Robust Parameter Design of an Adaptive Multi-Channel Active Noise Controller

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Introduction

Noise induced by machines or propellers can lead to high tonal sound pressure levels inside vehicle or aircraft cabins. Due to increasing comfort requirements, noise protection is a mandatory component of each modern technical system. With advances in technology, this challenge can be solved in some cases by the Active Noise Control (ANC) method.

The present paper describes two new controller parameter design methods which weight the error microphones and loudspeakers of the active system individually. The first design method is based on the nominal physical system. The second design method also includes the uncertainty of the physical system.

Problem Description

As outlined in Figure 1, an ANC system is considered where the N_{L} loudspeakers and N_{E} error microphones are mounted at the boundaries of the enclosure. The controller of the active noise control system is designed to reduce the noise at the error microphones. However, the proper objective of the active noise control system should be to reduce the noise in the center of the enclosure where typically the head of a sitting or standing person is. This problem occurs frequently during the design of ANC systems inside vehicles or aircrafts. This problem is usually solved by optimization of the loudspeaker and error microphone positions in such a way that the noise is reduced at the error microphones and some monitor microphones inside a monitor volume at the same time. However, this solution requires the ability to adjust the transducer positions which is only possible in an early state of the design process of an ANC system. At the end of the design process only controller software solutions are practical.

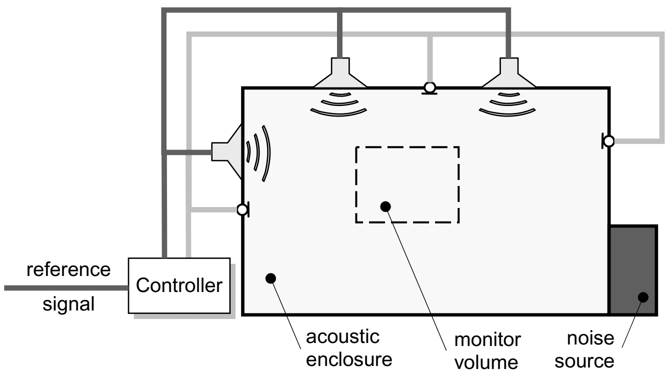

Figure 1: The acoustic enclosure of an aircraft or vehicle cabin with loudspeakers and error microphones mounted on the enclosure boundaries. The objective of the active noise control system is to reduce the noise inside the monitor volume.

In this paper a different solution is outlined to achieve the best noise reduction in the monitor volume. Here, controller weighting parameters of the loudspeaker and microphones are optimized.

If an active noise control system is designed for the industrial or consumer market, specified control performance and control reliability under the influence of uncertainties are important criteria. These requirements are above the requirements of a first laboratory demonstration. A control system is specified as a robust system when quantitative predictions can be made about tolerable model uncertainties.

In general, the controller design tasks are distinguished by the following points, listed in ascending order of the level of difficulty [1]:

- **Nominal Control Stability:** The controller is stable for a constant nominal plant model.
- **Nominal Control Quality:** The controller assures a defined control quality (e.g.: mean noise reduction in a certain volume of the cabin)
- **Robust Control Stability:** The controller is stable for all uncertain plant models which can alternate with a bounded norm around the nominal model.
- **Robust Control Quality:** The controller assures a defined control quality in the presence of uncertainties.

Therefore, the robust controller is able to deal with bounded uncertain plant models.

The paper is organized as follow: First, the control system is mathematically modeled. Then, the design method of the nominal controller and robust controller is described. Finally, the design method is applied to a semi-enclosed workstation; the experimental results will be discussed.

Model of the Control System

The adaptive noise control problem is assumed to be tonal with minor changes in time and will be discussed in the frequency domain at the steady state. Therefore, all variables are complex and frequency dependent. The explicit dependency on the frequency will be skipped in all equations for clarity.

In Figure 2 the block diagram of the control system is shown. The residual sound pressure at the error microphones $\mathbf{e}$ are the sum of the primary noise field $\mathbf{d}$ and the product of the transfer function matrix $\mathbf{G}$ with the loudspeaker excitation signal $\mathbf{u}$. Simultaneously, the loudspeaker excitation effects the sound pressure at the N_{M} monitor microphones via the transfer function matrix $\mathbf{G}_{\mathrm{M}}$. This leads to the following equations:

$$\mathbf{e} = \mathbf{G}\mathbf{u} + \mathbf{d} \quad \text{and} \quad \mathbf{e}_\mathrm{M} = \mathbf{G}_\mathrm{M}\mathbf{u} + \mathbf{d}_\mathrm{M} . \tag{1}$$

The primary noise $\mathbf{d}$ and $\mathbf{d}_\mathrm{M}$ are assumed to be excited by the same tonal noise source.

In the steady state, the actuation signal of the loudspeaker $\mathbf{u}_\mathrm{opt}$ is given by the product of $\mathbf{d}$ with the steady state controller matrix $\mathbf{C}$:

$$\mathbf{u}_\mathrm{opt} = \mathbf{C}\mathbf{d} . \tag{2}$$

For a steepest descent adaptive controller [2, 3], the steady state controller matrix $\mathbf{C}$ is given by the minimum of the controller cost function J_R :

$$J_\mathrm{R} = \mathbf{e}^\mathrm{H}\mathbf{Q}\mathbf{e} + \mathbf{u}^\mathrm{H}\mathbf{R}\mathbf{u} . \tag{3}$$

Differentiation of the controller cost function J_R according to $\mathbf{u}$ leads to the controller transfer function matrix $\mathbf{C}$

$$\mathbf{C} = -\left[\mathbf{G}^\mathrm{H}\mathbf{Q}\mathbf{G} + \mathbf{R}\right]^{-1} \mathbf{G}^\mathrm{H}\mathbf{Q} . \tag{4}$$

The weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ in equation (3) are diagonal matrices which can be used to weight the microphones and loudspeakers independently. These weightings have a similar mathematical effect as different loudspeaker and microphone positions and change the phase and amplitude of the loudspeaker actuation. Therefore, changing the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ also effects the residual noise at the monitor microphones.

Theses $N_\mathrm{E} + N_\mathrm{L}$ weighting parameters of $\mathbf{Q}$ and $\mathbf{R}$ have to be adjusted during the controller design process.

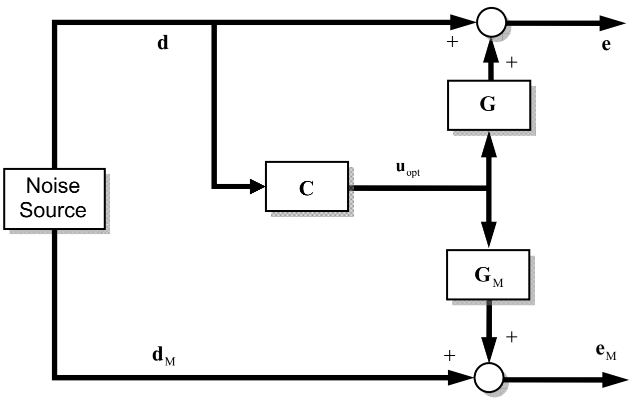

Figure 2: Block diagram of the control system.

Controller Design

The identification of the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ is in the focus of the controller design process. The nominal controller design is based on the nominal plant. The robust controller design includes in the design process additional uncertainties of the plant.

Nominal Controller Design

In the case of nominal controller design problem the transfer function matrices as well as the primary noise fields are assumed to be constant at their nominal values. Therefore, we can set

$$\mathbf{G}_\mathrm{M} = \mathbf{G}_\mathrm{M0},\ \mathbf{G} = \mathbf{G}_0,\ \mathbf{d} = \mathbf{d}_0,\ \mathbf{d}_\mathrm{M} = \mathbf{d}_\mathrm{M0} \text{ and } \mathbf{u}_\mathrm{opt} = \mathbf{u}_\mathrm{opt0} . \tag{5}$$

For this nominal plant, a nominal stability criterion and nominal performance criterion can be defined. The control stability of a steepest descent adaptive controller can be guarantied if the smallest eigenvalue λ_min of the matrix $\left[\mathbf{G}_0^\mathrm{H}\mathbf{Q}\mathbf{G}_0 + \mathbf{R}\right]$ is positive [3]. Hence, this fact will be used as the stability criterion.

The performance of the adaptive controller can be evaluated with the mean residual sound pressure level at the N_M monitor microphones. Hence, the residual mean squared error can be defined as the performance criterion which is to be minimized:

$$\text{minimize} \left\| \mathbf{e}_\mathrm{M0} \right\|_2^2 . \tag{6}$$

Moreover, the loudspeaker excitation should not be larger than the maximum allowed actuation amplitude $u_\mathrm{opt0}^\mathrm{max}$ which leads to another control quality criterion.

The optimal weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ are found when the controller stability criterion is satisfied and the best control performance is achieved. Using the equation (1), (2), (4), (5) and (6), the design problem can be written as an optimization problem:

$$\begin{aligned} &\underset{\mathbf{Q},\mathbf{R}}{\text{minimize}} && \left\| \mathbf{G}_\mathrm{M0} \cdot \mathbf{u}_\mathrm{opt0}\left(\mathbf{Q},\mathbf{R}\right) + \mathbf{d}_\mathrm{M0} \right\|_2^2 \\ &\text{subject to} && \mathbf{u}_\mathrm{opt0}\left(\mathbf{Q},\mathbf{R}\right) = -\left[\mathbf{G}_0^\mathrm{H}\mathbf{Q}\mathbf{G}_0 + \mathbf{R}\right]^{-1} \mathbf{G}_0^\mathrm{H}\mathbf{Q} \cdot \mathbf{d}_0 \\ & && \min\left\{\mathrm{eig}\left\{\mathbf{G}_0^\mathrm{H}\mathbf{Q}\mathbf{G}_0 + \mathbf{R}\right\}\right\} > 0 \\ & && \max_{i=1}^{N_\mathrm{L}} \left| u_{\mathrm{opt0}(i)} \right| < u_\mathrm{opt0}^\mathrm{max} . \end{aligned} \tag{7}$$

Here, $\mathbf{Q}$ and $\mathbf{R}$ are the optimization variables and the residual sound pressure in the monitor volume is the objective function. The stability and quality criterions are passed into the nonlinear constraints. This nonlinear constrained optimization problem can be solved using a genetic algorithm.

Robust Controller Design

The simplified mathematical models of the plant above are an abstraction of the dynamic behavior of real time-variant systems with weak nonlinearities. It is assumed that the real physical plant varies around the nominal plant. The implemented model in the controller equates the nominal model. Hence, according to (5), we set for the physical plant

$$\begin{aligned} \mathbf{G}_\mathrm{M} &= \mathbf{G}_\mathrm{M0} + \Delta\mathbf{G}_\mathrm{M} & \mathbf{d} &= \mathbf{d}_0 + \Delta\mathbf{d} \\ \mathbf{G} &= \mathbf{G}_0 + \Delta\mathbf{G} & \mathbf{d}_\mathrm{M} &= \mathbf{d}_\mathrm{M0} + \Delta\mathbf{d}_\mathrm{M} \\ \mathbf{u}_\mathrm{opt} &= \mathbf{u}_\mathrm{opt0} + \Delta\mathbf{u}_\mathrm{opt} \end{aligned} \tag{8}$$

where the variables labeled with Δ represents the additive modeling errors. Here, the modeling errors are arbitrarily and only known in their norm.

Similar to the optimization problem above, a robust optimization problem was defined which combines the robust control stability and robust control quality criterion. The solution of the optimization problem has to guarantee the stability and has to achieve the best performance for all norm-bounded uncertain plants. In other words, the objective function and the constraints should be evaluated for the worst-case stability and worst-case performance. The worst-case can be found by solving a maximization problem according to the variables $\Delta\mathbf{G}_{\mathrm{M}}$, $\Delta\mathbf{d}_{\mathrm{M}}$, $\Delta\mathbf{G}$ and $\Delta\mathbf{d}$. However, such a minimax optimization problem is not solvable in an efficient manner.

On the other hand, the worst-case can be estimated by algebraic formulas. For example, the worst-case loudspeaker actuation can be approximated by using the Sherman-Morrison-Woodbury formula [4] for an estimation of the variable $\Delta\mathbf{u}_{\mathrm{opt}}$

$$\Delta\mathbf{u}_{\mathrm{opt}}\left(\mathbf{Q},\mathbf{R}\right) \approx \mathbf{C}\Delta\mathbf{d} + \mathbf{C}\Delta\mathbf{G}\mathbf{u}_{\mathrm{opt0}}\left(\mathbf{Q},\mathbf{R}\right). \tag{9}$$

Moreover, using the triangle inequality and neglecting the quadratic terms leads to an estimation of the residual noise at the monitor microphones:

$$\max_{\Delta}\left\|\left(\mathbf{G}_{\mathrm{M0}}+\Delta\mathbf{G}_{\mathrm{M}}\right)\mathbf{u}_{\mathrm{opt}}\left(\mathbf{Q},\mathbf{R}\right)+\mathbf{d}_{\mathrm{M0}}+\Delta\mathbf{d}_{\mathrm{M}}\right\|_2^2$$
$$\le \left\{\begin{array}{l} \left\|\mathbf{G}_{\mathrm{M0}}\mathbf{u}_{\mathrm{opt}}+\mathbf{d}_{\mathrm{M0}}\right\|_2 + \ldots \\ \max\limits_{\Delta\mathbf{G}_{\mathrm{M}}}\left\|\Delta\mathbf{G}_{\mathrm{M}}\mathbf{u}_{\mathrm{opt}}\right\|_2 + \max\limits_{\Delta\mathbf{d}}\left\|\mathbf{G}_{\mathrm{M}}\mathbf{C}\Delta\mathbf{d}\right\|_2 + \ldots \\ \max\limits_{\Delta\mathbf{G}}\left\|\mathbf{G}_{\mathrm{M}}\mathbf{C}\Delta\mathbf{G}\mathbf{u}_{\mathrm{opt}}\right\|_2 + \max\limits_{\Delta\mathbf{d}_{\mathrm{M}}}\left\|\Delta\mathbf{d}_{\mathrm{M}}\right\|_2 \end{array}\right\}^2 . \tag{10}$$

Equation (10) can be easily evaluated if the norm of $\Delta\mathbf{G}_{\mathrm{M}}$, $\Delta\mathbf{d}_{\mathrm{M}}$, $\Delta\mathbf{G}$ and $\Delta\mathbf{d}$ are known.

The worst-case stability follows by the lower bound of the smallest eigenvalue of the matrix $\left[\mathbf{G}_0^{\mathrm{H}}\mathbf{Q}\left(\mathbf{G}_0+\Delta\mathbf{G}\right)+\mathbf{R}\right]$, see [3]. This eigenvalue can be estimated by

$$\hat{\lambda}_i\left(\mathbf{Q},\mathbf{R}\right) = \lambda_i\left(\mathbf{Q},\mathbf{R}\right) - \mathbf{x}_i^{\mathrm{H}}\mathbf{G}_0^{\mathrm{H}}\mathbf{Q}\Delta\mathbf{G}\mathbf{x}_i, \tag{11}$$

where λ_i are the eigenvalues and $\mathbf{x}_i$ are the eigenvectors of the matrix $\left[\mathbf{G}_0^{\mathrm{H}}\mathbf{Q}\mathbf{G}_0+\mathbf{R}\right]$.

According to the optimization problem (7), the robust optimization problem is based on the worst-case estimations:

$$\underset{\mathbf{Q},\mathbf{R}}{\text{minimize}} \quad \left\{\begin{array}{l} \left\|\mathbf{G}_{\mathrm{M0}}\mathbf{u}_{\mathrm{opt}}+\mathbf{d}_{\mathrm{M0}}\right\|_2 + \ldots \\ \max\limits_{\Delta\mathbf{G}_{\mathrm{M}}}\left\|\Delta\mathbf{G}_{\mathrm{M}}\mathbf{u}_{\mathrm{opt}}\right\|_2 + \max\limits_{\Delta\mathbf{d}}\left\|\mathbf{G}_{\mathrm{M}}\mathbf{C}\Delta\mathbf{d}\right\|_2 + \ldots \\ \max\limits_{\Delta\mathbf{G}}\left\|\mathbf{G}_{\mathrm{M}}\mathbf{C}\Delta\mathbf{G}\mathbf{u}_{\mathrm{opt}}\right\|_2 + \max\limits_{\Delta\mathbf{d}_{\mathrm{M}}}\left\|\Delta\mathbf{d}_{\mathrm{M}}\right\|_2 \end{array}\right\}^2$$
$$\text{subject to} \quad \mathbf{u}_{\mathrm{opt}} = -\left[\mathbf{G}_0^{\mathrm{H}}\mathbf{Q}\mathbf{G}_0+\mathbf{R}\right]^{-1}\mathbf{G}_0^{\mathrm{H}}\mathbf{Q}\mathbf{d}_0 + \Delta\mathbf{u}_{\mathrm{opt}}$$
$$\min_{i=1}^{N_{\mathrm{L}}}\hat{\lambda}_i > 0$$
$$\max_{i=1}^{N_{\mathrm{L}}}\left|u_{\mathrm{opt}(i)}\right| < u_{\mathrm{opt0}}^{\max}. \tag{12}$$

The optimization variables are still the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$. However, the optimization is now performed for the worst-case approximation. In addition to the nominal plant, the norm of perturbations of $\Delta\mathbf{G}_{\mathrm{M}}$, $\Delta\mathbf{d}_{\mathrm{M}}$, $\Delta\mathbf{G}$ and $\Delta\mathbf{d}$ are necessary.

Application Example

As application example, a workstation is used to demonstrate the efficiency of the new design methods. The workstation is semi-enclosed with one open boundary surface to the rest of the laboratory which is assumed to be the primary noise source. Two load cases were considered where the primary noise field was comprised of a fundamental frequency (BPF) and two higher harmonics. The fundamental frequencies are 92 Hz for load case I and 97 Hz for the load case II.

Figure 3: Workstation with monitor microphones around an artificial head and a diffracting object.

Experimental Setup

For the experiments, a wooden mock-up of the working station was equipped with an experimental ANC system consisting of eight loudspeakers and sixteen microphones. All loudspeakers (maximum allowed excitation amplitude $u_{\mathrm{opt0}}^{\max} = 2\mathrm{V}$) and microphones are installed at the exterior wall and the ceiling. Four public address loudspeakers, distributed 6 m away of the mock-up were used to excite the tonal primary noise field. An implementation of the frequency domain active noise control algorithm which can handle the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ was realized on a rapid prototyping board (dSPACE 1103), see [2]. Moreover, 12 monitor microphones were distributed around an artificial head with two ear microphones.

The transfer functions and the primary noise fields were measured for the nominal condition. To simulate the perturbation caused by different persons and objects inside the cabin, the transfer functions and primary noise fields were perturbed with three different sound hard diffracting objects suspended on the ceiling of the workstation, see Figure 3. The measured median norm of the uncertainty of the primary noise field and the transfer function is shown in Table 1 for both load cases.

	load case I	**load case II**
$\left\|\Delta\mathbf{G}\right\|_{\mathrm{F}} / \left\|\mathbf{G}_0\right\|_{\mathrm{F}} \cdot 100\%$	3.6 / 12.6 / 13.9	3.2 / 11.7 / 12.2
$\left\|\Delta\mathbf{d}\right\|_2 / \left\|\mathbf{d}_0\right\|_2 \cdot 100\%$	4.7 / 10.5 / 14.6	3.7 / 8.6 / 8.8
$\left\|\Delta\mathbf{G}_{\mathrm{M}}\right\|_{\mathrm{F}} / \left\|\mathbf{G}_{\mathrm{M0}}\right\|_{\mathrm{F}} \cdot 100\%$	3.4 / 4.9 / 12.1	3.4 / 5.9 / 8.6
$\left\|\Delta\mathbf{d}_{\mathrm{M}}\right\|_2 / \left\|\mathbf{d}_{\mathrm{M0}}\right\|_2 \cdot 100\%$	4.8 / 6.8 / 10.7	5.0 / 13.1 / 9.3

Table 1: Uncertainty norm of the primary noise field and the transfer functions for the three frequencies of each load case.

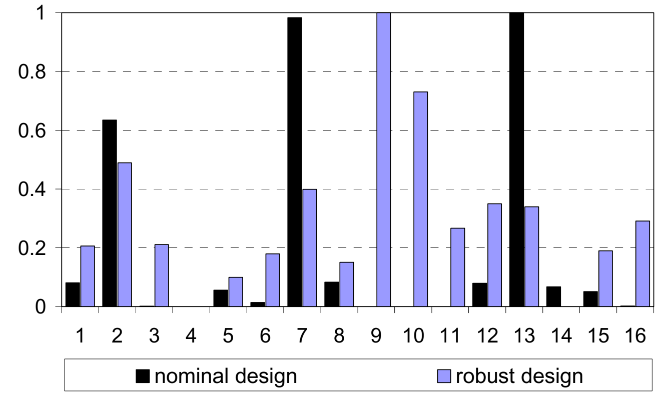

Figure 4: Diagonal elements of the weighting matrix **Q** of the fundamental frequency at load case I.

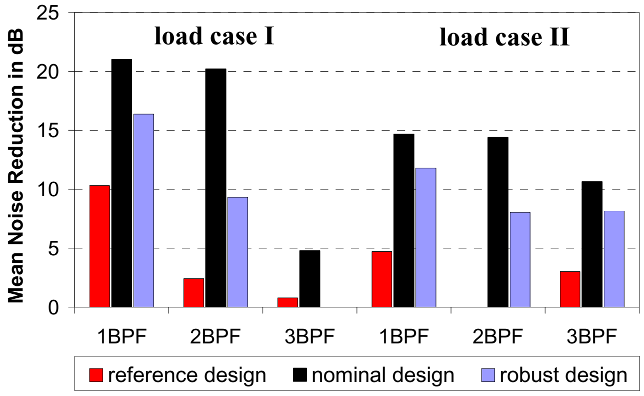

Figure 5: Reduction of the mean sound pressure level in the monitor volume.

Optimization Results

The controller weighting matrices **Q** and **R** were optimized according to (7) and (12) using Matlab® with the *Genetic Algorithm and Direct Search Toolbox*. The nonlinear constraints were included in the nonlinear cost function with the barrier function method.

As shown in Figure 4, the weighting matrix **Q** of the fundamental frequency (load case I) was chosen as an illustrative example for the optimized weighting matrices **Q** and **R** . The nominal design method results in a controller which only uses 10 of the 16 microphones. On the other hand, the robust design method leads to a controller which uses 14 of 16 microphones. Microphone 4 was excluded by both design methods. In comparison to an optimal transducer placement method, the loudspeakers and microphones are weighted differently. The differences of the robust controller weights are smaller than the controller weights of the nominal design. Similar results could be observed at other frequencies.

In general, the robust design includes more microphones in the controller cost function than the nominal design. This general observation also applies for the weighting matrix **R** where the loudspeaker excitation was more uniformly in the robust design method.

Experimental Active Noise Control Results

A reference design method was chosen to compare the control results of the two new design methods. For the reference method, the weighting matrices were set to $\mathbf{Q} = (1-\alpha)\mathbf{E}$ and $\mathbf{R} = \alpha\mathbf{E}$ where $\mathbf{E}$ is the identity matrix. Hence, all microphones and loudspeakers are equally weighted. The parameter α was chosen according to the maximum noise reduction in the monitor volume.

The experiment was performed for each frequency individually. The primary noise field was set to at least 75dB(A) at the ear microphones of the artificial head. The control profit was evaluated as the difference of the mean sound pressure level in the entire monitor volume:

$$\Delta L = 10\log_{10}\frac{\mathbf{d}_{\mathrm{M}}^{\mathrm{H}}\mathbf{d}_{\mathrm{M}}}{\mathbf{e}_{\mathrm{M}}^{\mathrm{H}}\mathbf{e}_{\mathrm{M}}} \quad (13)$$

The results are shown in Figure 5. In comparison to the reference method, the new methods achieved a significantly better noise reduction. More than 20 dB noise reduction could be achieved at the lowest frequencies. At 3BPF (load case I), no noise reduction with the robust design could be achieved. At all other frequencies, perceptible noise reduction could be measured. The weighting of the nominal design method achieved a better noise reduction than the robust design method. However, with uncertainties, the robust weighting will achieve a more reliable noise reduction.

Summary

An acoustic interior noise problem was considered in which the microphones of the active noise control system could only be installed at boundaries of the enclosure. For such design problems, two new design methods of the optimal parameterization of the controller weighting matrices **Q** and **R** were presented. The first design method takes only the nominal case into account. The second design method also considers the uncertainty of the primary noise field and the transfer functions in the design process. First experimental results have shown the capabilities of these new methods in comparison to the reference approach.

In future publications, the higher robustness of the robust weighting approach will be shown. Moreover, a non-diagonal structure of the weighting matrix **Q** and **R** will be analyzed.

References

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