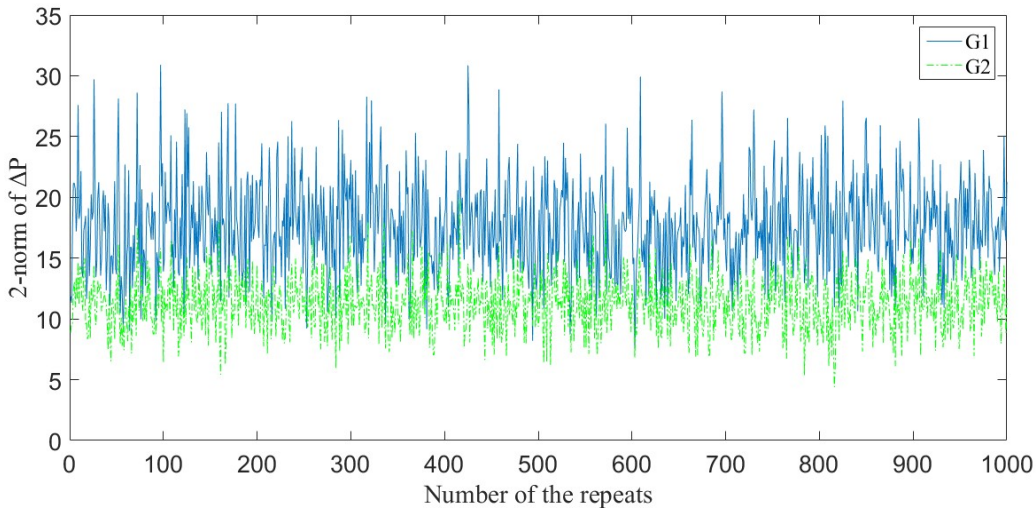

Figure 5 – Comparison of reproduction errors

4.2 Experimental example

The method of this paper was applied to sound field reproduction inside a simulated cabin as Figure 4. The out wall is made of alloy aluminum skin, the inside wall is micro-perforated panel, and the interlayer is melamine foam.

As seen in Figure 6, a loudspeaker was placed outside the cabin to produce the preliminary sound field. The sound pressure of three target points was measured by three microphones hanged inside the cabin.

Figure 6 – Photograph of the test setup

Five loudspeakers both inside and outside the cabin were used to reproduce this sound field, as shown in Figure 7. The method raised in this paper is used, and the reproduction result is shown in Figure8 (take point 1 for example). In all 1/3 octave between 40 to 5000Hz, the reproduction error is generally smaller than 2dB.

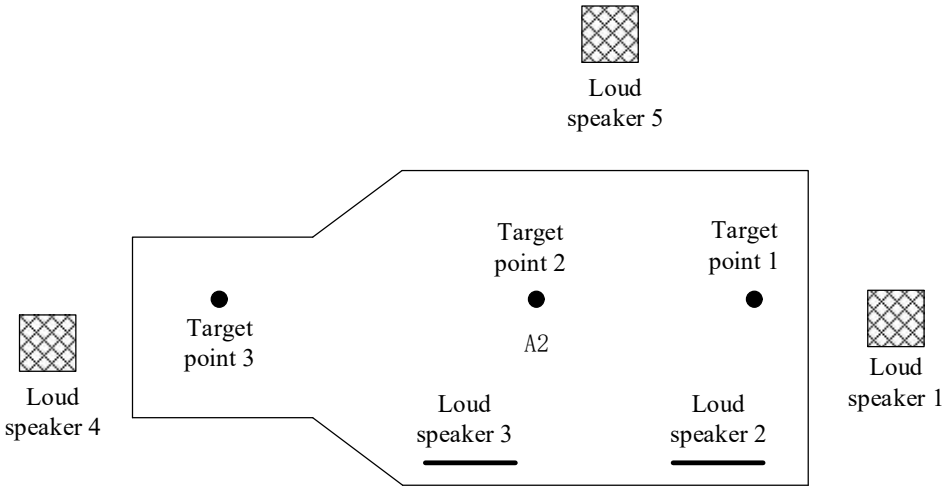

Figure 7 – The location of loud speakers and target points

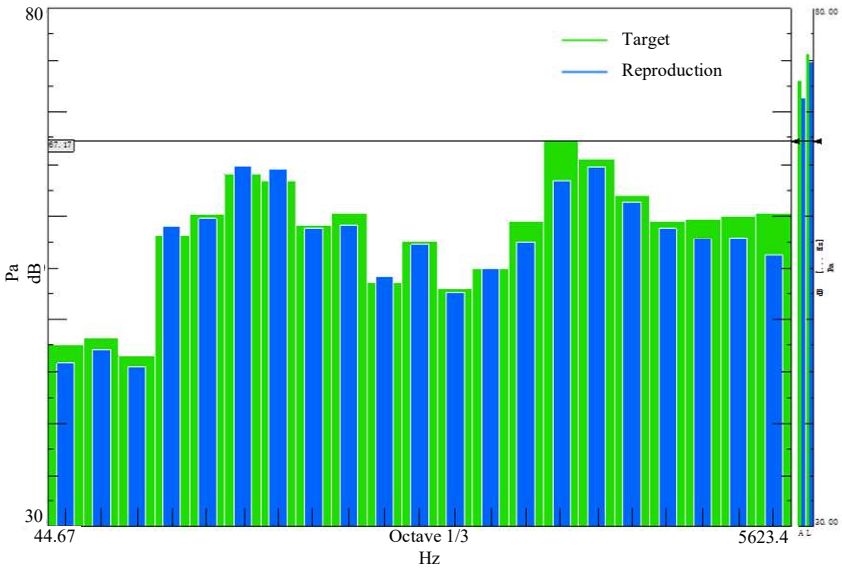

Figure 8 – Reproduction result

5. CONCLUSION

The sound pressure matching method is adopted to reproduce the sound field. The case of more sound sources than target points is studied. The advantage of more sound sources is its insensitivity to the measurement noises in transfer functions. The best choice in infinite solutions is given. The more sound sources used, the better. After the theory basis, the detailed reproduction procedure is given. The method is validated by both numerical and experimental examples.

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