ANC Concept study using a 1-D-Model

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Introduction

In propeller driven aircrafts the sound pressure level may reach high values caused by propeller blades producing disturbances when passing the fuselage. This excitation induces considerable tonal noise levels in the low frequency range. Due to fuel economisation corresponding with mass reduction as a major aspect in aircraft design, research has been carried out on methods like active noise control (ANC), active structural acoustic control (ASAC) and active vibration control (AVC). In this context it is of interest to evaluate different methods in terms of their effectiveness, e.g. their energy consumption.

This paper describes the concept and numerical implementation of a very simple model including fixed feedback ANC being able to serve as a reference for evaluating different methods of noise reduction. The aircraft cavity is represented by a one-dimensional (1-D) air column which can be seen as an acoustic duct and the vibrating aircraft structure is represented by a mass-spring-damper. The acoustic duct is discretized by finite elements using Comsol. The FE-model is transfered into state-space-form [4] and combined with the mass-spring-damper in Matlab/Simulink. Numerical results are compared with analytical ones based on [1] to verify the model. Thereafter the ANC concept as a representative noise reduction method is implemented in the Simulink model.

1-D-Model description

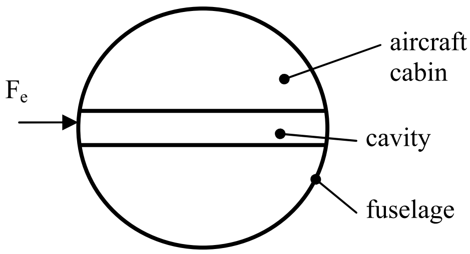

Figure 1: The considered part of the cavity

As shown in figure 1 the external sound pressure is taken into account by an external force F_e that excites the fuselage. Due to vibro-acoustical coupling this excitation causes an internal acoustic pressure. Only a tubular part of the cabin cavity is considered for the following investigations allowing the use of a very simple model reproducing the cavity's transverse natural modes as well as possible. The model of the tubular part is displayed in Figure 2. The excited panel of the fuselage is modeled as a mass-spring damper (M_s, D_s, K_s) linked with the air continuum (c_F, ρ_F, l).

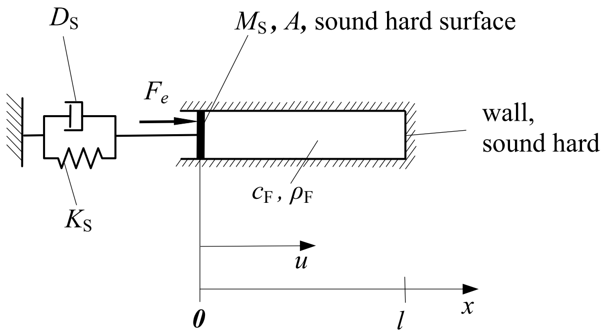

Figure 2: The model of the structure-air-coupling

Analytical Approach

To verify the Simulink-Model eigenfrequencies obtained by numerical and analytical calculations were compared based on the data of table 1 [1]:

Structure		
cross section area	A	0.000625 m²
spring stiffness	K_S	7474.75 N/m
panel mass	M_S	0.01 kg
damp coefficient	D_S	0.50 kg/s
excitation force	F_e	2.1885 N
Fluid		
duct length	l	1.25 m
(duct) cross section area	A	0.000625 m
speed of sound	c_F	344 m/s
density of air	ρ_F	1.205 kg/m

Table 1: Data for the model of figure 2

For the analytical solution the wave equation (PDE)

$$\frac{\partial^2 p(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p(x,t)}{\partial t^2} \tag{1}$$

had to be solved by using a separation approach which leads to 2 ODEs. Considering the boundary conditions

$$\dot{x}(0) = \dot{u}(0) \quad \text{and} \quad v(x = 1.25) = 0 \tag{2}$$

the eigenfrequencies ω_k for the coupled system were obtained from:

$$\tan(\frac{\omega_k}{c_F} l) = -\frac{A \cdot \omega_k \cdot c_F \cdot \rho_F}{k - m\omega_k^2} \tag{3}$$

The first eigenfrequencies of the structure, the fluid and the coupled systems are displayed in table 2. The parameters in table 1 are chosen in that way that the eigenfrequency of the structure exactly matches the first of the fluid, which leads to a splitted pair of frequencies for the coupled system.

eigenfrequencies ω		
Fluid	**Structure**	**coupled system**
137.6	137.6	128.3
		147.2
275.2	-	276.1
412.8	-	413.3
550.4	-	550.7
688.0	-	688.3
825.6	-	825.8
963.2	-	963.4

Table 2: Eigenfrequencies for the structure, fluid and the coupled system by analytical calculations

Numerical Approach

By means of Comsol, the cavity part of the coupled system was modeled. Comsol solves a more general form of the wave equation

$$\frac{1}{\rho_0 c^2}\frac{\partial^2 p}{\partial t^2} - d_a \frac{\partial p}{\partial t} + \nabla \cdot \left(-\frac{1}{\rho_0}(\nabla p - q) \right) = Q \qquad (4)$$

where d_a is a damping coefficient, q is a monopole and Q a dipole source. The damping coefficient d_a was set to 10^{-5} ms/kg. The acoustic duct was discretized with 15 (Lagrange-quadratic) Finite Elements. Figure 3 shows 4 of the calculated modes and eigenfrequencies for the duct. They match the ones of the fluid from table 2 with a high accordance. The number of displayed modes can be chosen higher but doesn't facilitate the depiction.

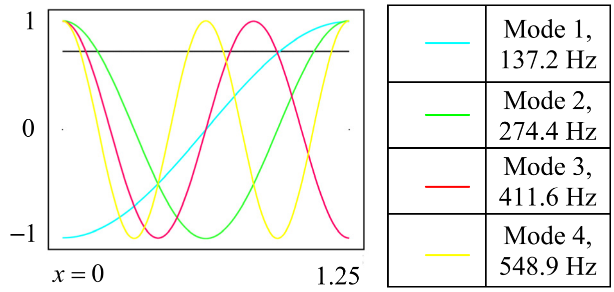

Figure 3: Numerical calculated modes and eigenfrequencies for the duct

The black line represents a "rigid body mode" of the fluid at 0 Hz. For the later simulations in Simulink some input and output variables were defined. Depending on the simulation model, a variable acceleration value at the left boundary and a displacement value in the middle of the duct enable Simulink to drive the FE-model. Variable pressure values at the same places let Simulink obtain information for the control systems. The FE-models with their specific variables are exported from Comsol as state space models. For the base model air damping was neglected and all of the modes were retained, but for the advanced Simulink models used for the active control methods the exported FE models contained damping and modal reduction.

Simulation of Coupled System

To compare the eigenfrequencies of the coupled system with the analytical results a base configuration was installed, containing only the simulink blocks shown in figure 4. Neither cavity nor the panel block included damping (D_S=0).

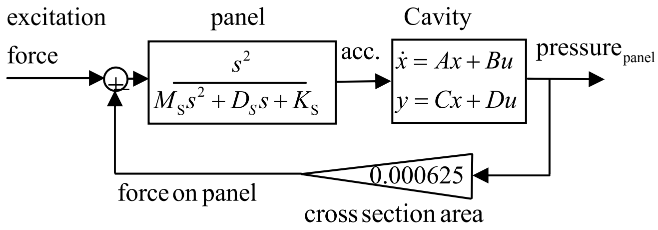

Figure 4: Simulink model for the base configuration

The panel is represented by a transfer function block. Its input is the sum of excitation force and the force due to the acoustic pressure. The output is an acceleration being the input to the block "cavity", which is the exported FE-model with the input variable acceleration and the output variable pressure at the left boundary. This pressure is multiplied with the value of the cross section area and fed back as a force to the summation point. The frequency response of the system is shown in figure 5.

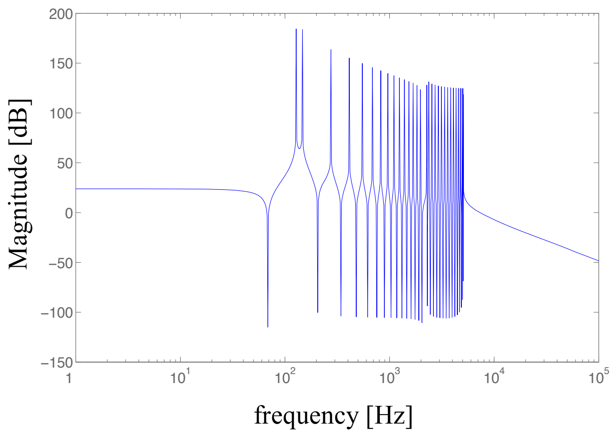

Figure 5: Frequency response of the undamped simulink model

With this cavity model the system has 32 eigenfrequencies, their first values are listed in table 3. They match the ones from table 2 perfectly.

frequencies [Hz]	
eigenfrequencies	**anti-resonances**
128.0	68.4
147.1	205.8
275.3	343.0
412.2	480.3
549.4	617.7
686.8	755.6
824.8	894.0
963.4	1032.9

Table 3: First eigenfrequencies and anti-resonances of the undamped simulink model

Comparing the numerical with the analytical solutions for the eigenfrequencies a good agreement can be found. The negligence of damping in the simulink model was necessary to compare the results with the analytical ones, which are also computed without regarding any damping. However this simplification is not valid because real systems always are provided with damping and the absence of damping leads to indifferent stability of the numerical system as you can see in the pole-zero-plot in figure 6:

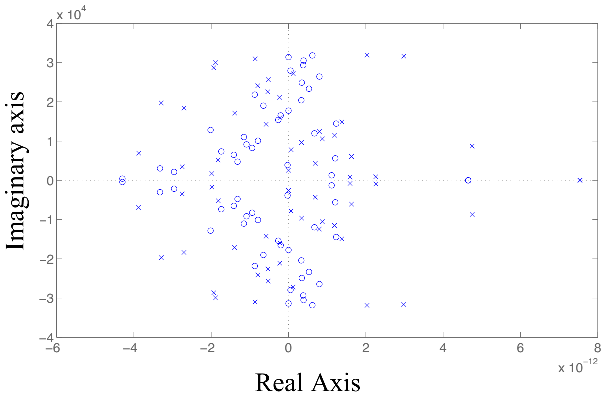

Figure 6: Pole-zero map of the undamped system

In order to retain a stable system an advanced model of the cavity was prepared with the damping coefficient d_a and exported to Simulink. The benefit of this method shows figure 7, in which the pole-zero-map of the system with damped cavity is displayed. All poles, except the ones for frequency 0 Hz, are located in the area's left side.

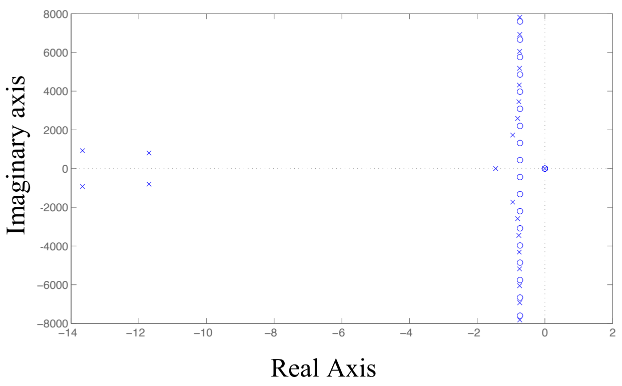

Figure 7: Pole-zero map of the system with damped cavity

Using the modal reduction option while exporting the FE-model to state space form the number of modes was reduced to 5 which seemed to be sufficient. In addition the state vector $\boldsymbol{x}$ with the modal coordinates (modal "displacements" and "velocities") was exported.

Analysis of Control Approach

For the representation of the ANC method in Simulink a Comsol model with additional input and output variables was prepared. This enables the state space block to receive a control displacement u_{ls} within the cavity and to emit a pressure value p at an arbitrary location. The principle sketch of the ANC system is shown in figure 8.

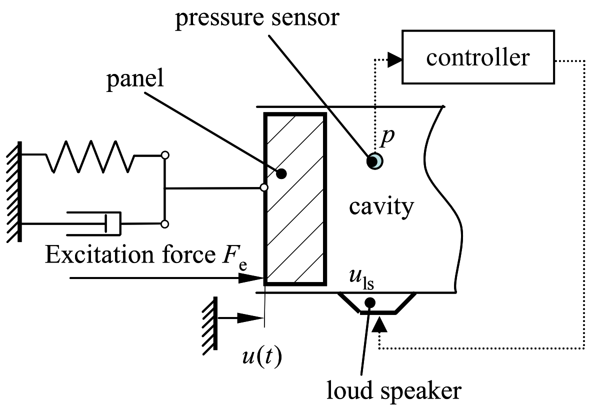

Figure 8: Principle sketch of the ANC system

The control-loop was designed as IMC (Internal Model Control) [2]. By means of the LTI-Viewer the transfer function between loud speaker and pressure sensor was obtained and transformed into a state space-object within the matlab workspace. While this function is the internal-model its negative inverse represents the IMC-law. This ANC Simulink model with control displacement and measured pressure at location x = 0.10m (fig. 2) is shown in figure 9.

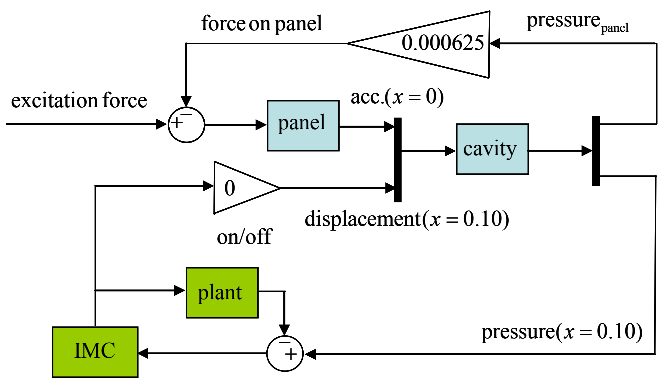

Figure 9: Simulink representation of the ANC system

For preparing the IMC- block it is necessary to calculate the inverse of the secondary path transfer function. To get a causal system two poles are added to the inverse representing a time delay of second order.

For the computation of the potential energy within the cavity the modal coordinates which are the state variables of the "cavity" state space model (fig. 9) are evaluated as shown in figure 10.

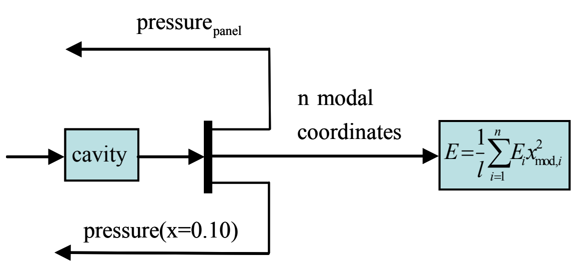

Figure 10: Extension of the simulink model to calculate the acoustic energy

For the energy calculation shown in the right block of figure 10 a normalized potential energy is defined

$$\left\| E_{pot} \right\| = \frac{E_{pot}}{E_{0,pot}} \quad (5)$$

where E_{pot} and $E_{0,pot}$ are the time-averaged potential energies [3]:

$$E_{pot} = \int_V \frac{|\mathrm{p(x)}|^2}{4\rho_0 c_0^2} dV, \quad E_{0,pot} = \int_V \frac{|\mathrm{p_0(x)}|^2}{4\rho_0 c_0^2} dV \quad (6)$$

depending on the actual pressure function p(x) and a reference pressure $p_0(x) = 1$ Pa. The normalized energy is then given by:

$$\left\| E_{pot} \right\| = \frac{1}{l} \int_l \mathrm{p(x)}^2 dx \quad (7)$$

Regarding that the pressure function p(x) can be seen as the sum of weighted modes:

$$\mathrm{p(x)} = \sum_{i=1}^{n} \Phi_i x_{mod,i} = \Phi^T x_{mod} \quad (8)$$

where $\Phi = [\Phi_1, \Phi_2, ..., \Phi_i]^T$ with the eigenvectors Φ_i and $x_{mod} = [x_{mod,1}, x_{mod,2}, ..., x_{mod,i}]^T$ with the modal pressure amplitudes $x_{mod,i}$. Then the normalized potential energy is:

$$\left\| E_{pot} \right\| = \frac{1}{l} \int_l \Phi^T x_{mod} \Phi^T x_{mod} dx = \frac{1}{l} \int_l x_{mod}^T \Phi \Phi^T x_{mod} dx \quad (9)$$

Assuming orthogonal eigenvectors Φ_i equation (9) can be transformed to

$$\left\| E_{pot} \right\| = \frac{1}{l} \int_l \sum_{i=1}^{n} \Phi_i^2 \, x_{mod,i}^2 dx = \sum_{i=1}^{n} \frac{1}{l} \int_l \Phi_i^2 dx \, x_{mod,i}^2 \quad (10)$$

and finally yields

$$E = \left\| E_{pot} \right\| = \sum_{i=1}^{n} E_i \cdot x_{mod,i}^2 \quad (11)$$

with the mode specific energy values E_i (“modal energies”) calculated by Comsol.

A plot of the normalized energy depending on the frequency for the controlled and uncontrolled model is shown in figure 11. In the frequency range of 100 to 1000 Hz there is an average attenuation of energy by ca. 15 dB. As expected the energy peaks of both curves are corresponding with the eigenfrequencies of the coupled system. For the controlled system there are additional peaks near the anti-resonances which means that a controlling attempt at such a frequency effects an increase of the energy.

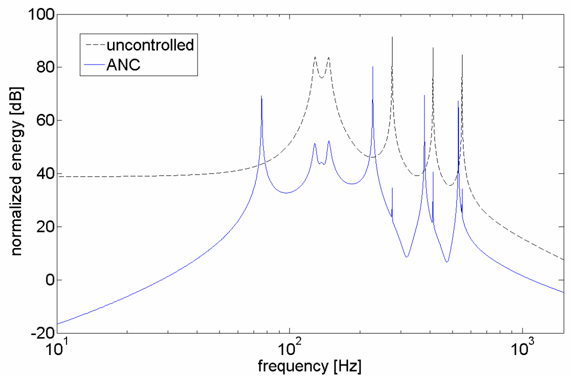

Figure 11: Normalized potential energy of the uncontrolled and the controlled system

Summary and outlook

This paper illustrates the concept and the preparing of a simulink model for an ANC system. After defining a simple base model with known analytical solutions a numerical base model is set up and evaluated. With the extension of this to a controlled system using an IMC-Controller a normalized energy calculation is made. Considering the peaks occurring at the calculated resonance frequencies and anti-resonances and the fact that the controlled system causes generally an attenuation of the energy figure 11 shows reasonable graphs.

In a future step additional passive and active methods will be investigated using the approach shown in this paper. Other effects like damping will be examined too. The theoretical work will be completed by experiments with a duct.

References

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