Measuring Frequencies with Historic Resonators from Schaefer

Rüdiger Hoffmann, Dieter Mehnert, Rolf Dietzel, Günther Fuder
TU Dresden, Institut für Akustik und Sprachkommunikation, 01062 Dresden, Germany
Email: ruediger.hoffmann@tu-dresden.de, di.mehnert@freenet.de

Introduction

The collection of historic acoustic-phonetic devices which exists at the Laboratory of Acoustics and Speech Communication of the TU Dresden represents the development of the experimental phonetics between approx. 1890 and 1960 [1]. This collection includes numerous devices for procedures, which enable the measurement of frequencies in the audible range. One of these measurement methods which comparably low expense will be described here. The resonators from K. L. Schaefer [4] were investigated to figure out their limits of applicability, their measurement uncertainties, and their application (Figure 1).

Figure 1: The Hamburg phonetician G. Panconcelli-Calzia demonstrating the resonators from K. L. Schaefer (historic photograph).

Principle of the Measuring Method

A resonator from K. L. Schaefer (1866 – 1931) consists of two tubes of equal length, the inner and the outer tube. They are sliding one into each other and can be shifted easily (Figure 2). Therefore the total length l of the resonator can be varied continuously. One end of the tube is open. The other, closed end of the tube is equipped with a thin pipe (earpiece) serving as connection for a short piece of rubber hose which is put into the auditory canal of the experimenter. The resonators are suited for the investigation of stationary sounds only.

If the experimenter aims to determine the frequency of single tones or of a partial tone of a composed sound (e. g., from an organ pipe), he changes the length of the tube l as long as a clear maximum of the loudness can

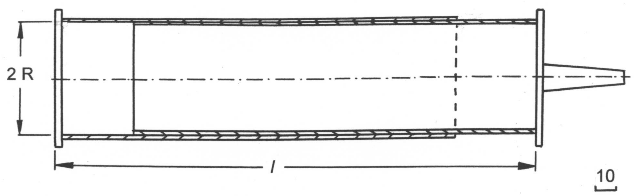

Figure 2: Longitudinal section of a resonator tube from K. L. Schaefer.

be detected at the hose at the end of the tube. In this case, the tube shows a length of $\lambda/4$. λ describes the wavelength of the investigated partial tone.

The tube length l can be read at an engraved scale at the inner tube. It is used to calculate the frequency $f = c/4{\cdot}l$ where c is the speed of sound $c = 343$ m/s. Additionally, the musical pitch is provided at the scale (Figure 3). This correspondence between frequency and musical pitch is basing on the older standard pitch with a^1 at $f = 435$ Hz which was valid until 1935.

A complete series of resonators consists of four tubes with equal diameters (approx. 60 mm) but different lengths l (cf. Table 1 and Fig. 4). This is sufficient for the frequencies which are characteristic for a speech field.

Figure 3: Resonator no. 4 with the length adjusted to $l =$ 107.5 mm.

Table 1: Resonator lengths, pitch and frequency ranges.

Tube No.	adjustable length l/mm	engraved musical pitches	frequency range in Hz
1	767 . . . 410	$A \dots g$	108 . . . 194
2	410 . . . 210	$g^\sharp \dots f^\sharp$	198 . . . 367
3	240 . . . 120	$f^1 \dots d^2$	345 . . . 580
4	110 . . . 60	$d^{2\sharp} \dots c^3$	643 . . . 1036

Figure 4: The complete series of resonators from K. L. Schaefer.

The Resonator Tube as an One–dimensional Acoustic Waveguide

Fundamentals

A tube, having a diameter which is sufficiently small compared to the wavelength of the sound, can be modelled as an one-dimensional acoustic waveguide. It can be described using the equations known from the theory of electrical lines in analogous way.

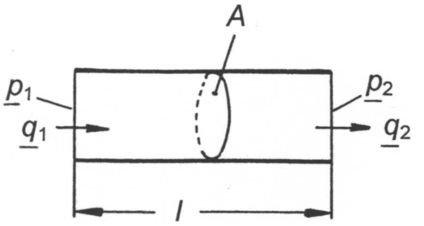

Figure 5: The cylindric tube as acoustical waveguide.

The correspondence between the acoustic pressures $\underline{p}_1$, $\underline{p}_2$ and the acoustical volume velocities $\underline{q}_1$, $\underline{q}_2$ at the input and the output of a lossless tube which is open at both ends with the cross-section area A and the length l (Figure 5) is expressed by the following four-pole equations [2]:

$$\underline{p}_1 = \cos\beta l\,\underline{p}_2 + jZ_{a_0}\sin\beta l\,\underline{q}_2 \qquad (1)$$

$$\underline{q}_1 = j\frac{1}{Z_{a_0}}\sin\beta l\,\underline{p}_2 + \cos\beta l\,\underline{q}_2, \qquad (2)$$

where $Z_{a_0} = \rho c/A$ is the acoustic impedance of the tube, and ρ = density of the air, c = speed of sound, λ = wavelength, and the wave number

$$\beta = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}.$$

From Eq. 1 and 2 follows the acoustic characteristic impedance $\underline{Z}_{a_2}$ at the right end of the tube:

$$\underline{Z}_{a_2} = \frac{\underline{p}_2}{\underline{q}_2} = \frac{\frac{\underline{p}_1}{\underline{q}_1}\cos\beta l + jZ_{a_0}\sin\beta l}{\frac{\underline{p}_1}{\underline{q}_1}\cdot\frac{j}{Z_{a_0}}\sin\beta l + \cos\beta l}. \qquad (3)$$

If the right end of the tube is closed, there holds $|\underline{q}_2| = 0$ for the acoustical volume velocity. At the open left end of the tube is $|\underline{p}_1| = 0$. From Eq. 3 results then

$$\frac{1}{\underline{Z}_{a_2}} = \frac{1}{jZ_{a_0}}\cdot\frac{\cos\beta l}{\sin\beta l} = 0. \qquad (4)$$

This equation is only fulfilled if $1/(\tan\beta l) = 0$, i. e., if the argument of the tangent function is an odd multiple of $\pi/2$:

$$\beta l = \frac{2\pi f}{c}\,l = \frac{2\pi l}{\lambda} = (2m+1)\frac{\pi}{2};\quad m = 0,1,2,\ldots \qquad (5)$$

It follows that $\frac{l}{\lambda} = \frac{1}{4}(2m+1)$. An at one side closed tube with the length l acts as resonator only if $l = \frac{1}{4}\lambda$, $l = \frac{3}{4}\lambda$, $l = \frac{5}{4}\lambda$, etc.

Such tubes can therefore be used not only for its deepest resonance frequency $f_1 = \frac{c}{4l}$, but also for the subsequent resonances $f_2 = \frac{3}{4}\frac{c}{l}$, $f_3 = \frac{5}{4}\frac{c}{l}$, etc. This means, that the applicable frequency range of each tube is clearly larger than indicated in Table 1.

Calculating the Resonance Frequencies Considering the Mouth Correction

If sound is propagated in a tube with $2R \ll \lambda$, a certain amount of air outside the open end of the tube is additionally vibrating in the direction of the tube axis. Therefore, a “mouth correction” Δl must be added to the geometric length l of the tube (e. g., [3]). The correction $\Delta l = \pi R/4$ holds for a one-sided open tube with the radius R. The acoustically effective length l_w of the resonator tubes with $2R$ = 59.5 mm which are investigated here, is

$$l_w = l + 23.3\,\mathrm{mm}. \qquad (6)$$

If a resonator is excited with its deepest, i. e. first resonance frequency f_1, the result is

$$f_1 = \frac{c}{4\cdot l_w}. \qquad (7)$$

Measuring Arrangements

All measurements were performed in the anechoic chamber of the Laboratory of Acoustics and Speech Communication of the TU Dresden where a free sound field can approximatively be assumed.

Measuring the Transfer Function of the Tube Resonators

The transfer function $|\underline{B}(f)|$ of the tube resonators is defined as the quotient of the sound pressures $|\underline{p}_2|$ at the earpiece and $|\underline{p}_1|$ at the open input of the tube.

Figure 6 shows the principle of the measuring setup. The sound board SoundMAX HD Audio in a PC produces an AC voltage with continuously tunable frequency which is amplified and fed to a loudspeaker box. A support with the tube resonator was provided in a distance of

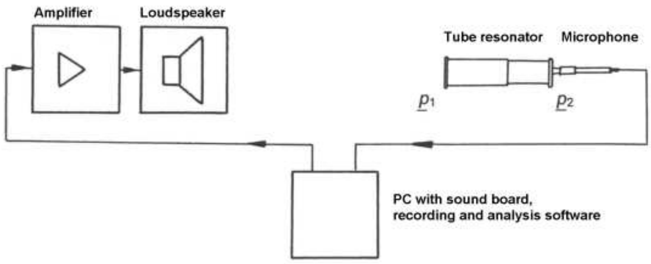

Figure 6: Arrangement for the objective measurement of the transfer function.

approx. 3 m. The measurement microphone for $|\underline{p}_2|$ was connected to the earpiece of the closed end of the tube using a short piece of hose. The microphone voltage was recorded by a PC with the recording software HEAD Recorder and the analysis software HEAD Artemis 8.00.

Subjective Measurements of the Resonance Frequencies

The experimental arrangement (Figure 7) was oriented towards the historic recordings (cf. Figure 1). The observer in the anechoic chamber targets with the open end of the tube resonator with a distance of 3 m to the loudspeaker box, which is supplied by a generator with continuously tunable frequency. The left ear of the observer is closed by an earplug to exclude the unavoidable crosstalking.

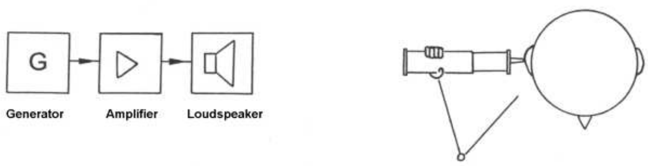

Figure 7: Arrangement for the subjective measurement of the resonance frequencies.

The observer determines the maximum of the loudness by changing the frequency according to the adjustment technique. Only the first (i. e. the deepest) resonance frequency f_1 was determined for each given length of the tubes.

Results of the Measurements

Transfer Function $\underline{B}(f)$

Figure 8 shows one of the measured transfer functions $|\underline{B}(f)|$ of the resonator tube 1 with the largest adjustable tube length $l = 767$ mm. As expected, the first resonance appears at $f_1 = 110$ Hz, followed by $f_2 = 330$ Hz, $f_3 = 550$ Hz, etc.

Due to the mouth correction, the acoustically effective tube length according to Eq. 6 is $l_w = 790.3$ mm, resulting in a computed first resonance frequency $f_{1c} = c/4 \cdot l_w = 108.5$ Hz. The relative difference from the measured resonance frequency amounts 1.4 %.

Numerous further transfer functions have been measured correspondingly for the four resonators down to $l = 62$

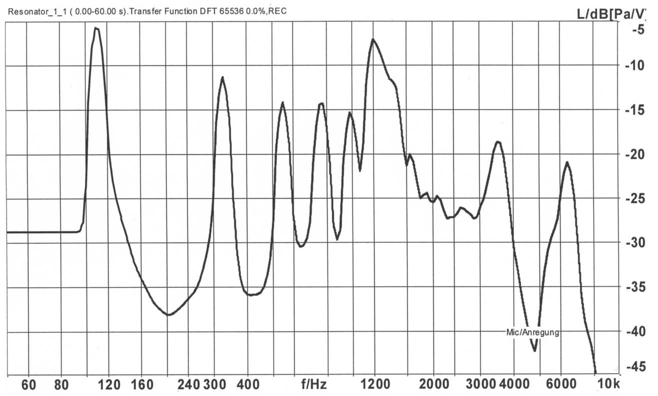

Figure 8: Transfer function of a resonator tube with $l_w = 790.3$ mm

mm. The measured first resonance frequencies of the resonators no. 1 – 3 differed from their calculated counterparts by approx. 0.2 % to 1.6 %. Only the shortest resonator (no. 4, $l = 62 \ldots 111$ mm) showed larger differences with (3…5) % higher measured resonance frequencies f_1. The tube diameter of this short resonator is no more sufficiently small compared to the wavelength λ.

Results of the Subjective Measurements of the Resonance Frequency

Five normal hearing subjects participated in the subjective measurements as test persons. The sound pressure level was $L = 38$ dB at the place of the test person. The four resonator tubes (cf. Table 1) were applied one after the other. The tube lengths – between 767 mm and 62 mm – were adjusted by the experimenter for all test persons in the same way.

The test person was requested to tune the frequency of the generator G (Fig. 7) following the adjustment technique until the maximum loudness was perceived. He was free to decide to start the measurement tuning the frequency either from higher to lower or from lower to higher values.

Let the difference between the frequencies f_{max} (frequency which the test subject tuned to approach the loudness maximum) and f_1 (physically measured resonance frequency) be $\Delta f_1 = f_{max} - f_1$. The quotient $\Delta f_1/f_1$ is called relative deviation. Figure 9 shows this relative deviation depending on the frequency f_1 for all involved test persons. Each person is characterized by a specific symbol. The results show an acceptable agreement between the physical measured and the subjective determined resonance frequencies. The mean of all amounts of the relative deviations of all single observations is $\overline{|\Delta f_1/f_1|} = 2.3$ %.

Additionally it should be investigated if the height of the sound pressure level influences the adjustment of the maximum of the loudness at the given frequency. Therefore, similar measurements were repeated with three test persons applying a sound pressure level of $L = 60$ dB. The tube lengths which were selected for this experiment agreed with the first series of measurements. In this case,

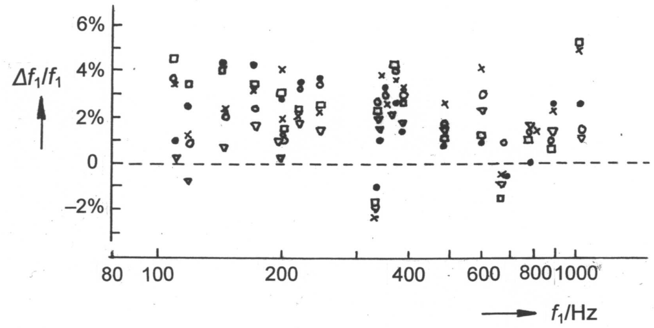

Figure 9: Relative deviation $\Delta f_1/f_1 = (f_{max} - f_1)/f_1$ of the frequency f_{max}, which is adjusted by the test person to obtain the loudness maximum, from the physically measured resonance frequency f_1.

the mean of the relative deviation over all persons and frequencies was $\overline{|\Delta f_1/f_1|} = 2.2$ %. This means that the loudness has no influence on the measuring uncertainty.

Figure 9 shows the surprising detail that all test persons adjusted very frequently a frequency f_{max} (the frequency where they observed the maximum of loudness), which was a little bit *higher* compared to f_1. The relative deviations $\Delta f_1/f_1$ occur predominantly in the positive range. The reason can be supposed as follows: In determining the maximal loudness, an error is caused due to the limited ability to judge simultaneously occurring, small changes of loudness and pitch.

Conclusion

The continuously adjustable resonators from K. L. Schaefer offered to the experimental acoustics and phonetics a versatile measuring method for quantitative statements on the frequencies of the partial tones of stationary sounds, e. g. from musical instruments, tongue pipes, or also from noise sources. The measuring method was easy to handle. Skilled observers could achieve remarkable measuring uncertainties of as low as 2.3 %.

The authors thank Dr. E. Altinsoy for his kind support of measuring the resonator transfer functions.

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